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SUBJECT:

INTRO TO STRUCTURAL DYNAMICS

&

EARTHQUAKE ENGG:

DATE:

29th-JUNE-2020:

Question No: (1):

Given Data:

$E = 29{,}000$ ksi , $I = 150\ in^4$

δ_{st} = Deflection due to 7395 lb static load.

Beam is pulled $\frac{1}{2}$" downwards.

Required:

→ Natural time period of system = ?

→ Develop and solve equation of motion = ?

→ Draw graphs to show variation of displacement with time and the variation of equivalent static forces with time.

Solution:

General EOM for SDOF system is;

$$ku + c\dot{u} + m\ddot{u} = p(t).$$

Since system is undamped = $C = 0$

Undergoing free vibration = $p(t) = 0$

Hence, general EOM becomes

$$ku + m\ddot{u} = 0 \quad \text{——— (1)}$$

$$K = \frac{3EI}{L^3} = \frac{3 \times 29000\ k/in^2 \times 150\ in^4}{(10 \times 12\ in)^3}$$

$K = 7.55 K/in \Rightarrow K = 7.55208.$

→ In order to eliminate chances of mistake during calculation, it is more appropriate to use fundamental units like lb, ft, sec or kg, m, sec.

$K = 7.55208 K/in \Rightarrow 90625\ lb/ft$

$m = W/g = 7395/32.12 \Rightarrow 230.23$ slug.

$$W_n = \sqrt{K/m} = \sqrt{90625/230.23}$$

$W_n = 19.84$ rad/sec.

$$T_n = 2\pi/W_n \Rightarrow 2\pi/19.84 \Rightarrow 0.316 \text{ sec}$$

Put m & K in eq (1)

