Department of Electrical Engineering
Final Exam Assignment
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Course Details

Course Title: Digital Signal Processing **Module: 6th**
Instructor: Pir Mehar Ali Shah **Total Marks: 50**

Student Details

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Q1.	(a)	Determine the response y(n), n ≥ 0, of the system described by the second order difference equation $$y(n) - 4y(n-1) + 4y(n-2) = x(n) - x(n-1)$$ To the input $(n) = (-1)^n u(n)$. And the initial conditions are y (-1) = y (-2) = 0.	**Marks 7** **CLO 2**
	(b)	Determine the impulse response and unit step response of the systems described by the difference equation. $$y(n) - 0.7y(n-1) + 0.1y(n-2) = 2x(n) - x(n-2)$$	**Marks 7** **CLO 2**
Q2.	(a)	Determine the causal signal x(n) having the z-transform $$x(z) = \frac{1}{(1-2z^{-1})(1-z^{-1})^2}$$ (Hint: Take inverse z-transform using partial fraction method)	**Marks 6** **CLO 2**
	(b)	Evaluate the inverse z- transform using the complex inversion integral $$X(z) = \frac{1}{1-az^{-1}} \qquad \lvert z\rvert > \lvert a\rvert$$	**Marks 6** **CLO 2**
		A two- pole low pass filter has the system response	**Marks 6**

Q.3	(a)	$$H(z) = \frac{b_o}{(1 - pz^{-1})^2}$$ Determine the values of b_o and p such that the frequency response $H(\omega)$ satisfies the condition $H(0) = 1$ and $\left\|H\left(\frac{\pi}{4}\right)\right\|^2 = \frac{1}{2}$.	**CLO 3**
	(b)	Design a two-pole bandpass filter that has the center of its passband at $\omega = \pi/2$, zero in its frequency response characteristics at $\omega = 0$ and $\omega = \pi$ and its magnitude response in $\frac{1}{\sqrt{2}}$ at $\omega = 4\pi/9$.	**Marks 6** **CLO 3**
Q 4	(a)	A finite duration sequence of Length L is given as $$x(n) = \begin{cases} 1, & 0 \le n \le L-1 \\ 0, & otherwise \end{cases}$$ Determine the N- point DFT of this sequence for $N \ge L$	**Marks 6** **CLO 2**
	(b)	Perform the circular convolution of the following two sequences. Solve the problem step by step $$x_1(n) = \{\underset{\uparrow}{2}, 1, 2, 1\}$$ $$x_2(n) = \{\underset{\uparrow}{1}, 2, 3, 4\}$$	**Marks 6** **CLO 2**

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(1)

Q1(a) Ans #

Sol#
$y(n) - 4y(n-1) + 4y(n-2) = x(n) - x(n-1)$

The character equation is.

$\lambda^2 - 4\lambda + 4 = 0$

$\lambda = 2, 2$ Hence

$y_h(n) = c_1 2^n + c_2 n^n$

The particular solution is

$y_p(n) = k(-1)^n u(n)$

Substituting this solution into different equation we obtain.

$k(-1)^n u(n) - 4k(-1)^{n-1} u(n-1) + 4k(-1)^{n-2} u(n-2) =$
$(-1)^n u(n) - (-1)^{n-1} u(n-1)$

for $n = 2, k(1+4+4) = 2 \Rightarrow k = \frac{2}{9}$ The total solution is

$y(n) \left[c_1 2^n + c_2 n 2^n + \frac{2}{9}(-9)^n \right] u(n)$

from the inital condition, we obtain $y(0) = 1, y(1) = 2$ Then

$c_1 + \frac{2}{9} = 1$

$\Rightarrow c_1 = \frac{7}{9}$

$2c_1 + 2c_2 - \frac{2}{9} = 2$

$\Rightarrow c_2 = \frac{1}{3}$

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1b # Sol #

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$y(n) - 0.7y(n-1) + 0.1y(n-2) = 2x(n) - x(n-2)$

The characteristic equation is

$$\lambda^2 - 0.7\lambda + 0.1 = 0$$

$\lambda = \frac{1}{2}, \frac{1}{5}$ Hence.

$$y_h(n) = C_1 \frac{1}{2}^n + C_2 \frac{1}{5}^n$$

With $x(n) = \delta(n)$, we have

$$y(0) = 2$$

$$y(1) = 0.7y(0) = 0 \Rightarrow y(1) = 1.4$$

Hence $C_1 + C_2 = 2$ and

$$\frac{1}{2} C_1 + \frac{1}{5} = 1.4 = \frac{7}{5}$$

$$\Rightarrow C_1 + \frac{2}{5} C_2 = \frac{14}{5}$$

These equation yield

$$C_1 = \frac{10}{3}, C_2 = -\frac{4}{3}$$

$$h(n) = \left[\frac{10}{3}\left(\frac{1}{2}\right)^n - \frac{4}{3}\left(\frac{1}{5}\right)^n\right] u[n]$$

