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SECTION : A

MODULE : 8TH SEMESTER

(Q1)

Sol: $f(t) = 1 + t \quad -\pi \le t \le \pi$

here we use the formula

$$\Rightarrow f(x) = a_0 + \sum_{n=1}^{\infty} a_n \cos t + \sum_{n=1}^{\infty} b_n \sin t, \quad \text{eq(1)}$$

$$\Rightarrow a_x = \frac{1}{2\pi} \int_{-\pi}^{\pi} f(t)\, dt$$

$$\Rightarrow a_0 = \frac{1}{2\pi} \int_{-\pi}^{\pi} (1+t)\, dt$$

$$\Rightarrow a_0 = \frac{1}{2\pi} \left| t + \frac{t^2}{2} \right|_{-\pi}^{\pi}$$

$$\Rightarrow a_0 = \frac{1}{2\pi} \left(\pi - (-\pi) + \frac{\pi^2}{2} - \left(-\frac{\pi^2}{2}\right) \right)$$

$$\Rightarrow a_0 = \frac{1}{2\pi} \left(2\pi + \frac{2\pi^2}{2} \right)$$

$$\Rightarrow \boxed{a_0 = \frac{1}{2\pi}\left(2\pi + \pi^2\right)}$$

$$\Rightarrow a_n = \frac{1}{\pi}\int_{-\pi}^{\pi}(1+t)\cdot(\cos nt)\,dt$$

$$\Rightarrow a_n = \frac{1}{\pi}\left[(1+t)\frac{\sin nt}{n}\right]_{-\pi}^{\pi} - \int \frac{\sin nt\, d(1+t)}{n\,dt}$$

$$\Rightarrow a_n = \frac{1}{\pi}\left[(1+t)\frac{\sin nt}{n}\right]_{-\pi}^{\pi} - \int \frac{\sin nt}{n}\frac{d}{dt}(1+t)$$

$$\Rightarrow a_n = \frac{1}{\pi}\left[(1+t)\frac{\sin nt}{n} - \frac{\cos nt}{n^2}\right]_{-\pi}^{\pi}$$

$$\Rightarrow a_n = \frac{-1}{n^2\pi}\left(\cos n\pi - \cos n(-\pi)\right)$$

$$\Rightarrow a_n = \frac{-1}{n^2\pi}\left(-1-(-1)\right)$$

$$\Rightarrow \boxed{a_n = 0}$$

(Q2)

Sol : $A = \begin{vmatrix} 1 & 0 & -1 \\ 3 & 1 & 4 \\ 0 & 2 & 2 \end{vmatrix}$

Eigen values = ?

$\Rightarrow$ Step = 1 we have

$\Rightarrow (A - \lambda I) X = 0$ A = Square even Matrix

Step = 2

Characteristic Equation given

$|A - \lambda I| = 0$

$\Rightarrow \begin{vmatrix} 1 & 0 & -1 \\ 3 & 1 & 4 \\ 0 & 2 & 2 \end{vmatrix} - \lambda = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$

$$\Rightarrow \begin{vmatrix} 1-\lambda & 0 & -1 \\ 3 & 1-\lambda & 4 \\ 0 & 2 & 2-\lambda \end{vmatrix} = 0$$

Step # 3

$\Rightarrow \lambda^3 - |\text{Sum of diagonal}|\lambda^2 + |\text{Sum of diagonal minors}|\lambda - |A| = 0 \quad (B)$

$\Rightarrow$ Sum of diagonal Element $= 1 + 1 + 2 = 4$

$\Rightarrow$ Sum of diagonal minors $= \begin{vmatrix} 1 & 4 \\ 2 & 2 \end{vmatrix} + \begin{vmatrix} 1 & -1 \\ 0 & 2 \end{vmatrix} + \begin{vmatrix} 1 & 0 \\ 3 & 1 \end{vmatrix}$

$\Rightarrow (-6) + (2) + (1)$

$\Rightarrow -6 + 2 + 1 = -3$

By putting values in eq (B)

$\Rightarrow \lambda^3 - 4\lambda^2 - 3\lambda - |A| = 0 \quad \text{---} \quad (C)$

$$|A| = \begin{vmatrix} 1 & 0 & -1 \\ 3 & 1 & 4 \\ 0 & 2 & 2 \end{vmatrix} = 1 \begin{vmatrix} 1 & 4 \\ 2 & 2 \end{vmatrix} - 0 \begin{vmatrix} 3 & 4 \\ 0 & 2 \end{vmatrix} + 1 \begin{vmatrix} 3 & 1 \\ 0 & 2 \end{vmatrix}$$

$= 1(2-8) - 0 + 1(6-0)$

$= -6 + 6$

$= 0$ by putting values in (c)

$\lambda^3 = 4\lambda^2 - 3\lambda = 0$

$\lambda^3 - 4\lambda^2 - 3\lambda = 0$

$\lambda^2 = 4\lambda - 3 = 0$

Using Quadratic formula

$$\lambda = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$a = 1$

$b = 4$

$c = -3$

$$\Rightarrow \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(-3)}}{2a}$$

$$\Rightarrow \frac{4 \pm \sqrt{16 + 12}}{2} = \frac{4 \pm \sqrt{28}}{2}$$

Q # 3

$$5x + 4z + 2m = 3$$
$$x - y + 2z + m = 1$$
$$4x + y + 2z = 1$$
$$x + y + z + m = 0$$