$$90625u + 230.23\ddot{u} = 0$$

where K is in lb/ft and m is in

in/ lb/sec/ft²

→ General solution to EOM for undamped free vibration is

$$u(t) = u(0)\cos(W_n t) + \frac{\dot{u}(0)}{2u_n}\sin(W_n t).$$

$$u(0) = 1/2'' = 1/24 \text{ ft}$$

$$\dot{u}(0) = 0$$

$$u(t) = \frac{1}{24} \times \cos(19.84) + 0$$

$$= \left(\frac{1}{24}\right)\cos(19.84)^t$$

Equivalent static force at any time t is

$$f = (t) = k \cdot u(t) = \frac{90625 \times \cos(19.84t)}{24.}$$

$$= 3776\cos(19.84t)$$

Amplitude of dynamic displacement, u_o for undamped free vibration is

$$u_o = \sqrt{\left[(u(0))^2 + \left(\frac{\dot{u}(0)}{W_n}\right)^2\right.}$$

$$= \sqrt{\left(\left(\frac{1}{24}\right)\right)^2 + 0}$$

$$= \frac{1}{24} \text{ ft}$$

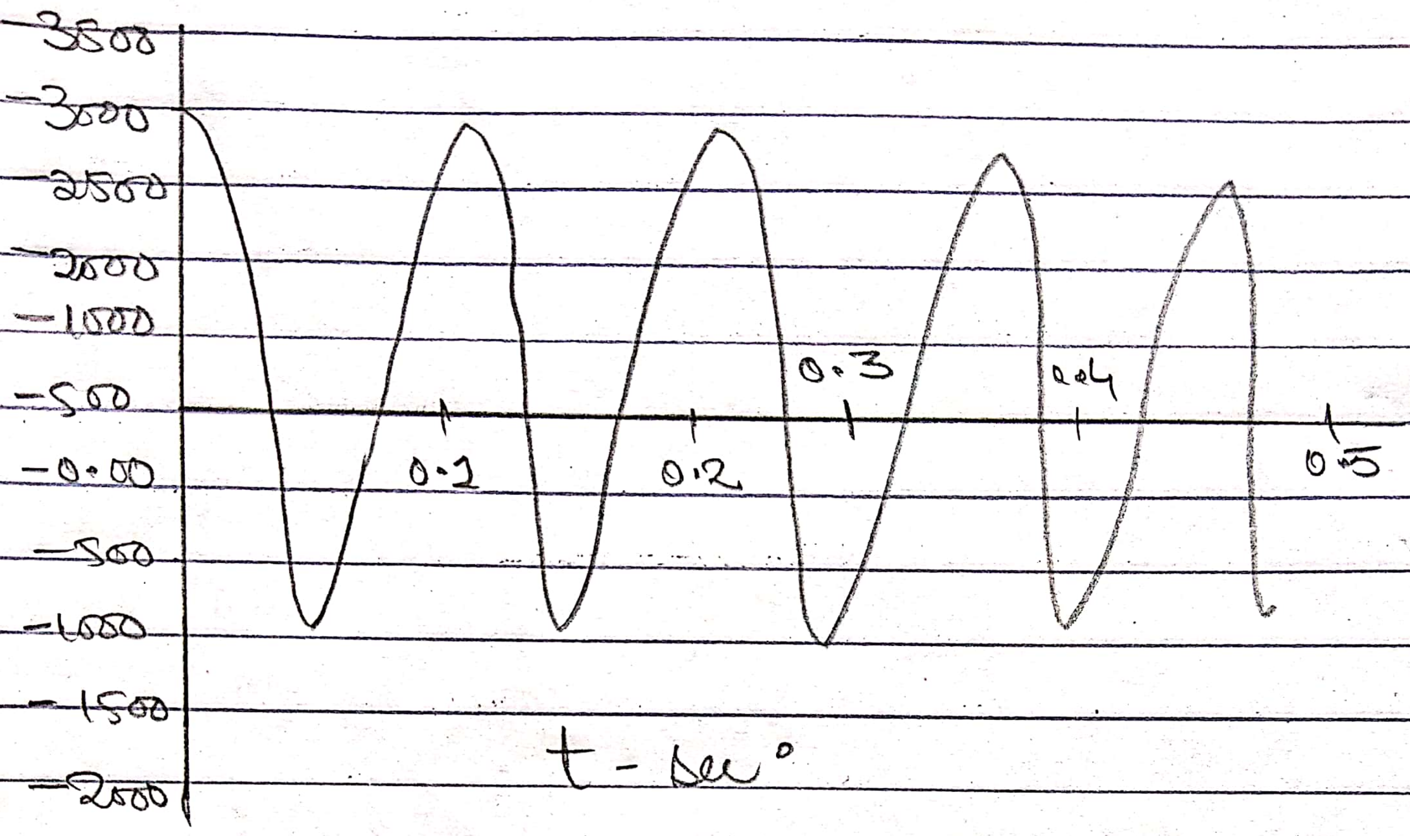
Amplitude of equivalent static force,

so,

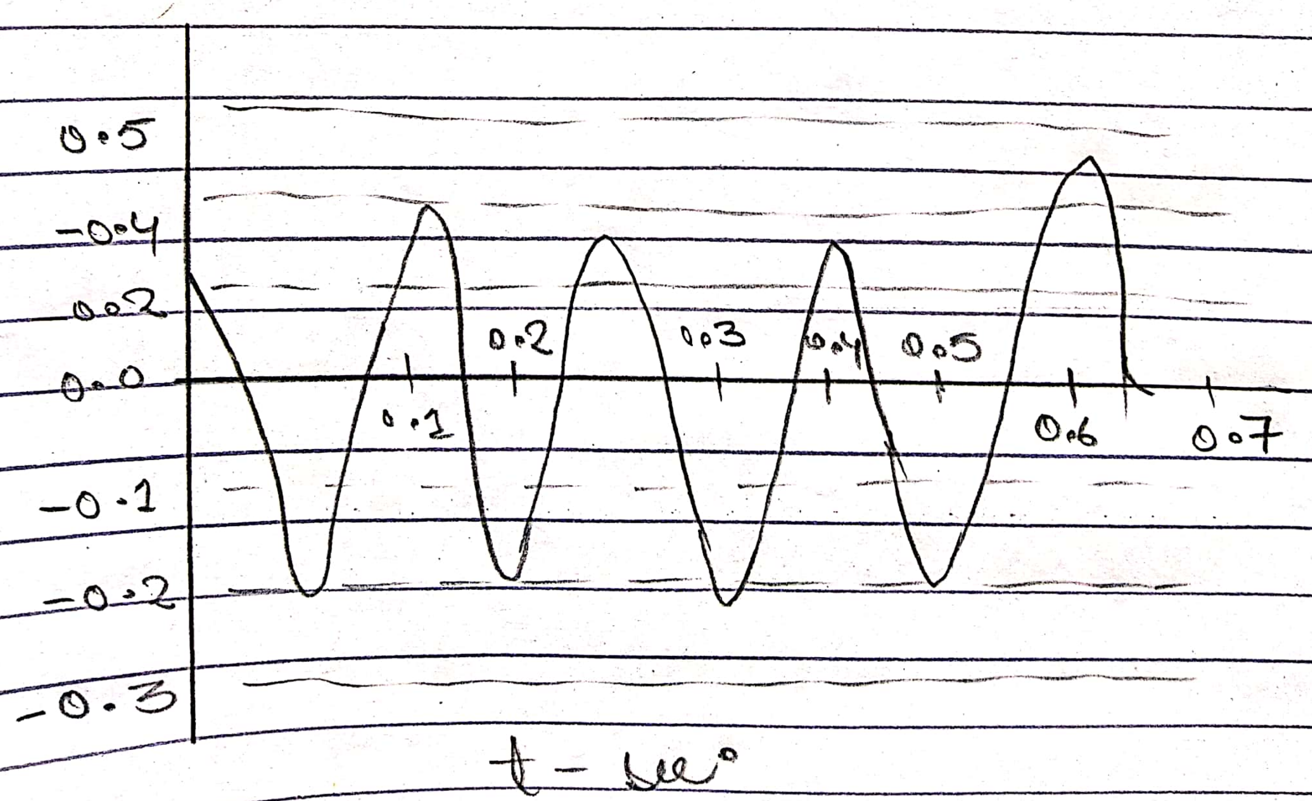
$$K y_0 = 90625 \times \frac{1}{24}$$

$$K y_0 = 3776.$$

Ans



UNDAMPED FREE-VIBRATIONS:



Question No 2:

Given Data:

ξ (Damping ratio) of reinforced concrete with considerable cracking = 3 – 5%
= 3%

Using Data of beam given in Question 1:

Required Data:

→ Develop and solve the equation showing variation in equivalent static force with time.

→ Draw graph to show variation of displacement with time and the variation of equivalent static force with time.

Solution:

E.O.M damped free vibration is

$$ku + c\dot{u} + m\ddot{u} = 0 \quad \text{——— (1)}$$

from Question 1,

$k = 90625 \text{ lb/ft}$ and $m = 230.23 \text{ lb·sec}^2/\text{ft}$

$W_n = 19.84$

$c = \xi \times 2mW_n$

$c = (0.03) \times 2(230.23)(19.84)$

$c = 274.06$ lb.sec/ft

Put values in eq (1)

$90625 + 274.06 + 230.23 = 0$

Solution to the EOM for damped free vibration is.

$$u(t) = e^{-\xi W_n t}\left[u(0)\cos(W_D t) + \frac{1}{W_D}\left[\dot{u}(0) + u(0)\xi W_n\right]\sin W_D t\right]$$

$W_D = 19.84$ rad/sec.