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$$y(n) = \sum_{k=0}^{n} h(n-k)$$

$$= \frac{10}{3} \sum_{k=0}^{n} \left(\frac{1}{2}\right)^{n-k} - \frac{4}{3} \sum_{k=0}^{n} \left(\frac{1}{5}\right)^{n-k}$$

$$= \frac{10}{3} \left(\frac{1}{2}\right)^{n} \sum_{k=0}^{n} 2^{k} - \frac{4}{3} \left(\frac{1}{5}\right)^{n} \sum_{k=0}^{n} 5^{k}$$

$$= \frac{10}{3} \left(\frac{1}{2}\right)^{n} (2^{n+1} - 1)u(n) - \frac{1}{3} \left(\frac{1}{5}\right)(5^{n+1} - 1)u(n)$$

Answer

Q2(4) Sol#

$$X(z) = \frac{1}{(1-2z^{-1})(1-z^{-1})^2}$$

$$= \frac{A}{(1-2z^{-1})} + \frac{B}{(1-z^{-1})} + \frac{Cz^{-1}}{(1-z^{-1})^2}$$

$A = 4, B = -3, C = -1$

Hence, $x(n) = [4(2)^n - 3 - n]u(n)$.

Ans.

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Ans 2(b) #

sol #

$$X(z) = \frac{1}{1-az^{-1}} \quad |z| > |a|$$

Using complex inversion integral

we have

$$x(n) = \frac{1}{2\pi j} \oint_c \frac{z^{n-1}}{1-az^{-1}} dz = \frac{1}{2\pi j} \oint_c \frac{z^n dz}{z-a}$$

Where c is a circle of radius greater than $|a|$. We shall evalute this integral using $f(z) = z^n$. We distinguish two cases

1. if $n \geq 0$, $f(z)$ has only zero and hence no poles inside c. The only pole inside c is $z = a$ Hence

$$x(n) = f(z_0) = a^n \quad n \geq 0$$

2. if $n < 0$, $f(z) = z^n$ has an nth-order pole at $z = 0$ which is also inside there are contributions from both poles

For $n = -1$ we have

$$x(-1) = \frac{1}{2\pi j} \oint_c \frac{1}{z(z-a)} dz = \frac{1}{z-a}\Big|_{z=0} + \frac{1}{z}\Big|_{z=0} = 0$$

if $n = -2$ we have

$$x(-2) = \frac{1}{2\pi j} \oint_c \frac{1}{z^2(z-a)} dz = \frac{d}{dz}\left(\frac{1}{z-a}\right)\Big|_{z=0} + \frac{1}{z^2}\Big|_{z=0} = 0$$

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By continuing in the same way, we can show that

$x(n) = 0$ for $n < 0$ thus

$x(n) = a^n u(n)$

Answer

Q3(a) Soln

At $w = 0$ we have

$$H(0) = \frac{b_0}{(1-p)^2} = 1$$

Hence $\quad b_0 = (1-p)^2$

At $w = \pi/4$

$$H\left(\frac{\pi}{4}\right) = \frac{(1-p)^2}{(1-pe^{-j\pi/4})^2}$$

$$= \frac{(1-p)^2}{(1-p\cos\left(\frac{\pi}{4}\right) + jp\sin\left(\frac{\pi}{4}\right))^2}$$

$$= \frac{(1-p)^2}{(1-p/\sqrt{2} + jp/\sqrt{2})^2}$$

Hence $$\frac{(1-p)^4}{[(1-p/\sqrt{2})^2 + p^2/2]^2} = \frac{1}{2}$$

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$$\sqrt{2}\,(1-p)^2 = 1+p^2-\sqrt{2}\,p$$

The value of $p = 0.32$ satisfied this equation

$$H(z) = \frac{0.46}{(1-0.32z^{-1})^2}$$ Answer.

