

**Department of Electrical Engineering**  
**Final Exam Assignment**  
**Date: 27/06/2020**

**Course Details**

**Course Title: Digital Signal Processing**  
**Instructor: Pir Mehar Ali Shah**

**Module: 6th**  
**Total Marks: 50**

**Student Details**

**Name: Junaid Ur Rehman**

**Student ID: 11484**

Q1.	(a)	Determine the response $y(n)$ , $n \geq 0$ , of the system described by the second order difference equation  $y(n) - 4y(n - 1) + 4y(n - 2) = x(n) - x(n - 1)$ <p>To the input <math>x(n) = (-1)^n u(n)</math>. And the initial conditions are <math>y(-1) = y(-2) = 0</math>.</p>	<b>Marks</b> <b>7</b>
			<b>CLO</b> <b>2</b>
Q1.	(b)	Determine the impulse response and unit step response of the systems described by the difference equation.  $y(n) - 0.7y(n - 1) + 0.1y(n - 2) = 2x(n) - x(n - 2)$	<b>Marks</b> <b>7</b>
			<b>CLO</b> <b>2</b>
Q2.	(a)	Determine the causal signal $x(n)$ having the z-transform  $X(z) = \frac{1}{(1 - 2z^{-1})(1 - z^{-1})^2}$ <p>(Hint: Take inverse z-transform using partial fraction method)</p>	<b>Marks</b> <b>6</b>
			<b>CLO</b> <b>2</b>
	(b)	Evaluate the inverse z- transform using the complex inversion integral  $X(z) = \frac{1}{1 - az^{-1}} \quad  z  >  a $	<b>Marks</b> <b>6</b>
			<b>CLO</b> <b>2</b>
		A two- pole low pass filter has the system response	<b>Marks</b> <b>6</b>

Q.3	(a)	$H(z) = \frac{b_0}{(1 - pz^{-1})^2}$ <p>Determine the values of <math>b_0</math> and <math>p</math> such that the frequency response <math>H(\omega)</math> satisfies the condition <math>H(0) = 1</math> and <math> H(\pi) ^2 = 1</math>.</p>	<b>CLO</b> <b>3</b>
	(b)	<p>Design a two-pole bandpass filter that has the center of its passband at <math>\omega = \pi/2</math>, zero in its frequency response characteristics at <math>\omega = 0</math> and <math>\omega = \pi</math> and its magnitude response in <math>\frac{1}{\sqrt{2}}</math> at <math>\omega = 4\pi/9</math>.</p>	<b>Marks</b> <b>6</b> <b>CLO</b> <b>3</b>
	(a)	<p>A finite duration sequence of Length <math>L</math> is given as</p> $x(n) = \begin{cases} 1, & 0 \leq n \leq L - 1 \\ 0, & \text{otherwise} \end{cases}$ <p>Determine the <math>N</math>- point DFT of this sequence for <math>N \geq L</math></p>	<b>Marks</b> <b>6</b> <b>CLO</b> <b>2</b>
Q 4	(b)	<p>Perform the circular convolution of the following two sequences. Solve the problem step by step</p> $x_1(n) = \{2, 1, 2, 1\}$ $x_2(n) = \{1, 2, 3, 4\}$	<b>Marks</b> <b>6</b> <b>CLO</b> <b>2</b>

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Q10 Ans #

Sol<sup>n</sup>  $y(n) - 4y(n-1) + 4y(n-2) = x(n) - x(n-1)$

The character equation is:

$$\lambda^2 - 4\lambda + 4 = 0$$

$\lambda = 2, 2$  hence

$$y_h(n) = C_1 2^n + C_2 n 2^n$$

The particular solution is:

$$y_p(n) = k (-1)^n u(n)$$

Substituting this solution into different equation  
we obtain:

$$k (-1)^n u(n) - 4k (-1)^{n-1} u(n-1) + 4k (-1)^{n-2} u(n-2) =$$
$$(-1)^n u(n) - (-1)^{n-1} u(n-1)$$

for  $n = 2, k(1 + 4 + 4) = 2 \Rightarrow k = \frac{2}{9}$  The total solution is

$$y(n) \left[ C_1 2^n + C_2 n 2^n + \frac{2}{9} (-1)^n \right] u(n)$$

from the initial condition, we obtain  $y(0) = 1, y(1) = 2$  Then

$$C_1 + \frac{2}{9} = 1$$

$$\Rightarrow C_1 = \frac{7}{9}$$

$$2C_1 + 2C_2 - \frac{2}{9} = 2$$

$$\Rightarrow C_2 = \frac{1}{3}$$

(2)

