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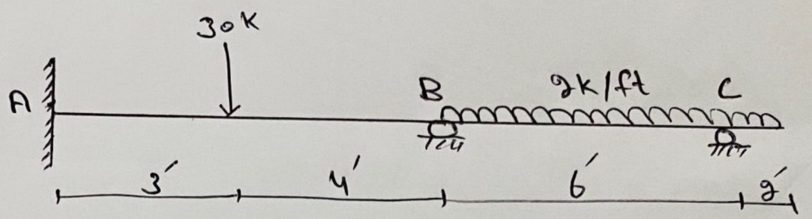
Semester : 10^{th}

Subject: Structure Analysis II

Submitted to: Engr. Adeed Khan

Department: Civil Engineering.

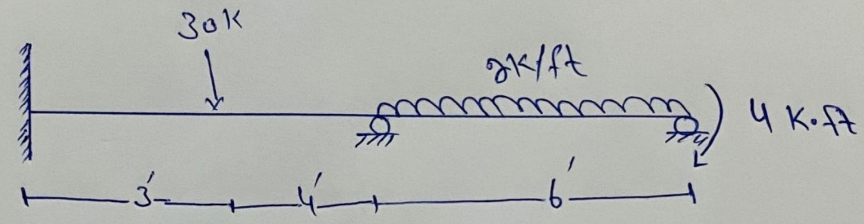
Q1:- Analyze the beam by Stifness methode. Assume E is constant.



Solution: Step 1 Determing Kinematic Indeterminancy

$K.I = 5^\circ$

So we have reduce the extended portion.

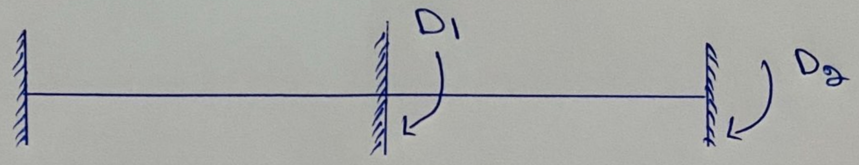


$$\Rightarrow \frac{2(2)}{2} = 2 K.ft.$$

Now:-

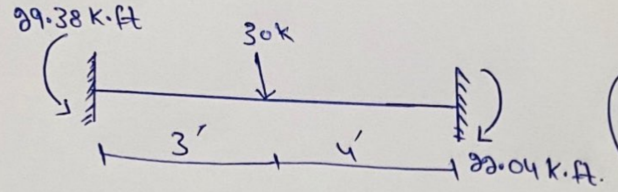
$K.I = 2^\circ$

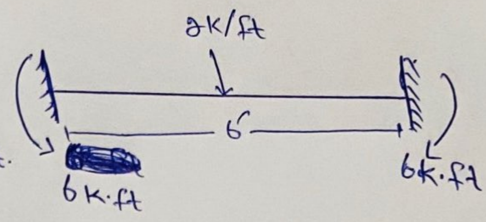
Step 2:- Determine unknown Joint displacement



$$\begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \begin{bmatrix} ? \\ ? \end{bmatrix} \qquad \begin{bmatrix} AD_1 \\ AD_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 4 \end{bmatrix}$$

Step 3: Compute [ADL] matrix





=> For Pointed load (not at mid)

-> for left end

$$\frac{Pab^2}{L^2} = \frac{(30)(3)(4)^2}{(7)^2} = 29.38 \text{ k.ft}$$

-> For Right end:

$$\frac{Pa^2b}{L^2} = \frac{(30)(3)^2(4)}{(7)^2} = 22.04 \text{ k.ft}$$

=> For UDL

$$\frac{wL^2}{12} \rightarrow \frac{(2)(6)^2}{12} \Rightarrow 6 \text{ k.ft.}$$

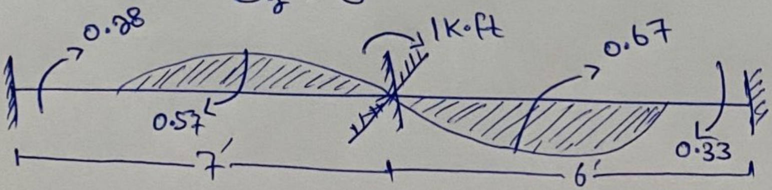
$ADL_1 = +22.04 - 6 = 16.04$ k.ft

$ADL_2 = 6$ k.ft

Step 4 Compute [S] matrix

$$S = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}$$

a) $D_1 = K, \quad D_2 = 0$



$$\frac{4EI}{7} = 0.57 \qquad \frac{2EI}{6} = 0.33$$

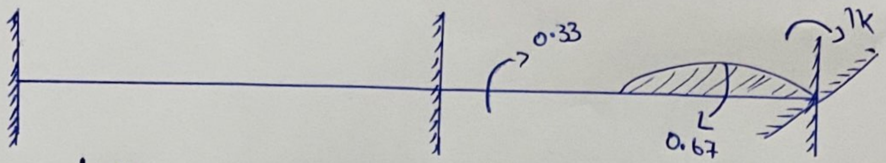
$$\frac{4EI}{6} = 0.67 \qquad \frac{2EI}{7} = 0.28$$

$S_{11} = 0.57 + 0.67$

$= 1.24\ EA$

$S_{21} = 0.33\ EA$

b) $D_1 = 0, \quad D_2 = 1k$



$\frac{4EI}{6} = 0.67$

$\frac{2EI}{6} = 0.33$

$S_{12} = 0.33$

$S_{22} = 0.67$

$$S = \begin{bmatrix} 1.24 & 0.33 \\ 0.33 & 0.67 \end{bmatrix}$$

Step 5 Compute $[D]$ matrix

$$\begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}^{-1} \times \begin{bmatrix} AD_1 \\ AD_2 \end{bmatrix} - \begin{bmatrix} ADL_1 \\ ADL_2 \end{bmatrix}$$

$$\Rightarrow \frac{1}{\begin{bmatrix} 1.24 & 0.33 \\ 0.33 & 0.67 \end{bmatrix}} \times \text{Adj}A \times \begin{bmatrix} AD_1 \\ AD_2 \end{bmatrix} - \begin{bmatrix} ADL_1 \\ ADL_2 \end{bmatrix}$$

$|S| = (1.24 \times 0.67) - (0.33 \times 0.33)$

$= 0.8308 - 0.1089$

$|S| = 0.7219$

$$\text{Adj } A = \begin{bmatrix} 0.67 & -0.33 \\ -0.33 & 1.24 \end{bmatrix}$$

Now:

$$\begin{bmatrix} AD_1 & - & ADL_1 \\ AD_2 & & ADL_2 \end{bmatrix} = \begin{bmatrix} 0 & - & 16.04 \\ 4 & - & 6 \end{bmatrix} = \begin{bmatrix} -16.04 \\ -2 \end{bmatrix} E$$

$$\begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \frac{\begin{bmatrix} 0.67 & -0.33 \\ -0.33 & 1.24 \end{bmatrix}}{0.7219} \times \begin{bmatrix} -16.04 \\ -2 \end{bmatrix}$$

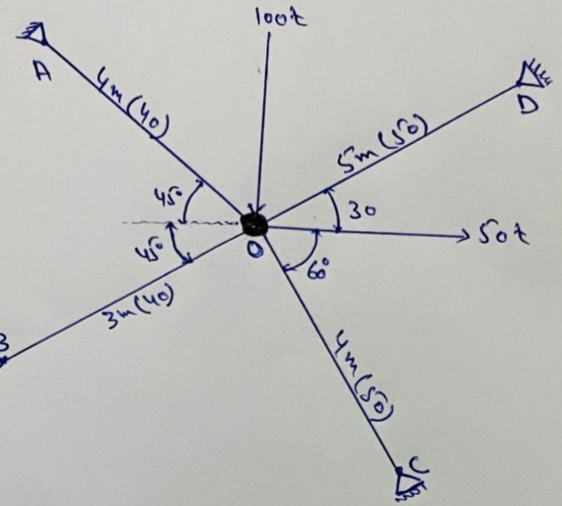
$$\begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \frac{\begin{bmatrix} -10.7468 & 0.66 \\ 5.2932 & -2.48 \end{bmatrix}}{0.7219}$$

$$\begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \frac{\begin{bmatrix} -10.0868 \\ 2.8132 \end{bmatrix}}{0.7219}$$

$$\begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \begin{bmatrix} \frac{-10.0868}{0.7219} \\ \frac{2.8132}{0.7219} \end{bmatrix}$$

$$\begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \begin{bmatrix} 13.9725 \\ 3.8969 \end{bmatrix}$$