Q3(b) Solution # the filter must have poles at

$$p_{1,2} = re^{\pm j\pi/2}$$

and zero at $z=1$ and $z=-1$. Consequently the system is

$$H(z) = G\,\frac{(z-1)(z+1)}{(z-jr)(z+jr)}$$

$$= G\,\frac{z^2-1}{z^2+r^2}$$

The gain factor is determine by evaluating the frequency $H(\omega)$ of the filter at $\omega = \pi/2$. Thus we have

$$H\left(\frac{\pi}{2}\right) = G\,\frac{2}{1-r^2} = 1$$

$$G = \frac{1-r^2}{2}$$

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The value of r is determined by evaluating $H(\omega)$ at $\omega = 4\pi/9$. Thus we have

$$\left|H\left(\frac{4\pi}{9}\right)\right|^2 = \frac{(1-r^2)^2}{4} \frac{2-2\cos(8\pi/9)}{1+r^2+2r^2\cos(8\pi/9)} = \frac{1}{2}$$

or equivalently

$$1.94(1-r^2)^2 = 1-1.88r^2+r^4$$

The value of $r^2 = 0.7$ satisfied this equation. Therefore the system function for the desired filter is

$$H(z) = 0.15 \frac{1-z^{-2}}{1+0.7z^{-2}}$$

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Q 4(A) :- Sol#

The Fourier transform of the sequence is

$$X(\omega) = \sum_{n=0}^{L-1} x(n) e^{-j\omega n}$$

$$= \sum_{n=0}^{L-1} e^{-j\omega n} = \frac{1-e^{-j\omega L}}{1-e^{-j\omega}} = \frac{\sin(\omega L/2)}{\sin(\omega/2)} e^{-j\omega(L-1)/2}$$

Hence

$$X(k) = \frac{1-e^{-j2\pi k L/N}}{1-e^{-j2\pi k/N}} \qquad k = 0, 1, \cdots N-1$$

$$= \frac{\sin(\pi k L/N)}{\sin(\pi k/N)} e^{-j\pi k(L-1)/N}$$

if N is selected such that N = L, then the DFT becom

$$X(k) = \begin{cases} L & k = 0 \\ 0 & k = 1, 2, \ldots L-1 \end{cases}$$

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Q 4(b) Solution # each Sequence consist of four non zero points for the purpose of illustrating the operation involved in circular convolution.

Now $x_3(m)$ is obtained by circularly convolving $x_1(n)$ with $x_2(n)$ as specified by. Beginning with $m=0$ we have.

$$x_3(0) = \sum_{n=0}^{3} x_1(n)\, x_2((-n))_N$$

The folded sequence is simply $x_2(n)$ graphed in a clockwise direction.

The product sequence is obtained by multiplying $x_1(n)$ with $x_2((-n))_4$. point by point. Finally we sum the values in the product sequence is obtained

$$x_3(0) = 14$$

For $m=1$ we have

$$x_3(1) = \sum_{n=0}^{3} x_1(n)\, x_2((1-n))_4$$

it is easily verified that $x_2((1-n))_4$ is simply the sequence $x_2((-n))_4$. rotated counter clockwise by one unit in time. Finally we sum the value is the product sequence to obtain

$x_3(1) = 16$ 10

For $m=2$ we have

$$x_3(2) = \sum_{n=0}^{3} x_1(n)\, x_2((2-n))_4$$

Now $x_2((2-n))_4$ is the folded sequence is rotated two units of times in the counterclock wise.

$$x_3(2) = 14$$

For $m=3$ we have

$$x_3(3) \sum_{n=0}^{3} x_1(n)\, x_2((3-n))_4$$

$$x_3(3) = 16$$

$$x_3(n) = \{14, 16, 14, 16\}$$

$$x_3(m) = \sum_{n=0}^{N-1} x_2(n)\, x_1((m-n))_N \quad m = 0, 1, \dots N-1$$

The following example series to illustrate the computation of $x_3(n)$ by means of the DFT & IDFT

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(a)

$x_1(1)=1$

$x_1(2)=2$ — $x_1(n)$ — $x_1(0)=2$

$x_1(3)=1$

$x_2(1)=2$

$x_2(2)=2$ — $x_2(n)$ — $x_2(0)=1$

$x_2(3)=4$

(b)

$x_2(3)=4$

$x_2(2)=3$ — $x_2((-n))_4$ — $x_2(0)=1$

$x_2(1)=2$

Folded sequency

4

6 — $x_1(n)\, x_2((-n))_4$ — 2

2

product sequence

(c)

$x_2(0)=1$

$x_2(3)=4$ — $x_2((1-n))_4$ — $x_2(1)=2$

$x_2(1)=2$

Folded sequence rotated by one unit in time

1

8 — $x_1(n)x_2((1-n))_4$ — 4

3

Product sequence

(d)

$x_2(1)=2$

$x_2(0)=1$ — $x_2((2-n))_4$ — $x_2(2)=3$

$x_2(3)=4$

2

2 — $x_1(n)x_2((2-n))_4$ — 6

4

produt sequence

$x_2(2)=3$

$x_2(1)=2$ — $x_2((3-n))_4$ — $x_2(3)=4$

$x_2(0)=1$

(e)

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3

4 — $x_1(n)x_2((3-n))_4$ — 8

1

product sequence

Folded sequence rotated by three in time.