1b # Sol#

~~$y(n) - 0.7y(n-1) + 0.1y(n-2) = 2x(n) - x(n-2)$~~

$$y(n) - 0.7y(n-1) + 0.1y(n-2) = 2x(n) - x(n-2)$$

The characteristic equation is

$$\lambda^2 - 0.7\lambda + 0.1 = 0$$

$\lambda = \frac{1}{2}, \frac{1}{5}$  hence.

$$y_h(n) = c_1 \left(\frac{1}{2}\right)^n + c_2 \left(\frac{1}{5}\right)^n$$

With  $x(n) = \delta(n)$ , we have

$$y(0) = 2$$

$$y(1) = 0.7y(0) = 0 \Rightarrow y(1) = 1.4$$

Hence  $c_1 + c_2 = 2$  and

$$\frac{1}{2}c_1 + \frac{1}{5}c_2 = 1.4 = \frac{7}{5}$$

$$\Rightarrow c_1 + \frac{2}{5}c_2 = \frac{14}{5}$$

These equations yield

$$c_1 = \frac{10}{3}, c_2 = -\frac{4}{3}$$

$$h(n) = \left[ \frac{10}{3} \left(\frac{1}{2}\right)^n - \frac{4}{3} \left(\frac{1}{5}\right)^n \right] u(n)$$

(3)

$$\begin{aligned} f(n) &= \sum_{k=0}^n h(n-k) \\ &= \frac{10}{3} \sum_{k=0}^n \left(\frac{1}{2}\right)^{n-k} - \frac{4}{3} \sum_{k=0}^n \left(\frac{1}{3}\right)^{n-k} \\ &= \frac{10}{3} \left(\frac{1}{2}\right)^n \sum_{k=0}^n 2^k - \frac{4}{3} \left(\frac{1}{3}\right)^n \sum_{k=0}^n 3^k \\ &= \frac{10}{3} \left(\frac{1}{2}\right)^n (2^{n+1} - 1)u(n) - \frac{4}{3} \left(\frac{1}{3}\right)^n (3^{n+1} - 1)u(n) \end{aligned}$$

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Q2 (a) Soln

$$\begin{aligned} X(z) &= \frac{1}{(1-2z^{-1})(1-z^{-1})^2} \\ &= \frac{A}{(1-2z^{-1})} + \frac{B}{(1-z^{-1})} + \frac{C}{(1-z^{-1})^2} \\ A &= 4, B = -3, C = -1 \end{aligned}$$

Hence,  $x(n] = [4(2)^n - 3 - n] u(n)$ .

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(4)

ANS 2(b) #

sol#

$$X(z) = \frac{1}{1-az^{-1}} \quad |z| > |a|$$

Using complex inversion integral

we have

$$x(n) = \frac{1}{2\pi j} \oint_C \frac{z^{n-1}}{1-az^{-1}} dz = \frac{1}{2\pi j} \oint_C \frac{z^n dz}{z-a}$$

where  $C$  is a circle of radius greater than  $|a|$ . We shall evaluate this integral using  $f(z) = z^n$ . We distinguish two cases

1- i)  $n \geq 0$ ,  $f(z)$  has only zero and hence no poles inside  $C$ .

The only pole inside  $C$  is  $z = a$  hence

$$x(n) = f(a) = a^n \quad n \geq 0$$

2) if  $n < 0$ ,  $f(z) = z^n$  has an  $m$ th-order pole at  $z=0$  which is also inside there are contributions from both poles

For  $n = -1$  we have

$$x(-1) = \frac{1}{2\pi j} \oint_C \frac{1}{z(z-a)} dz = \frac{1}{z-a} \left[ \frac{1}{z} \right]_{z=0}^{z=0} = 0$$

if  $n = -2$  we have

$$x(-2) = \frac{1}{2\pi j} \oint_C \frac{1}{z^2(z-a)} dz = \frac{d}{dz} \left( \frac{1}{z-a} \right) \left[ \frac{1}{z^2} \right]_{z=0}^{z=0} = 0$$

By continuing in the same way, we can show that  
 $x(n) = 0$  for  $n < 0$  thus  
 $x(n) = a^n u(n)$ .