Q 2:-



E = 2000 t/cm²

Solution:

For A:

$$\sin 45° = \frac{P}{H} = \frac{P}{4}$$

$\rightarrow P = 2.828\ m$

$$\cos 45° = \frac{b}{H} = \frac{P}{4}$$

$\rightarrow b = 2.828\ m$

For B:

$$\sin 45° = \frac{P}{H} = \frac{P}{3}$$

$\rightarrow P = 2.12\ m$

$$\cos 45 = \frac{b}{H} = \frac{b}{3}$$

$\rightarrow b = 2.12\ m$

For C:

$$\sin 60 = \frac{P}{H} = \frac{P}{4}$$

$$(\sin 60)(4) = P$$

$\rightarrow P = 3.46$

$$\cos 60 = \frac{b}{H} = \frac{b}{4} \Rightarrow \cos 60 \times 4 = b, \Rightarrow b = 2$$

For D

$$\sin 30 = \frac{P}{5}$$

$$P = 2.5m$$

$$\cos 30 = \frac{b}{3}$$

$$b = 4.33m.$$

Now,

$$EA(a) = 2000 \times 40 = 80{,}000t$$

$$EA(b) = 2000 \times 40 = 80{,}000t$$

$$EA(c) = 2000 \times 50 = 100{,}000t$$

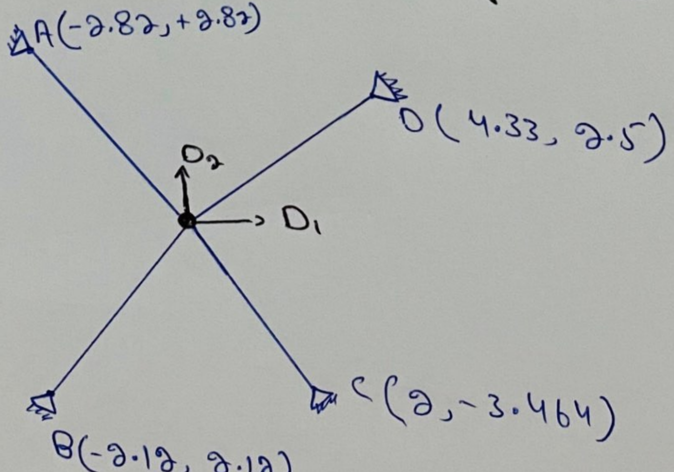
$$EA(D) = 2000 \times 50 = 100{,}000t$$

Step 1: K.I

$$K.I = 2j - r$$

$$= 2(5) - 8 = 2^\circ$$

Step 2: Select unknown Joint displacement.



$$\begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \begin{bmatrix} ? \\ ? \end{bmatrix}, \quad \begin{bmatrix} AD_1 \\ AD_2 \end{bmatrix} = \begin{bmatrix} 50 \\ -100 \end{bmatrix}$$

Step 3:-

$[AMD]_{4\times 2}$ $[S]_{2\times 2}$

i) $D_1 = 1k$, $D_2 = 0$

$$AMD = \frac{EA}{L^2}(xk - xj)$$

$$AMD_{11} = \frac{80,000}{(400)^2} \times (0 + 282) = 141$$

$$AMD_{21} = \frac{80,000}{(300)^2} \times (0 + 212) = 188.44$$

$$AMD_{31} = \frac{100,000}{(500)^2} \times (0 - 433) = -173.2$$

$$AMD_{41} = \frac{100,000}{(400)^2} \times (0 - 200) = -125$$

Now

$$S_{11} = \sum_{i=1}^{m} \frac{EA}{L^3}(xk - xj)^2$$

$$= \frac{80,000}{(400)^3} \times (282)^2 + \frac{80,000}{(300)^3} \times (212)^2 + \frac{100,000}{(500)^3} \times (-433)^2 + \frac{100,000}{(400)^3} \times (-200)^2$$