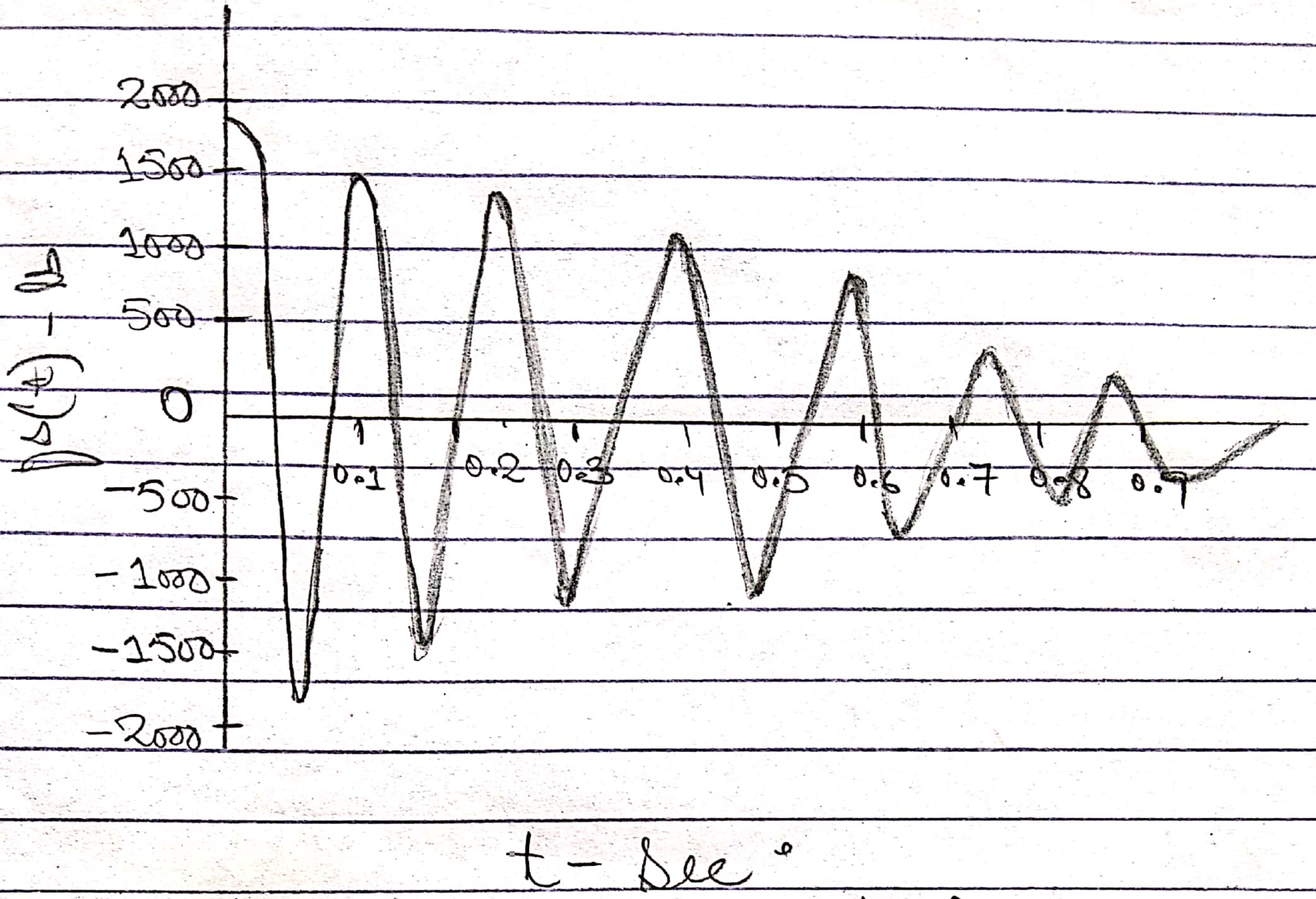
$$u(t) = e^{-0.03 \times 19.84 t}\left[\frac{1}{24} \times \cos(19.84t) + \frac{1}{19.84} \times \left[0 + \frac{1}{24} \times 0.03 \times 19.84 \times \sin(19.84t)\right)\right.$$

$$\Rightarrow = e^{-0.5952}\left[0.041 \times \cos(19.84t) + 0.0124 \times \sin(19.84t)\right]$$