Answer.

3(a) Soln

At  $\omega = 0$  we have

$$H(0) = \frac{b_0}{(1-p)^2} = 1$$

Hence  $b_0 = (1-p)^2$

At  $\omega = \pi/4$

$$H\left(\frac{\pi}{4}\right) = \frac{(1-p)^2}{(1 - pe^{-j\pi/4})^2}$$

$$= \frac{(1-p)^2}{(1 - p \cos(\frac{\pi}{4}) + jp \sin(\frac{\pi}{4}))^2}$$

$$= \frac{(1-p)^2}{(1 - p/\sqrt{2} + jp/\sqrt{2})^2}$$

Hence  $\frac{(1-p)^4}{[(1 - p/\sqrt{2})^2 + p^2/2]} = \frac{1}{2}$

$\frac{1}{2}$



(6)

$$\sqrt{2} (1-p)^2 = 1+p^2 - \sqrt{2}p$$

The value of  $p = 0.32$  satisfied this equation

$$H(z) = \frac{0.46}{(1-0.32z^{-1})^2} \text{ answer.}$$

Q 3(b) Solution # the filter must have poles at

$$p_{1,2} = re^{n\pi/2}$$

and zero at  $z=1$  and  $z=-1$ . (consequently the system is

$$\begin{aligned} H(z) &= G \frac{(z-1)(z+1)}{(z-jr)(z+jr)} \\ &= G \frac{z^2-1}{z^2+r^2} \end{aligned}$$

The gain factor is determine by evaluating the frequency  $H(\omega)$  of the filter at  $\omega = \pi/2$ . Thus we have

$$H\left(\frac{\pi}{2}\right) = G \frac{2}{1-r^2} = 1$$

$$G = \frac{1-r^2}{2}$$



(7)

The value of  $r$  is determined by evaluating  $H(\omega)$  at  
 $\omega = 4\pi/9$ . Thus we have

$$\left| H\left(\frac{4\pi}{9}\right) \right|^2 = \frac{(1-r^2)^2}{4} \frac{2-2\cos(8\pi/9)}{1+r^2+2r^2\cos(8\pi/9)} = \frac{1}{2}$$

or equivalently

$$1.94(1-r^2)^2 = 1 - 1.88r^2 + r^4$$

The value of  $r^2 = 0.7$  satisfied this equation. Therefore  
the system function for the desired filter is

$$H(z) = 0.15 \frac{1-z^{-2}}{1+0.7z^{-2}}$$

(8)

Q 4(A) :- Sol#

The Fourier transform of the sequence is

$$\begin{aligned} X(\omega) &= \sum_{n=0}^{L-1} x(n) e^{-j\omega n} \\ &= \sum_{n=0}^{L-1} e^{-j\omega n} = \frac{1 - e^{-j\omega L}}{1 - e^{-j\omega}} = \frac{\sin(\omega L/2)}{\sin(\omega/2)} e^{-j\omega(L-1)/2} \end{aligned}$$

Hence

$$X(k) = \frac{1 - e^{-j2\pi k L/N}}{1 - e^{-j2\pi k/N}} \quad k = 0, 1, \dots, N-1$$

$$= \frac{\sin(\pi k L/N)}{\sin(\pi k/N)} e^{-j\pi k(L-1)/N}$$

if  $N$  is selected such that  $N=L$ , then the DFT becomes

$$X(k) = \begin{cases} L & k=0 \\ 0 & k=1, 2, \dots, N-1 \end{cases}$$

(9)

Q4(b) Solution # each sequence consist of four non-zero points for the purpose of illustrating the operation involved in circular convolution.

New  $x_3(m)$  is obtained by circularly convolving  $x_1(n)$  with  $x_2(n)$  as specified by. Beginning with  $m=0$  we have.

$$x_3(0) = \sum_{n=0}^3 x_1(n) x_2((1-n))_4$$

The folded sequence is simply  $x_2(n)$  graphed in a clockwise direction.

The product sequence is obtained by multiplying  $x_1(n)$  with  $x_2((1-n))_4$  point by point. Finally we sum the values in the product sequence is obtained

$$x_3(0) = 14$$

For  $m=1$  we have

$$x_3(1) = \sum_{n=0}^3 x_1(n) x_2((1-n))_4$$

it is easily verified that  $x_2((1-n))_4$  is simply the sequence  $x_2((1-n))_4$  rotated counter clockwise by one unit in time.