$$S_{11} = 99.405 + 133.107 + 149.991 + 62.5$$

$$\boxed{S_{11} = 445.062}$$

$$S_{12} = S_{21} = \sum_{i=1}^{m} \frac{EA}{L^3} \times (xk - xj)(yk - yj)$$

$$= \frac{80,000}{(400)^3} \times (282)(-282) + \frac{80,000}{(300)^3} \times (212)(212) + \frac{100,000}{(500)^3} \times ((-433)(0-250) + \frac{100,000}{(400)^3} \times (-200)(0+346)$$

$$\boxed{S_{12} = S_{21} = 12.237}$$

ii) $D_1 = 0$, $D_1 = 1k$

$$AMD = \frac{EA}{L^2}(y_k - y_j)$$

$$AMD_{12} = \frac{80{,}000}{(400)^2}(-282) = -141$$

$$AMD_{22} = \frac{80{,}000}{(300)^2}(212) = 188.44$$

$$AMD_{32} = \frac{100{,}000}{(500)^2}(-250) = -100$$

$$AMD_{42} = \frac{100{,}000}{(400)^2}(346) = 216.25$$

Now,

$$S_{22} = \sum_{i=1}^{m} \frac{EA}{L^3}(y_k - y_j)^2$$

$$= \frac{80{,}000}{(400)^3}(-282)^2 + \frac{80{,}000}{(300)^3}(212)^2 + \frac{100{,}000}{(500)^3}(-250)^2 +$$

$$\frac{100{,}000}{(400)^3}(346)^2$$

$$\boxed{S_{22} = 469.628}$$

Step 4 :-

$$[D] = [S]^{-1} \times [AD]$$

$$\begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \begin{bmatrix} 445.003 & 12.237 \\ 12.237 & 469.628 \end{bmatrix}^{-1} \times \begin{bmatrix} 50 \\ -100 \end{bmatrix}$$

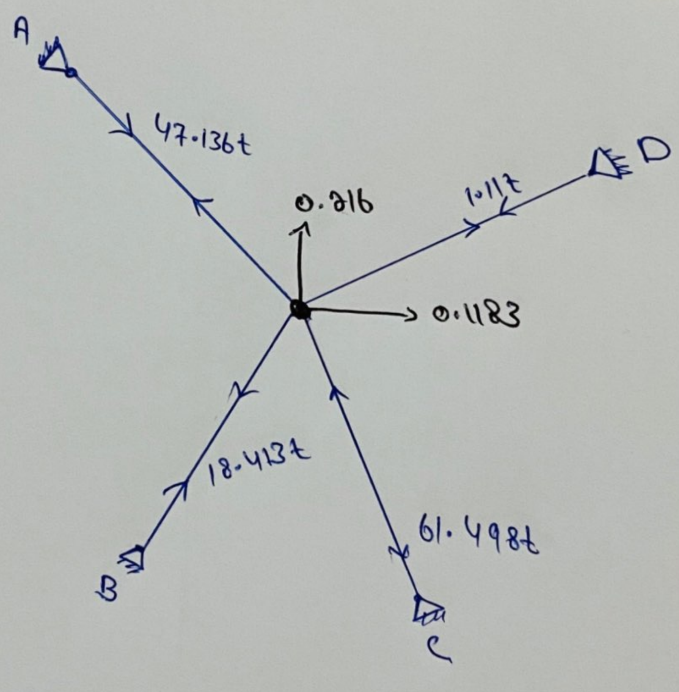
$$\begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \begin{bmatrix} 0.1183 \\ -0.216 \end{bmatrix}$$

