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SECTION: A

CIVIL ENGINEERING

DEPARTMENT

APPLIED CALCULUS

MID TERM EXAM

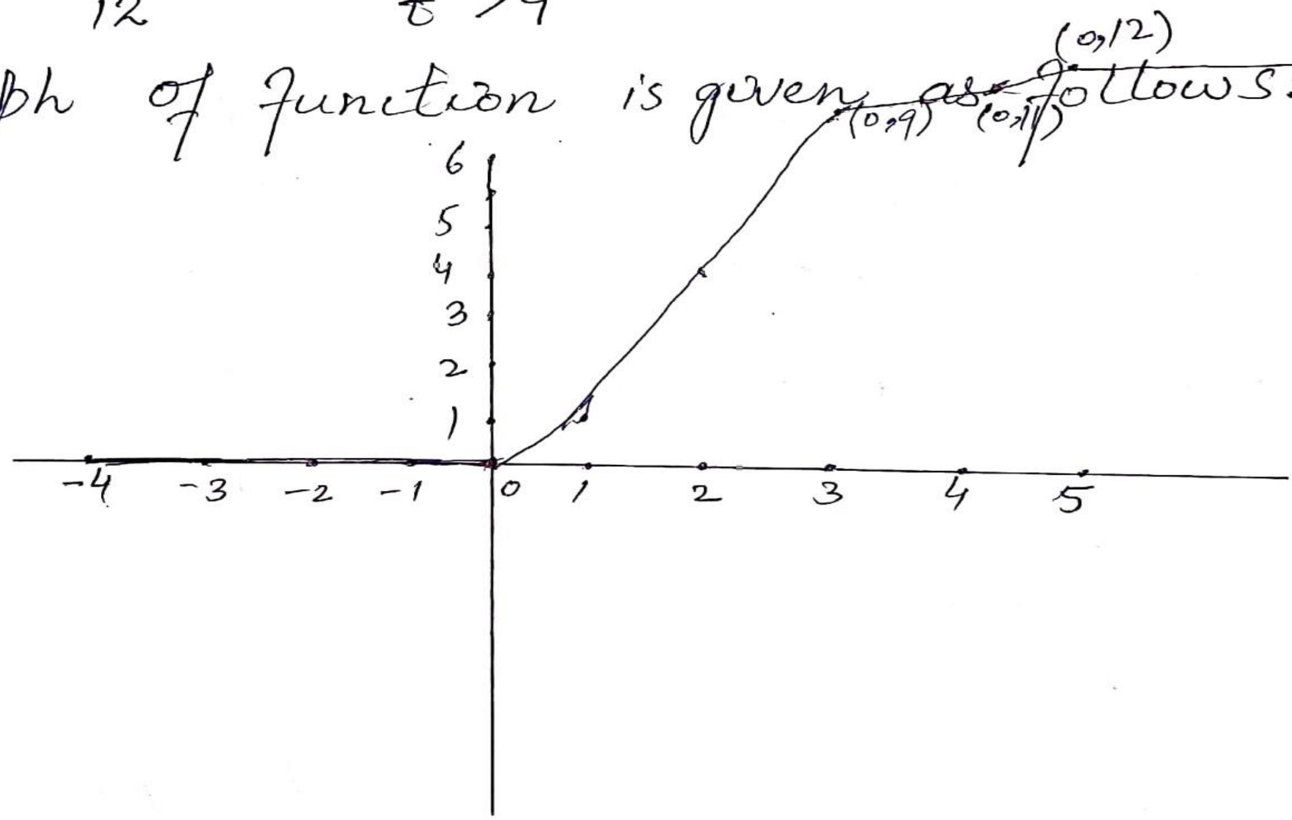
DATED: 21/08/2020

Q NO1

The function $g(t)$ is defined by

$$g(t) = \begin{cases} 0 & t < 0 \\ t^2 & 0 \leq t \leq 3 \\ 2t+3 & 3 < t \leq 4 \\ 12 & t > 4 \end{cases}$$

Graph of function is given as follows:



(a) from above figure, it can be concluded that there is no point of discontinuity in function.