Finally we sum the value is the product sequence to obtain

~~14~~



$$x_3(1) = 16$$

(10)

For  $m=2$  we have

$$x_3(2) = \sum_{n=0}^3 x_1(n) x_2((2-n))_4$$

Now  $x_2((2-n))_4$  is the folded sequence is rotated two units of times in the counterclockwise.

$$x_3(2) = 14$$

For  $m=3$  we have

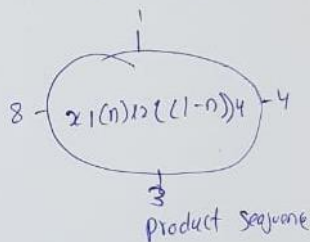
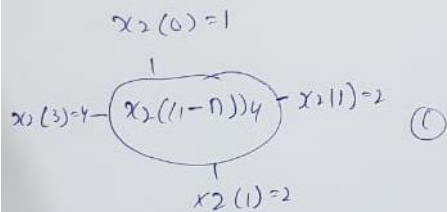
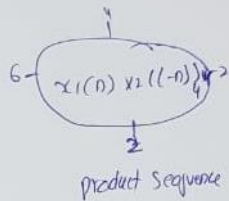
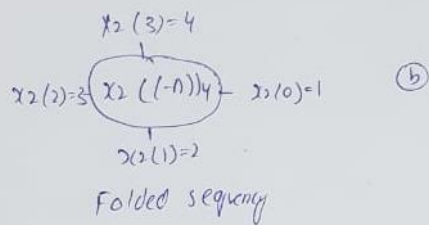
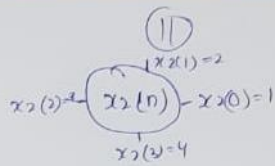
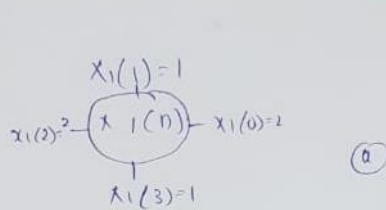
$$x_3(3) = \sum_{n=0}^3 x_1(n) x_2((3-n))_4$$

$$x_3(3) = 16$$

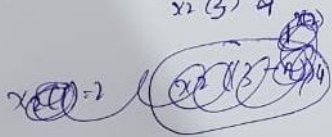
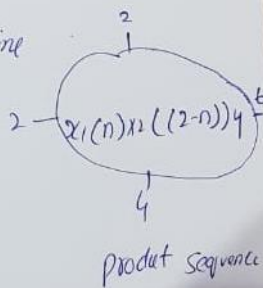
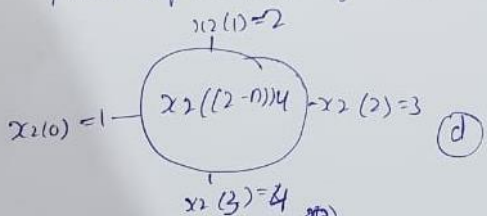
$$x_3(n) = \{14, 16, 14, 16\}$$

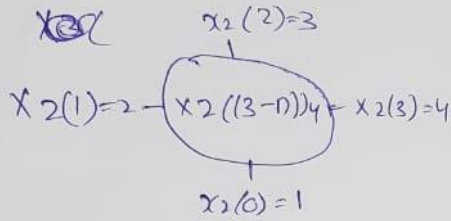
$$x_3(m) = \sum_{n=0}^{N-1} x_2(n) x_1((m-n))_{N-m} \quad m = 0, 1, \dots, N-1$$

The following example series is illustrate the computation of  $x_3(n)$  by means of the DFT & [DFT]

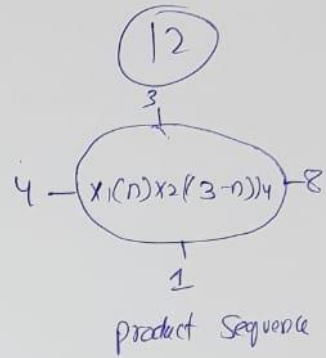


Folded sequence rotated by one unit in time





(e)



Folded Sequence related by three  
 in time.

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