Step 5 :- $[AM]$

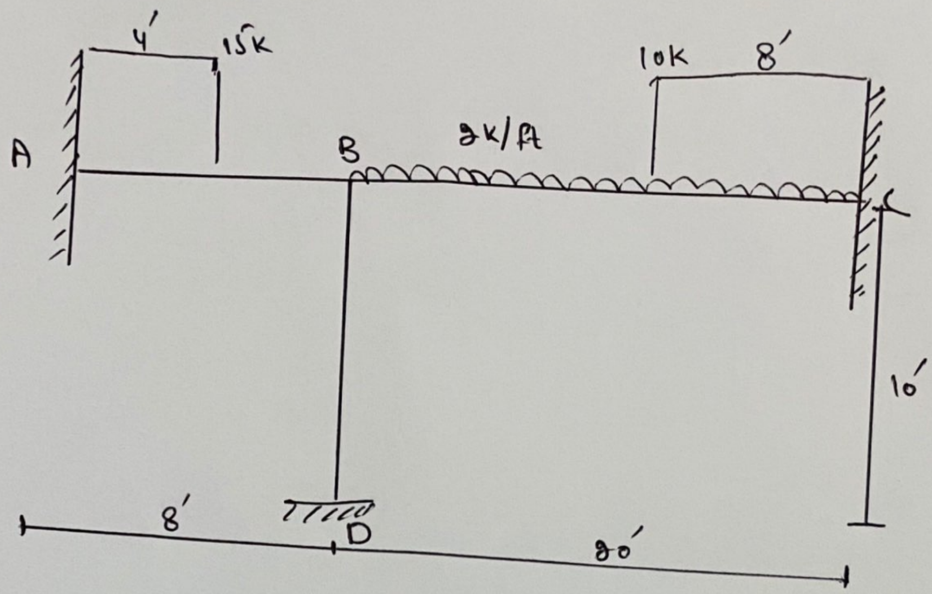
$$\begin{bmatrix} AM_1 \\ AM_2 \\ AM_3 \\ AM_4 \end{bmatrix} = \begin{bmatrix} 141 & -141 \\ 188.44 & 188.44 \\ -173.2 & -100 \\ -125 & 216.25 \end{bmatrix} \times \begin{bmatrix} 0.1183 \\ -0.216 \end{bmatrix}$$

$$= \begin{bmatrix} 141 \times 0.1183 + (-141) \times (-0.216) \\ 188.44 \times 0.1183 + 188.44 \times (0.216) \\ -173.2 \times 0.1183 + (-100) \times (-0.216) \\ 125 \times 0.1183 + 216.25 \times (0.216) \end{bmatrix}$$

$$\begin{bmatrix} AM_1 \\ AM_2 \\ AM_3 \\ AM_4 \end{bmatrix} = \begin{bmatrix} 16.68 & +30.46 \\ 22.29 & -40.70 \\ -20.49 & +21.6 \\ 14.79 & -46.7 \end{bmatrix} = \begin{bmatrix} 47.136t \\ -18.413t \\ 1.11t \\ 61.498t \end{bmatrix}$$



Q3.

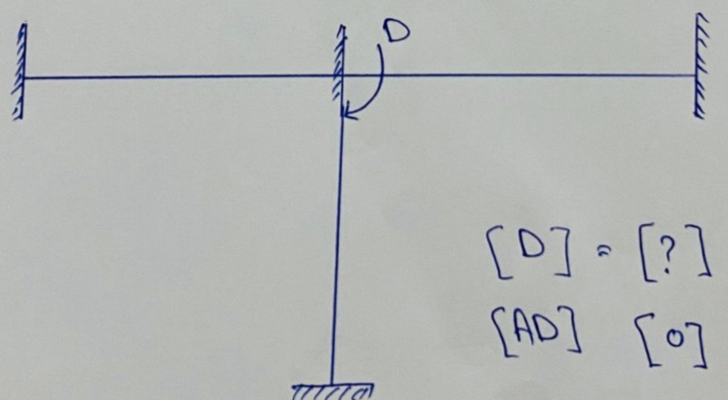


Solution:-

Step 1:- Determine Kinematic Indeterminancy

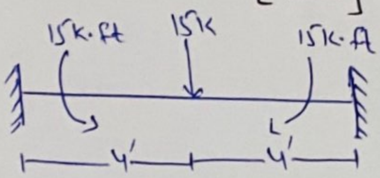
K.I = 1°

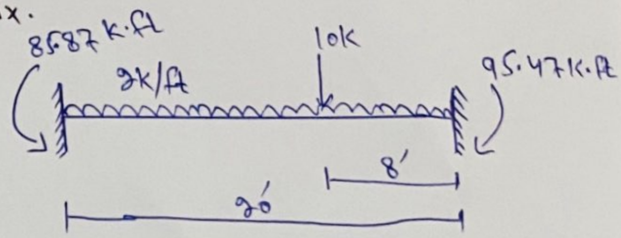
Step 2:- Determine unknown Joint Displacement.

