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Section (A)

Q1) The function $g(t)$ is defined

$$g(t) = 0 \quad t < 0$$

$$t^2 \quad 0 \leq t \leq 3$$

$$2t + 3 \quad 3 < t \leq 4$$

$$12 \quad t > 4$$

(a) State any point of discontinuity.

To check the possibility of discontinuity of the function

at $x = 4$

$$g(4) = 2(4) + 3$$

$$= 8 + 3$$

$$\boxed{= 11} \rightarrow (1)$$

L.H.L

(2)

$$\begin{aligned}\lim_{t \rightarrow 4} g(t) &= \lim_{t \rightarrow 4} (2t + 3) \\ &= 2(4) + 3 \\ &= 11\end{aligned}$$

R.H.L $\lim_{t \rightarrow 4} g(t) = 12$

L.H.L \neq R.H.L

So the function is discontinuous at $x=4$

(b) Find if they exist $\lim_{t \rightarrow 3} g$

$$\begin{aligned}\lim_{t \rightarrow 3} g(t) &= \text{limit } t^2 \quad 0 \leq t \leq 3 \\ &= 3^2 = 9.\end{aligned}$$

R.H.L

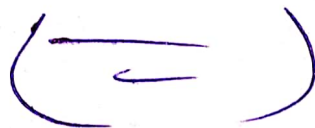
$$\lim_{t \rightarrow 3} g(t) = \lim_{t \rightarrow 3} (2t + 3)$$

$$= 2(3) + 3$$

$$= 9$$

$$L.H.L = R.H.L$$

So limit exist
 $t \rightarrow 3$.



(3)

Q2) Find the Maclaurin's series from. (4)

$$Y(x) = x^2 + \sin x$$

Solr.

$$f(x) = x^2 + \sin x \quad \Rightarrow \quad f(0) = 0$$

taking derivate.

$$\frac{dy}{dx} = \frac{d}{dx} (x^2 + \sin x)$$

$$y' = \frac{dx^2}{dx} + \frac{d}{dx} \sin x$$

$$y' = 2x + \cos x \quad \Rightarrow \quad f'(0) = 1$$

taking second derivatives

$$y'' = \frac{d}{dx} (2x + \cos x)$$

$$y'' = 2 - \sin x \quad \Rightarrow \quad f''(0) = 2$$

taking third derivate.

$$y''' = \frac{d}{dx} (2 - \sin x)$$

$$y''' = -\cos x \quad \Rightarrow \quad f'''(0) = -1$$

taking fourth derivate.

$$y'' = \frac{d}{dx} (1 - \cos x)$$

$$y'' = -(-\sin x)$$

$$y'' = \sin \Rightarrow f''(0) = 0$$

taking fifth derivative

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$$y'' = \frac{d}{dx} \sin x$$

$$y'' = \cos x$$

$$\Rightarrow f''(0) = 1$$

taking sixth derivatives

$$y^{vi} = \frac{d}{dx} \cos x$$

$$y^{vi} = -\sin x$$

$$\Rightarrow f^{vi}(0) = 0$$

taking seventh derivatives

$$y^{vii} = \frac{d}{dx} -\sin x$$

$$y^{viii} = -\cos x$$

$$\Rightarrow f^{viii}(0) = -1$$

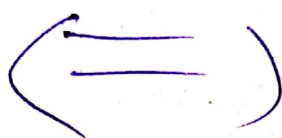
Mac laurin's series

$$f(x) = f(0) + xf'(0) + \frac{x^2}{2!} f''(0) + \frac{x^3}{3!} f'''(0) + \dots$$

$$\frac{x^4}{4!} f^{iv}(0) + \frac{x^5}{5!} f^{v}(0) + \frac{x^6}{6!} f^{vi}(0) + \frac{x^7}{7!} f^{vii}(0) + \dots$$

$$x^2 + \sin x = x + x^2 - \frac{x^3}{6} + \frac{x^5}{120} - \frac{x^7}{5040} + \dots$$

Ans.



Q3 (i) Find y'' given.

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$$1 + xy = x^2 + y^2$$

Taking derivatives on B.S

$$\frac{d}{dx} (1 + xy) = \frac{d}{dx} (x^2 + y^2)$$

$$0 + \frac{xdy}{dx} + y \frac{dx}{dx} = \frac{dx^2}{dx} + \frac{dy^2}{dx}$$

$$x \frac{dy}{dx} + y(1) = 2x + 2y \frac{dy}{dx}$$

$$x \frac{dy}{dx} - 2y \frac{dy}{dx} = 2x - y$$

$$\frac{dy}{dx} (x - 2y) = (2x - y)$$

$$\frac{dy}{dx} = \frac{2x - y}{x - 2y}$$

$$y' = \frac{2x - y}{x - 2y}$$

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Now taking second derivatives.

$$y'' = \frac{1}{(x-2y)^2} \left[(x-2y) \frac{d}{dx} (2x-y) - (2x-y) \frac{d}{dx} (x-2y) \right]$$
$$= \frac{\left[(x-2y) \left(2 - \frac{dy}{dx} \right) - (2x-y) \left(1 - 2 \frac{dy}{dx} \right) \right]}{(x-2y)^2}$$

Put $\frac{dy}{dx} = \frac{2x-y}{x-2y}$

$$y''(x) = \frac{\left[(x-2y) \left(2 - \frac{2x-y}{x-2y} \right) - (2x-y) \left(1 - 2 \left(\frac{2x-y}{x-2y} \right) \right) \right]}{(x-2y)^2}$$

$$= \frac{1}{(x-2y)^2} \left[\frac{(x-2y) [2x-4y-2x+y]}{x-2y} \right]$$
$$- 2x-y \left[\frac{x-2y-4x+2y}{x-2y} \right]$$

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$$= \frac{1}{(x-2y)^2} \left[-3y - (2x-y) \frac{(-3y)}{x-2y} \right]$$

$$= \frac{1}{(x-2y)^2} \left[\frac{-3y(x-2y) + 3y(2x-y)}{(x-2y)} \right]$$

Ans.

(3ii) Find y' by using logarithmic differentiation.

$$y = x^3 (1+x)^9 e^{6x}$$

Soln.

$$y = x^3 (1+x)^9 e^{6x}$$

taking \ln on B.S

$$\ln y = \ln (x^3 (1+x)^9 e^{6x})$$

$$\ln y = \ln x^3 + \ln (1+x)^9 + \ln e^{6x}$$

$$\ln y = 3 \ln x + 9 \ln (1+x) + 6x$$

$$\frac{d(\ln y)}{dx} = 3 \frac{d}{dx} (\ln x) + 9 \frac{d}{dx} (\ln(1+x)) + b \frac{dx}{dx}$$

$$\frac{1}{y} \frac{dy}{dx} = 3 \frac{1}{x} + \frac{9}{1+x} + b$$

$$\boxed{\frac{dy}{dx} = \left(\frac{3}{x} + \frac{9}{1+x} + b \right) y} \text{ Ans.}$$

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