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Section (A)

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Q1) The function g(t) is defined

$g(t) = 0 \qquad t < 0$

$t^2 \qquad 0 \le t \le 3$

$2t + 3 \qquad 3 < t \le 4$

$12 \qquad t > 4$

(a) State any point of discontinuity.

To check the possibility of discontinuity of the function

at $n = 4$

$g(4) = 2(4) + 3$

$= 8 + 3$

$\boxed{= 11} \rightarrow (1)$

L·H·L (2)

$\lim g(t) = \lim\limits_{t \to 4} (2t+3)$

$= 2(4) + 3)$

$= 4$

R·H·L $\lim\limits_{t \to 4} g(t) = 12$

L·H·L $\neq$ R·H·L

So the function is dirContinous at $x=4$

(b) Find if they exist $\lim\limits_{t \to 3} g$

$\lim\limits_{t - 3} g(t) = \text{limit } t^2 \quad 0 \leq t \leq 3.$

$= 3^2 = 9.$

R·H·L

$\lim\limits_{t \to 3} g(t) = \lim\limits_{t \to 3} (2t+3)$

3

$= 2(3) + 3$

$= 9$

L.H.L = R.H.L

so limit exist
$t \to 3.$

$(=)$

Q2) Find the Maclaurins series from. (4)

$$Y(x) = x^2 + \sin x$$

Solr.

$f(x) = x^2 + \sin x$, $f(0) = 0$

taking derivate.

$$\frac{dy}{dx} = \frac{d}{dx}(x^2 + \sin x)$$

$$y' = \frac{dx^2}{dx} + \frac{d}{dx}\sin x$$

$y' = 2x + \cos x$ $\Rightarrow f'(0) = 1$

taking second derivaties

$$y'' = \frac{d}{dx}(2x + \cos x)$$

$y'' = 2 - \sin x$ $\Rightarrow f''(0) = 2.$

taking third derivate.

$$y''' = \frac{d}{dx}(2 - \sin x)$$

$y''' = -\cos x$ $\Rightarrow f'''(0) = -1$

taking fourth derivate.

$$y^{iv} = \frac{d}{dx} 1 - (\cos x)$$

$$y^{iv} = -(-\sin x)$$

$$y^{iv} = \sin \Rightarrow f^{iv}(0) = 0$$

taking fifth derivative

$$y^{v} = \frac{d}{dx} \sin x.$$

$\boxed{y^{v} = \cos x}$ $\Rightarrow f^{v}(0) = 1$

taking sixth derivates

$$y^{vi} = \frac{d}{dx} \cos x.$$

$\boxed{y^{vi} = -\sin x.}$ $\Rightarrow f^{vi}(0) = 0$

taking seventh derivatives

$$y^{vii} = \frac{d}{dx} - \sin x$$

$\boxed{y^{viii} = -\cos x}$ $\Rightarrow f^{vii}(0) = -1$

Mac laurins. series

$$f(x) = f(0) + x f'(0) + \frac{x^2}{2!} f''(0) + \frac{x^3}{3!} f'''(0) +$$

$$\frac{x^4}{4!} f^{iv}(0) + \frac{x^5}{5!} f^{v}(0) + \frac{x^6}{6!} f^{vi}(0) + \frac{x^7}{7!} f^{vii}(0)$$

. . . . .

$$\boxed{x^2 + \sin x = x + x^2 - \frac{x^3}{6} + \frac{x^5}{120} - \frac{x^7}{5040} \cdots\cdots}$$

Ans.

$(\Leftarrow)$

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Q3 (i) Find y'' given.

$$1 + xy = x^2 + y^2$$

Taking derivatives on B·S

$$\frac{d}{dx}(1+xy) = \frac{d}{dx}(x^2+y^2)$$

$$0 + \frac{x\,dy}{dx} + y\frac{dx}{dx} = \frac{dx^2}{dx} + \frac{dy^2}{dx}$$

$$x\frac{dy}{dx} + y(1) = 2x + 2y\frac{dy}{dx}$$

$$x\frac{dy}{dx} - 2y\frac{dy}{dx} = 2x - y$$

$$\frac{dy}{dx}(x-2y) = (2x-y)$$

$$\frac{dy}{dx} = \frac{2x-y}{x-2y}$$

$$\boxed{y' = \frac{2x-y}{x-2y}}$$

Now taking second derivaties.

$$y'' = \frac{1}{(x-2y)^2}\left[(x-2y)\frac{d}{dx}(2x-y) - (2x-y)\frac{d}{dx}(x-2y)\right]$$

$$= \frac{\left[(x-2y)\left(2-\frac{dy}{dx}\right) - (2x-y)\left(1-2\frac{dy}{dx}\right)\right]}{(x-2y)^2}$$

Put $dy/dx = \dfrac{2x-y}{x-2y}$

$$y''(x) = \left[(x-2y)\left(2-\frac{2x-y}{x-2y}\right) - 2x-y \left(1-2\left(\frac{2x-y}{x-2y}\right)\right)\right](x-2y)^2$$

$$= \frac{1}{(x-2y)^2}\left[\cancel{(x-2y)}\left[\frac{2x-4y-2x+y}{\cancel{x-2y}}\right]\right.$$

$$\left. - 2x-y\left[\frac{x-2y-4x+2y}{x-2y}\right]\right]$$

$$= \frac{1}{(x-2y)^2}\left[-3y-(2x-y)\frac{(-3y)}{x-2y}\right]$$

$$\boxed{= \frac{1}{(x-2y)^2}\left[\frac{-3y(x-2y)+3y(2x-y)}{(x-2y)}\right]}$$

Ans.

(3ii) Find y' by using logorithimic differentiation.

$$y = x^3(1+x)^9 e^{6x}$$

Soln.

$$y = x^3(1+x)^9 e^{6x}$$

taking ln on B.S

$$\ln y = \ln (x^3(1+x)^9 e^{6x}$$

$$\ln y = \ln x^3 + \ln (1+x)^9 + \ln e^{6x}$$

$$\ln y = 3\ln x + 9\ln (1+x) + 6x$$

$$\frac{d \ln y}{dx} = 3\frac{d}{dx}\ln x + 9\frac{d}{dx}\ln(1+x) + 6\frac{dx}{dx}$$

$$\frac{1}{y}\frac{dy}{dx} = 3\frac{1}{x} + \frac{9}{1+x} + 6$$

$$\boxed{\frac{dy}{dx} = \left(\frac{3}{x} + \frac{9}{1+x} + 6\right)y} \text{ Ans.}$$