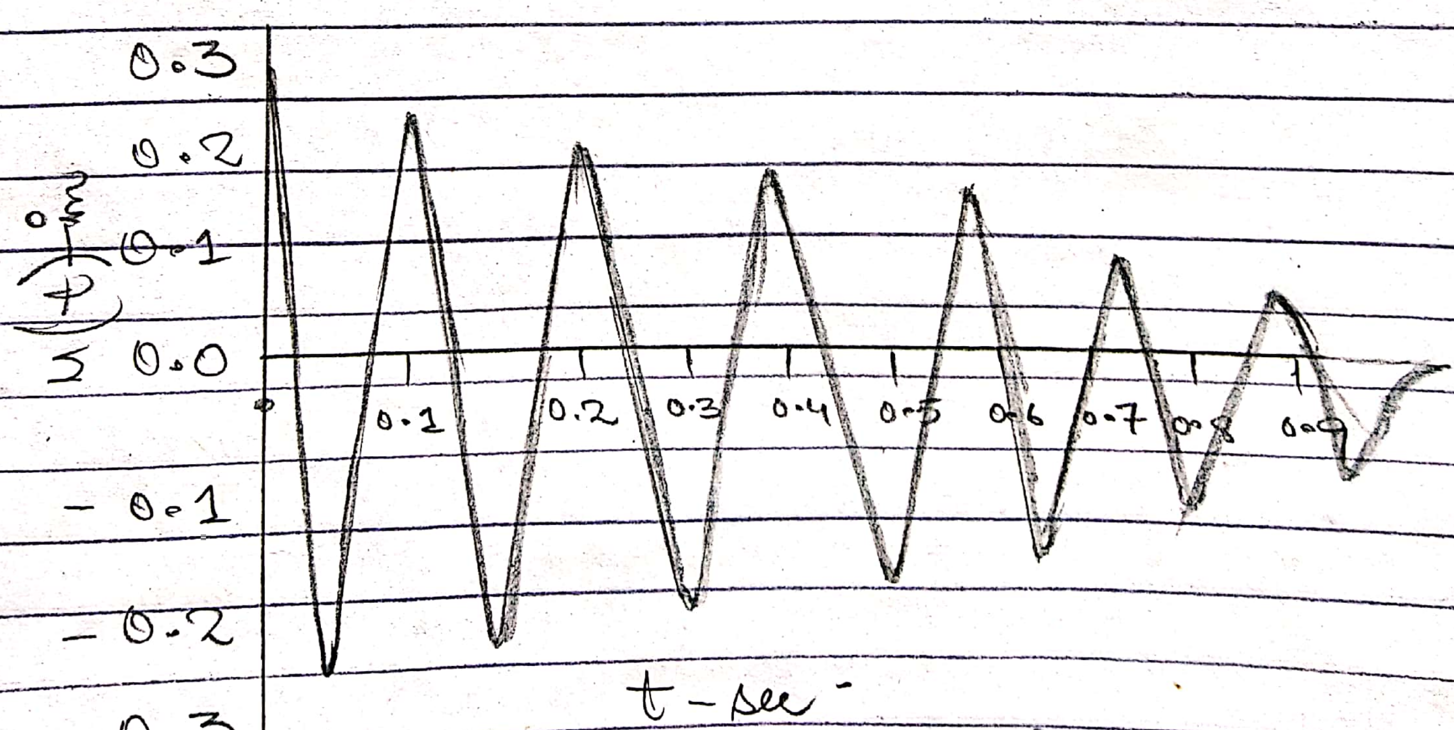
$$f_s(t) = k \cdot u(t) \Rightarrow 90625 \times u(t).$$

$$\Rightarrow e^{-0.5952}\left[(90625 \times 0.041)\cos(19.84t) + (90625 \times 0.0124)\sin(19.84t)\right]$$

$$\Rightarrow f_s(t) = e^{-0.5952}\left[3715.62 \cos(19.84t) + 1123.75 \sin(19.84t)\right]$$



Variation of Equivalent Static Forces:



Variation of displacement with time:

Question No 3:

Given:

Force = 60 kips

Displacement of Tank = $\left(\frac{ID}{1000}\right) = \left(\frac{7395}{1000}\right)''$

$= 7.395$

Cycles = 7.

