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SECTION	#	"B"
SUBJECT	#	CALCULUS
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Q1:- The function $g(t)$ is defined by

$$g(t) = \begin{cases} 0 & t < 0 \\ t^2 & 0 \le t \le 3 \\ 2t+3 & 3 < t \le 4 \\ 12 & t > 4 \end{cases}$$

a) State any point of discontinuity

b) find if they exist

1. $\lim_{t \to 3} g$

Sol To check possibility of the discontinouty of the function is at $t = 0$ & 4.

→ first at $t = 0$

$g(t) = t^2$
$g(0) - 0^2 = 0$

For R.H.L

$\lim g(1+h) = \lim_{h \to 0} (1+h^2)$

$\lim_{h \to 0} 1+h^2+2h$

Apply limit

$1+0^2+2(0)$

$= 1$

For L.H.L

$\lim_{h \to 0} g(1-h) = 2t+3$

$= \lim_{h \to 0} 2(1-h)+3.$

$\lim_{h \to 0} 2-2h+3$

Applying limit

$2-2(0)+3$

$= 5$

R.H.L $\neq$ L.H.L $= g(t) = 5$

Now at $t=4$

③

$g(4) = 2(4) + 3$

$= 8 + 3$

$= 11$

For R.H.L

$\lim_{h \to 0} g(1+h) = \lim_{h \to 0} 2(1+h) + 3$

$\lim_{h \to 0} 2 + 2h + 3$

Apply limit

$= 2 + 2(0) + 3 \Rightarrow 5$

For L.H.L

$\lim_{h \to 0} g(1-h) = 11$

$g(4) = R.H.L \neq L.H.L$

point of discontinuity is at $t = 4$

Q2:- find the Maclaurin's Series for

$y(x) = x^2 + \sin x$

Sol

$y(x) = x^2 + \sin x$

By Maclaurin's Series expansion

$$f(x) = f(0) + x f'(0) + \frac{x^2 f''(0)}{2!} + \frac{x^3 f'''(0)}{3!} + \dots$$

Now

$$f(x) = y(x) = x^2 + \sin x$$

$$f'(x) = 2x + \cos x$$

$$f''(x) = 2 - \sin x$$

$$f'''(x) = -\cos x.$$

Thus

$$f(0) = (0)^2 + \sin(0) = 0$$

$$f'(0) = 2(0) + \cos(0) = 1$$

$$f''(0) = 2 - \sin(0) = 2$$

$$f'''(0) = -\cos(0) = -1.$$

By Maclaurin's Series expansion.

$$f(x) = y(x) = 0 + x(1) + \frac{x^2(2)}{2!} + \frac{x^3(-1)}{3!}$$

$$= 0 + x + x^2 - \frac{x^3}{3!} + \cdots$$

$$= x + x^2 - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \frac{x^9}{9!} \cdots$$

The required Maclaurins Series expansion.

Q3 :-
(1) find y'' given.

$$1 + xy = x^2 + y^2.$$

Sol

$$1 + xy = x^2 + y^2 \quad \text{— ①}$$

diff ① w.r.t "x"

$$0 + x\, dy/dx + y(1) = 2x + 2y\, dy/dx$$

$$x \frac{dy}{dx} - 2y \frac{dy}{dx} = 2x - y$$

$$(x - 2y)\, dy/dx = 2x - y$$

$$\Rightarrow \frac{dy}{dx} = \frac{2x - y}{x - 2y} \quad \text{— Ⓐ}$$

again diff w.r.t "x"

$$\frac{d^2y}{dx^2} = \frac{(x-2y)\, d/dx\,(2x-y) - (2x-y)\, d/dx\,(x-2y)}{(x-2y)^2}.$$

(7)

$$= \frac{(x-2y)(2(1) - dy/dx) - (2x-y)(1-2\,dy/dx)}{(x-2y)^2}$$

$$= \frac{(x-2y)\left(2 - \frac{2x-y}{x-2y}\right) - (2x-y)\left(1 - 2\left(\frac{2x-y}{x-2y}\right)\right)}{(x-2y)^2}$$

$$= \frac{(x-2y)\left(\frac{2(x-2y) - (2x-y)}{x-2y}\right) - \left((2x-y)\frac{(x-2y-2(2x-y)}{x-2y}\right)}{(x-2y)^2}$$

$$= \frac{(x-2y)(\cancel{2x} - 4y - \cancel{2x} + y) - (2x-y)(x - \cancel{2y} - 4x + \cancel{2y})}{(x-2y)(x-2y)^2}$$

$$= \frac{(x-2y)(-3y) - (2x-y)(-3x)}{(x-2y)^3}$$

$$= \frac{-3xy + 6y^2 + 6x^2 - 3xy}{(x-2y)^3}$$

$$= \frac{6x^2 + 6y^2 - 6xy}{(x-2y)^3} \quad \text{ANS.}$$

Q3
(ii) $y = x^3 (1+x)^9 e^{6x}$.

find y' by using Lagarithmic differentiation ?

Sol

$y = x^3 (1+x)^9 e^{6x}$.

Taking ln to both side.

$\ln y = \ln [x^3 (1+x)^9 . e^{6x}]$

$\ln y = \ln x^3 + \ln (1+x)^9 + \ln e^{6x}$.

$\ln y = 3 \ln x + 9 \ln (1+x) + 6x \ln e$.

$\ln y \; 3 \ln x + 9 \ln (1+x) + 6x$

Taking derivative again "x"

$1/y \cdot dy/dx = 3(1/x) + 9(1/1+x) \, d/dx (1+x) + 6($

$dy/dx = y [3/x + 9/1+x \, (1) + 6]$

(9)

$$\frac{dy}{dx} = x^3 (1+x)^9 \cdot e^{6x} \left[3/x + 9/_{1+x} + 6 \right].$$

$$= 3x^2 (1+x)^9 e^{6x} + 9x^3 (1+x)^8 e^{6x} + 6x^3 (1+x)^9 \cdot e^{6x}.$$