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I.D : 7351

Subject : Differential Equation

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Q1: The of matrix A is $m \times p$ and the and the order of B is $p \times n$. Then the order of matrix AB is?

Sol: The order of matrix AB is $m \times n$.

(ii) The no of non zero rows in Echelon form?

The no of non zero rows in Echelon form is one

(iii) If $B = \begin{bmatrix} 1 & 4 \\ 2 & a \end{bmatrix}$ is singular matrix then a=?

If $B = \begin{bmatrix} 1 & 4 \\ 2 & a \end{bmatrix}$ is a singular matrix

Then $a = 8$.

(iv) ~~If~~ $\cancel{A = \begin{bmatrix} a & 0 \\ 0 & a \end{bmatrix}}$ ~~is~~

iv) If $A = \begin{bmatrix} 2i & i \\ i & -i \end{bmatrix}$

$|A| = \begin{bmatrix} 2i & i \\ i & -i \end{bmatrix}$ $\qquad i^2 = -1$

$= -2i^2 - i^2$

$= -2(-1) - (-1)$

$= 2 + 1 \Rightarrow \underline{3}$

(v) The Matrix $A = \begin{bmatrix} 9 & 0 \\ 0 & 9 \end{bmatrix}$ is Scalar Matrix.

The given because The diagnol element are same and non dignol are are zero

(VI) Solution of $\frac{dy}{dx} + 2xy = y$?

Sol: $\frac{dy}{dx} + 2xy = y$

$\frac{dy}{dx} = y - 2xy$

$\frac{dy}{dx} = y(1-2x)$ $\qquad$ y taking common

$\frac{dy}{dx} = y(1-2x)$

$\frac{dy}{dx} = (1-2x)\,dx$

taking integration

$\int \frac{1}{y}\,dy = \int (1-2x)\,dx$

$\ln y = x - \frac{\cancel{2}x^2}{\cancel{2}} + c$

$\ln y = x - x^2 + c$

$\log y = x - x^2 + c$

$e^{\ln y} = e^{x - x^2 + c}$

$\boxed{y = e^{x(1-x)+c}}$ Ans

(vii) The order and degree of differential equation

$$\left(\frac{dy}{dx}\right)^2 = \sqrt{1 + \left(\frac{dy}{dx}\right)^3} \text{ is}$$

order $= \underline{1}$

Degree $= \underline{3}$

(viii) The order and degree of differential equation

$$\frac{dy^2}{dx^2} - 4xy = \sin\left(\frac{dy}{dx^2}\right) \text{ is}$$

Sol: Order = two

Degree = one.

(ix) The differential equation $2\frac{dy}{dx} + x^2 y = 2x+3$

$y = (0) = 5$ is

Soln

$2y' + x^2 y = x^2+3, \; y(0) = 5$

$y' + \left(\frac{x^2}{2}\right) y = \frac{x^2+3}{2}$

$y' + \left(\frac{x^2}{2}\right) y = \frac{1}{2}\left(x^2+3\right)$

$u = x^2/2$

$e^{\int x^2/2 \, dx} = e^{x^3/6}$

$e^{x^3/6} y' + e^{x^3/6}\left(x^2/2\right) y = \frac{1}{2} e^{x^3/6}\left(x^3+3\right)$

$y(x) = \frac{e^{x^3/6} x^2 + 3e^{x^3/6}}{2e^{x^3/6}}$

$y(0) = \frac{0+3}{2} = 3/2$

$y(x) = \frac{e^{x^3/6} x^2 + 3e^{x^3/6}}{2e^{x^3/6}} + 3/2$ Ans

~~(X) $y(x) =$~~

(x) $\begin{bmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{bmatrix}$

Expand by C_1

$$1\begin{bmatrix} b & b^2 \\ c & c^2 \end{bmatrix} - 1\begin{bmatrix} a & a^2 \\ c & c^2 \end{bmatrix} + 1\begin{bmatrix} a & a^2 \\ b & b^2 \end{bmatrix}$$

$$1(bc^2 - cb^2) - 1(ac^2 + a^2c) + 1(ab^2 - a^2b)$$

$$= bc^2 - cb^2 - ac^2 - a^2c + ab^2 - a^2b$$

$$= ab^2 - cb^2 + a^2c - a^2b - ac^2 - a^2b$$

$$= a^2c - a^2b + ab^2 - cb^2 + bc^2 - ac^2$$

$$= a^2(c-b) + b^2(a-c) + c^2(b-a) \text{ Ans}$$

Q2(A) Express the Determine

$$\begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \end{vmatrix}$$

as the product of factor which are linear in a, b, c

Sol

$$a\begin{vmatrix} b^2 & c^2 \\ b^3 & c^3 \end{vmatrix} - b\begin{vmatrix} a^2 & c^2 \\ a^3 & c^2 \end{vmatrix} + c\begin{vmatrix} a^2 \\ a^3 \end{vmatrix}$$

$$= a(b^2c^3 - b^3c^2) - b(a^2c^3 - a^3c^2) + c(a^2b^3 - a^3b^3)$$

$$= ab^2c^3 - ab^3c^2 - a^2bc^3 - a^3bc^3 + a^2cb^3 - a^3b^2c$$

Common abc

$$= abc\left(bc^2 - b^2c - ac^2 - a^2c + ab^2 - a^2b\right)$$

$$= abc\left[bc(c-b) - ac(c-a) + ab(b-a)\right]$$

Ans

Q2B.