Time Taken to complete 7 cycles = 3.57 sec.

Amplitude of displacement = 2.286 cm.

$= 0.9''$

Required:

(A) Damping Ratio
(B) Natural period of undamped vibrations
(C) Stiffness of structure.
(d) Weight of Tank.
(E) Damping co-efficient.
(F) Number of cycles to reduce the displacement amplitude to 0.5"

Solution:

→ Displacement of Tank $u_1 = 7.395''$

→ After 7 cycles

i-e After $j = 7$, $u_{j+1} = u_8 = 0.9''$

(A) DAMPING RATIO = ζ = ?

$$j = \frac{1}{2\pi\zeta} \ln\left[\frac{u_1}{u_{j+1}}\right]$$

$$\zeta = \frac{1}{2\pi 7} \ln\left[\frac{7.395}{0.9}\right]$$

$$\zeta = 0.0477 = 4.77\%$$

(B) NATURAL PERIOD OF UNDAMPED VIBRATIONS = T_n = ?

As the 7 cycles of vibrations are completed in 3.57 sec.

→ Time required to complete one cycle

$$T_D = 3.57/7 = 0.51 \text{ sec.}$$

$$T_D = 0.51 \text{ sec.}$$

Now $W_D = W_n\sqrt{(1-\zeta)^2}$

$$\frac{2\pi}{W_D} = \frac{2\pi}{(W_n\sqrt{(1-\zeta^2)})}$$

$$= T_D = \frac{T_n}{(1-\zeta)^2}$$

$$T_n = T_D \times \sqrt{1-\zeta^2}$$

$$T_n = 0.51 \times \sqrt{1-(0.0477)^2}.$$

$$T_n = 0.5094 = 0.51 \text{ sec}$$

$$T_n = 0.51 \text{ sec}$$

(C) STIFFNESS OF STRUCTURE, K = ?

$$K = 60 \times \cos 60^\circ / 7.395 = 4.056 \text{ K/in}$$

$$K = 4.05 \text{ K/in} = 48600 \text{ lb/ft}$$

(D) WEIGHT OF TANK W = ?

$$W_n = \sqrt{K/m} = \sqrt{K/(W/g)} = \sqrt{Kg/W}$$

$$\Rightarrow W_n^2 = K \times g / W$$

$$W = K \times g / W_n^2$$

Also $W_n = 2\pi / T_n$

$$W = Kg / (4\pi^2 / T_n^2)$$

$$= Kg \times T_n^2 / 4\pi^2$$

$$W = \left[\frac{48600 \text{ lb}}{\text{ft}} \times \frac{32.2}{\text{sec}} \text{ ft} \right] \times \left(\frac{0.51 \text{ sec}}{4\pi^2} \right)^2$$

$$= 10320.79 \text{ lb} = 10.32 \text{ K}$$

(E) DAMPING CO-EFFICIENT, C = ?

It is know that

$$\zeta = \frac{c}{2mW_n}$$

$$c = \zeta \times 2mW_n$$

$$= \zeta \times 2m \times \left(\frac{2\pi}{T_n}\right)$$

$$c = \frac{(0.0477) \times 4 \times 3.14 \left(\frac{1032.79}{32.2}\right)}{0.51}$$

$$C = 376.52 \text{ lb.sec/ft}$$

(F) NUMBER OF CYCLES TO REDUCE THE DISPLACEMENT AMPLITUDE TO 0.5"

$$j = \frac{1}{2\pi\zeta} \ln\left[\frac{u_1}{u_{j+1}}\right]$$

$$j = \frac{1}{2\pi \times 0.0477} \ln\left[\frac{7.395}{0.5}\right]$$

$$j = 8.97 \text{ or } 9 \text{ cycles}$$