(b) $\lim_{t \to 3} g(t)$.

(i) $\lim_{t \to 3} (0) = 0$, (ii) $\lim_{t \to 3} (t^2) = 9$

(iii) $\lim_{t \to 3} (2t+3) = 9$ (iv) $\lim_{t \to 3} (12) = 12$.

Q NO 2

Maclaurin series of $(x^2 + \sin x)$.
it is defined by taylor series of function
$f(x)$ at $f(a) \Rightarrow 0 = a$

$$f(x) = f(a) + \frac{f'(a)}{1!}(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f'''a}{3!}(x-a)^3 + \text{---}$$

Putting $a = 0$.

$$f(x) = f(0) + \frac{f'(0)}{1!}(x) + \frac{f''(0)}{2!}x^2 + \text{---}$$

Applying the Maclaurin formula.

$$= 0 + \frac{\frac{d}{dx}(\sin x + x^2)(0)}{1!}(x) + \frac{\frac{d^2}{dx^2}(x^2+\sin x)(0)}{2!}x^2$$

$$+ \frac{\frac{d^3}{dx^3}(x^2+\sin x)(0)}{3!}x^3 + \text{———}$$

Evaluating derivatives.

$$= 0 + \frac{1}{1!}x + \frac{2}{2!}x^2 + \frac{-1}{3!}x^3 + \frac{0}{4!}x^4$$

$$+ \frac{1}{5!}x^5 + \frac{0}{6!}x^6 + \frac{-1}{7!}x^7 + \text{———}$$

Simplifying.

$$= 0 + x + x^2 + \frac{-1}{6}x^3 + \frac{1}{120}x^5 - \frac{1}{5040}x^7 + \text{———}$$

$$= x + x^2 - \frac{1}{6}x^3 + \frac{1}{120}x^5 - \frac{1}{5040}x^7 + \text{———}$$

Q No 3(a)

(i) Find y'' of $(1+xy) = x^2+y^2$

Implicit solving the derivatives. and treat y as $y(x)$.

$$x\frac{d}{dx}(y) + \frac{dy}{dx}(1) = 2x + 2y\frac{dy}{dx}$$

Isolating $\frac{dy}{dx}$

$$x\frac{dy}{dx} - 2y\frac{dy}{dx} = 2x - y.$$

$$\frac{dy}{dx} = \frac{2x-y}{x-2y}$$

Also,

$$\frac{d^2y}{dx^2} = \frac{dy}{dx}\frac{(2x-y)}{x-2y}.$$

Treat y as a constant and apply quotient rule.

$$= \frac{\frac{d}{dx}(2x-y)(x-2y) - \frac{d}{dx}(x-2y)(2x-y)}{(x-2y)^2}$$

As, $\frac{d}{dx}(2x-y) = 2$

$\frac{d}{dx}(x-2y) = 1$

Putting values:

$$y'' = \frac{2(x-2y) - 1(2x-y)}{(x-2y)^2}$$

$$= \frac{\cancel{2x} - 2y - \cancel{2x} + y}{(x-2y)^2}$$

$$\boxed{y'' = \frac{-3y}{(x-2y)^2}}$$

(b) y' by logarethmic differentiation.

$y = x^3(1+x)^9 e^{6x}$

Taking ln on both sides.

$$\ln y = \ln\left(x^3 (1+x)^9 e^{6x}\right).$$

$$\ln y = \ln x^3 + \ln(e^{6x}) + \ln(1+x)^9$$

$$= 3\ln x + 6x\ln(e) + 9\ln(1+x)$$

$$\ln y. = 3\ln x + 6x + 9\ln(1+x).$$

taking d/dx.

$$\frac{1}{y}y' = \frac{3}{x} + \frac{9}{x+1} + 6$$

$$\frac{1}{y}y' = \frac{12x+3}{x(x+1)} + 6$$

$$y' = (\ln x)^x(6) + (\ln x)^x \frac{12x+3}{x(x+1)}.$$