Solution :- System of equation can be writing in Matrix form

$$\left|\begin{array}{cccc|c} 5 & 0 & 4 & 2 & 3 \\ 1 & -1 & 2 & 1 & 1 \\ 4 & +1 & 2 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 \end{array}\right| \quad R_4 \; R_2$$

$$\left|\begin{array}{cccc|c} 5 & 0 & 4 & 2 & 3 \\ 1 & -1 & 2 & 1 & 1 \\ 4 & 1 & 2 & 0 & 1 \\ 0 & 2 & -1 & 0 & -1 \end{array}\right|$$

$$\left|\begin{array}{cccc|c} 5 & 0 & 4 & 2 & 3 \\ 1 & -1 & 2 & 1 & 1 \\ 0 & -1 & +6/5 & +4/5 & 3/5 \\ & & & & -1 \end{array}\right| \quad -1/5 \times R_2$$

$$\left|\begin{array}{cccc|c} 1 & 0 & 4/5 & 2/5 & 3/5 \\ 0 & -5 & 6 & 1 & 2 \\ 0 & 0 & 1 & 1 & -4/21 \\ 0 & 0 & 0 & 1 & 1/3 \end{array}\right| \quad (2x - 5$$

$$\left|\begin{array}{cccc|c} 1 & 0 & 4/5 & 2/5 & 3/5 \\ 0 & 1 & 6 & 1 & 2 \\ 0 & 0 & 1 & 1 & -4/21 \\ 0 & 0 & 0 & 1 & 1/3 \end{array}\right|$$

$$\left|\begin{array}{cccc|c} 1 & 0 & 4/5 & 2/5 & 3/5 \\ 0 & 1 & 0 & -5 & 26/21 \\ 0 & 0 & 1 & 0 & -11/21 \\ 0 & 0 & 0 & 1 & 1/3 \end{array}\right|$$

$$\left|\begin{array}{cccc|c} 1 & 0 & 4/5 & 2/5 & 3/5 \\ 0 & 1 & 0 & 0 & 31/21 \\ 0 & 0 & 1 & 0 & -11/21 \\ 0 & 0 & 0 & 1 & 1/3 \end{array}\right|$$

$$\left|\begin{array}{cccc|c} 1 & 0 & 1 & 1/2 & 3/4 \\ 0 & 1 & 0 & 0 & 31/21 \\ 0 & 0 & 1 & 0 & -11/21 \\ 0 & 0 & 0 & 1 & 1/3 \end{array}\right| \quad \frac{5}{4} \times R_1$$

$$\left|\begin{array}{cccc|c} 1 & 0 & 0 & 1/2 & 126/84 \\ 0 & 1 & 0 & 0 & 31/21 \\ 0 & 0 & 1 & 0 & -11/21 \\ 0 & 0 & 0 & 1 & 1/3 \end{array}\right|$$

$$\left|\begin{array}{cccc|c} 5 & 0 & 4 & 2 & 3 \\ 1 & -1 & 2 & 1 & 1 \\ 0 & -1 & 6/5 & 4/5 & 3/5 \\ 0 & 0 & 7/5 & 8/5 & 1/5 \end{array}\right|$$ 5x R_3 and 5x R_2

$$\left|\begin{array}{cccc|c} 5 & 0 & 4 & 2 & 3 \\ 1 & -1 & 2 & 1 & 1 \\ 0 & -5 & 6 & 4 & 3 \\ 0 & 0 & 7 & 6 & 1 \end{array}\right|$$ 5 R_3 and 5 R_4

$$\left|\begin{array}{cccc|c} 1 & 0 & 4/5 & 2/5 & 3/3 \\ 1 & -1 & 2 & 1 & 1 \\ 0 & -5 & 6 & 4 & 3 \\ 0 & 0 & 7 & 8 & 1 \end{array}\right|$$ $\frac{1}{5} \times R_1$

$$\left|\begin{array}{cccc|c} 1 & 0 & 4/5 & 2/5 & 3/5 \\ 1 & -1 & 6/5 & 1/5 & 2/5 \\ 0 & -5 & 6 & 4 & 3 \\ 0 & 0 & 7 & 8 & 1 \end{array}\right|$$ $R_2 \times 5$

$$\left|\begin{array}{cccc|c} 1 & 0 & 4/5 & 2/5 & 3/5 \\ 0 & -5 & 6 & 1 & 2 \\ 0 & 0 & 0 & 3 & 1 \\ 0 & 0 & 7 & 8 & 1 \end{array}\right|$$ $R_3 - R$

$$\left|\begin{array}{cccc|c} 1 & 0 & 4/5 & 2/5 & 3/5 \\ 0 & -5 & 6 & 1 & 2 \\ 0 & 0 & 1 & 8/7 & 1/7 \\ 0 & 0 & 0 & 1 & 1/3 \end{array}\right| \quad \begin{array}{l} R_3 \leftrightarrow R_4 \\ 1/7 \times R_2 \\ 1/3 \times R_4 \end{array}$$

$$\Rightarrow \left|\begin{array}{cccc|c} 1 & 0 & 0 & 1/2 & 1/2 \\ 0 & 1 & 0 & 0 & 31/21 \\ 0 & 0 & 1 & 0 & -11/21 \\ 0 & 0 & 0 & 1 & 1/2 \end{array}\right|$$

$$\left|\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 3/4 \\ 0 & 1 & 0 & 0 & 31/21 \\ 0 & 0 & 1 & 0 & -11/21 \\ 0 & 0 & 0 & 1 & 1/2 \end{array}\right|$$

$$(x, y, z, m) = \left(\frac{3}{7}, \frac{31}{21}, \frac{-11}{21}, \frac{1}{3}\right)$$

$$x = 3/4$$

$$y = 31/21$$

$$z = -11/21$$

$$m = 1/3$$