$$\begin{bmatrix} 2 & -1 & -1 & 0 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ 0 & -1 & -1 & 2 \end{bmatrix}$$

Sol.

$$\begin{bmatrix} 2 & -1 & -1 & 0 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ 0 & -1 & -1 & 2 \end{bmatrix}$$

characteristic eq $|A - \lambda I| = 0 \rightarrow (1)$

$$\begin{bmatrix} 2 & -1 & -1 & 0 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 5 & -1 \\ 0 & -1 & -1 & 2 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Now take determinant.

$$|A - \lambda I| = 0$$

$$\begin{vmatrix} 2-\lambda & -1 & -1 & 0 \\ -1 & 3-\lambda & -1 & -1 \\ -1 & -1 & 3-\lambda & -1 \\ 0 & -1 & -1 & 2-\lambda \end{vmatrix} = 0$$

Expand by R_1

$$2-\lambda \begin{vmatrix} 3-\lambda & -1 & -1 \\ -1 & 3-\lambda & -1 \\ -1 & -1 & 2-\lambda \end{vmatrix} - (-1) \begin{vmatrix} -1 & -1 & -1 \\ -1 & 3-\lambda & -1 \\ 0 & -1 & 2-\lambda \end{vmatrix}$$

$$-1 \begin{vmatrix} -1 & 3-\lambda & -1 \\ -1 & -1 & -1 \\ 0 & -1 & 2-\lambda \end{vmatrix} = 0 \rightarrow 2$$

Again

$$\begin{vmatrix} 3-\lambda & -1 & -1 \\ -1 & 3-\lambda & -1 \\ -1 & -1 & 2-\lambda \end{vmatrix} \quad \text{Expand by } R_1$$

$$\Rightarrow 3-\lambda \begin{vmatrix} 3-\lambda & -1 \\ -1 & 2-\lambda \end{vmatrix} - (-1) \begin{vmatrix} -1 & -1 \\ -1 & 2-\lambda \end{vmatrix} - 1 \begin{vmatrix} -1 & 3-\lambda \\ -1 & -1 \end{vmatrix}$$

$$= (3-\lambda) \big((3-\lambda)(2-\lambda) - (-1)(-1)\big) + 1(-1)(2-\lambda) - (-1)(-1)\big)$$

$$= (3-\lambda)(6-3\lambda-2\lambda+\lambda^2-1) + (-2+\lambda-1) - (+1+3-\lambda)$$

$$= (3-\lambda)(\lambda^2-5\lambda+5) + (-3\lambda) - (4-\lambda)$$

$$= 3\lambda^2 - 15\lambda + 15 - \lambda^3 + 5\lambda^2 - 5\lambda - 3 + \lambda - 4 + \lambda$$

$$= \boxed{-\lambda^3 + 8\lambda^2 - 18\lambda + 8} \longrightarrow 1$$

$$+1 \begin{vmatrix} -1 & -1 & -1 \\ -1 & 3-\lambda & -1 \\ 0 & -1 & 2-\lambda \end{vmatrix}$$

Expand C_1

$$= -1\begin{vmatrix} 3-\lambda & -1 \\ -1 & 2-\lambda \end{vmatrix} - (-1)\begin{vmatrix} -1 & -1 \\ -1 & 2-\lambda \end{vmatrix} + 0$$

$$\Rightarrow -1(6-3\lambda-2\lambda+\lambda^2-1)+1(-2+\lambda-1)$$

$$= -\lambda^2+5\lambda-5-3+\lambda$$

$$= \boxed{-\lambda^2+6\lambda-8} \rightarrow 2$$

$$-1\begin{vmatrix} -1 & 3-\lambda & -1 \\ -1 & -1 & -1 \\ 0 & -1 & 2-\lambda \end{vmatrix}$$