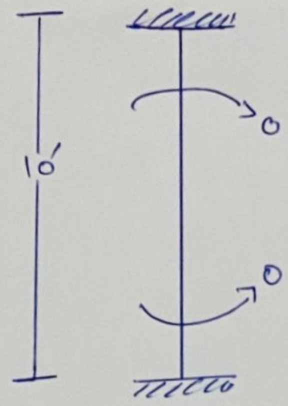


$$[D] = [?]$$
$$[AD] \quad [0]$$

Step 3:- Compute [ADL] Matrix.







⇒ Point load at centre

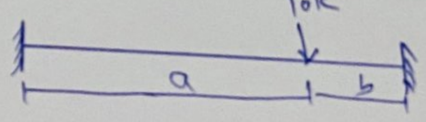
$$\frac{PL}{8} \Rightarrow \frac{(15)(8)}{8} = 15 \text{ kip.ft.}$$

⇒ Uniformly distributed load

$$\frac{wL^2}{12} \Rightarrow \frac{(2)(20)^2}{12} = 66.67 \text{ k.ft}$$

⇒ Point load (Not at mid)

Suppose:-



→ For left end

$$\frac{Pab^2}{L^2} \Rightarrow \frac{(10)(12)(8)^2}{(20)^2} = 19.2 \text{ k.ft}$$

⇒ For Right end

$$\frac{Pa^2b}{L^2} \Rightarrow \frac{(10)(12)^2(8)}{(20)^2} = 28.8 \text{ k.ft.}$$

So Total moment at left end:

$$19.2 + 66.67 \Rightarrow 85.87 \text{ k.ft.}$$

Similarly at Right end:

$$28.8 + 66.67 \Rightarrow 95.47 \text{ k.ft.}$$

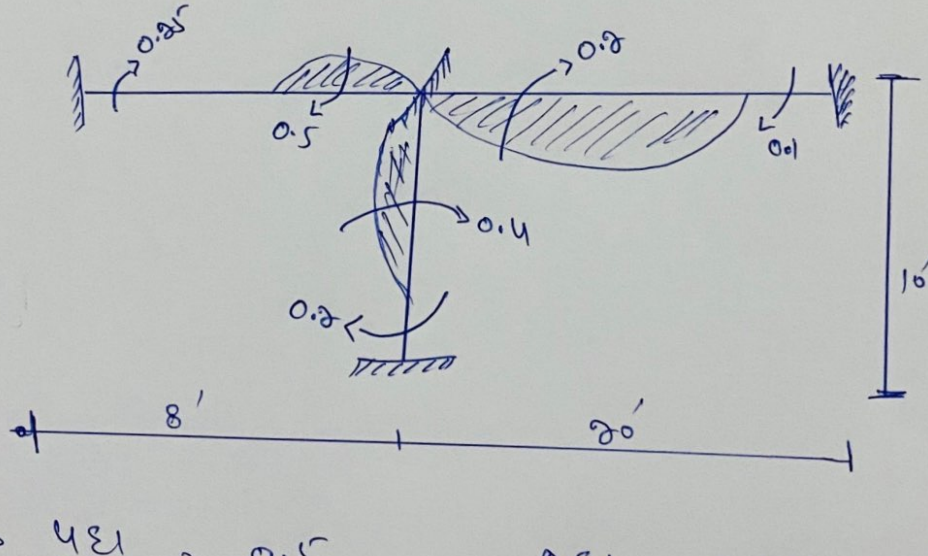
So $[ADL] = -85.87 + 15 = -70.87$ k.ft

Step 4: Determine $[S]$ Matrix

$$[S] = [S_{11}]$$

Now:

$D = 1K$



$\Rightarrow \frac{4EI}{8} = 0.5$ $\qquad \frac{2EI}{8} = 0.25$

$\Rightarrow \frac{4EI}{20} = 0.2$ $\qquad \frac{2EI}{20} = 0.1$

$\Rightarrow \frac{4EI}{10} = 0.4$ $\qquad \frac{2EI}{10} = 0.2$

$[S] = (0.5 + 0.4 + 0.2)\, EI$

$= 1.1\ EI$

$[S] = 1.1\ EI$

Step 5: Compute $[D]$ Matrix

$[D] = [S]^{-1} \times [AD] - [ADL]$

$[D] = \frac{1}{1.1} \times [0] - [-70.87]$

$= \frac{70.87}{1.1}$

$[D] = [64.42]\ ^{1}/_{EI}$

The End.