Expand by C_2

$$-\left[-1\begin{vmatrix} -1 & -1 \\ -1 & 2-\lambda \end{vmatrix} - (-1)\begin{vmatrix} 3-\lambda & -1 \\ -1 & 2-\lambda \end{vmatrix} + 0\right]$$

$$-\left[-(-2+\lambda-1)+1(6-3\lambda-2\lambda+\lambda^2-1)\right.$$

$$= -(3-\lambda+\lambda^2-5\lambda+5$$

$$-\lambda^2+5\lambda-5-3+\lambda$$

$$= -\lambda^2+5\lambda-5-3+\lambda$$

$$\lambda^2+6\lambda-8 \rightarrow c$$

put ~~ans~~ a, b and c in B

$$(2-\lambda)\left[-\lambda^3+3\lambda^2-18\lambda+8\right]-\lambda^2+6\lambda-8-\lambda^2+6\lambda$$
$$-8$$

$$= -2\lambda^3+16\lambda^2-36\lambda+\lambda^4-8\lambda^3+16x^2-8\lambda-\lambda^2$$
$$+6\lambda-8-\lambda^2+16\lambda-8$$

$$\Rightarrow \lambda^4-2\lambda^3-8\lambda^3+16\lambda^2+16\lambda^2-\lambda^2-\lambda^2-36\lambda-$$
$$-8\lambda+6\lambda+6\lambda+16-16$$

$\lambda^4 - 10\lambda^3 + 32\lambda^2 - 32\lambda = 0$

By synthatic division

We know that

$\lambda(\lambda-2)(\lambda^2 - 8\lambda + 16) = 0$

$(\lambda = 0)$

$= \lambda - 2 = 0$

$= \boxed{\lambda = 2}$

$\lambda^2 - 8\lambda + 16 = 0$

By Factorigation Method

$\lambda^2 - 4\lambda - 4\lambda + 16 = 0$

$\lambda(\lambda-4) - 4(\lambda-4) = 0$

$(\lambda-4)(\lambda-4) = 0$

$\lambda = 0, \lambda = 4$

$\lambda_1 = 0, \lambda_2 = 2, \lambda_3 = 3,$
$\lambda_4 = 4$ Ans

Q3: $(x^2 + 3y^2)dx - 2xy\,dy = 0.$

$x = 2, Y = 6$

Sol: $(x^2 + 3y^2)dx - 2xy\,dy = 0$

$\Rightarrow (x^2 + 3y)dx = 2xy\,dy$

Divide both side by $2xy\,dx$

We get

$\frac{dy}{dx} = \frac{x^2 + 3y^2}{2xy}$

$$\frac{dy}{dx} = \frac{x^2}{\cancel{2}xy} + \frac{3y^2}{2xy}$$

$$\frac{dy}{dx} = \frac{1}{2}\left[x/y + \frac{3y}{x} \right] \rightarrow 1$$

Let $y = vx$

Diff:

$dy = vdx + xdv$

Dividing by dx

$$\frac{dy}{dx} = v + \frac{dv}{dx} \rightarrow a$$

put a in 1

$$v + x\frac{dv}{dx} = \frac{1}{2}$$

$$\left[\frac{\cancel{x}}{\cancel{x}v} + 3\frac{v\cancel{x}}{\cancel{x}} \right]$$

$$v + \frac{xdv}{dx} = \frac{1}{2}\left[\frac{1}{v} + 3v \right]$$

Multiply Both side by "2"

We know that

$$2v + 2x\frac{dv}{dx} = \frac{1}{v} + 3v$$

$$2x\frac{dv}{dx} = 1/v + 3v - 2v$$

$$2x\frac{dv}{dx} = \frac{1}{v} + v \qquad \text{Take L.C.M.}$$

$$2x\frac{dv}{dx} = \frac{1 + v}{v}$$

xing both side by dx/dv
We know that

$$2xdv = \frac{1+v^2}{v} dx$$

Multiplying both side by $\frac{v}{x(1+v^2)}$
We know that

$$\frac{v}{1+v^2} dv = \frac{1}{x} dx$$

Taking "$\int$" on both side

$$\int \frac{2v}{1+v^2} dv = \int \frac{1}{x} dx + c$$

$$\ln |1+v^2| = \ln x + \ln c$$

Taking "e" on both side

$$e^{\ln |1+v^2|} = e^{\ln |xc|}$$

$$1+v^2 = xc$$

$$1+v^2 = xc$$

put $v = y/x$

$$1+(y/x)^2 = xc$$

$$\frac{x^2+y^2}{x^2} = xc$$

$$x^2+y^2 = x^3c \rightarrow (1)$$

put $x = 2, y = 6$ in eq (1).

$$x^2+y^2 = x^3c$$

$$4+36 = 8c$$

$$c = \frac{40}{8} = \boxed{c = 5}$$

Scanned with CamScanner

So

$$x^2 + y^2 = 5x^3$$

$$y^2 = 5x^3 - x^2$$

$$y^2 = x^2(5x-1)$$

Taking "$\sqrt{\ }$" on both side

$$y = +x\sqrt{5x-1} \quad , \quad y = -x\sqrt{5x-1}$$

or

$$y = \pm x\sqrt{5x-1}$$ Ans