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I.D :-

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SEC :-

"C"

SUBMITED TO :-

AMJAD ISLAM

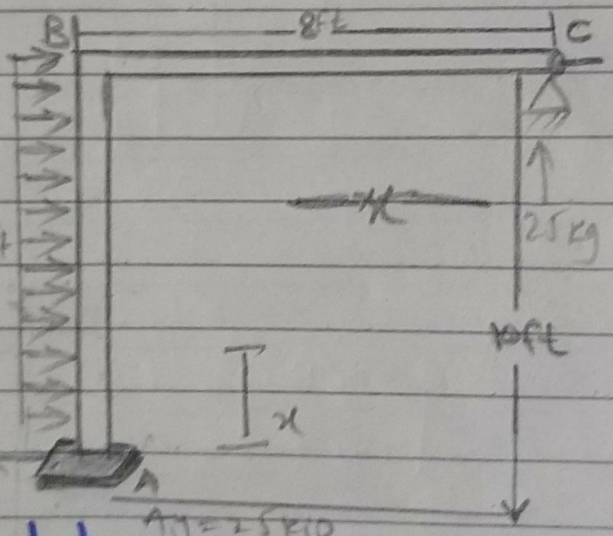
SUBJECT :-

STRUCTURAL ANALYSIS

Question # 1

Determine the vertical displacement of free end point C on the frame shown in figure Take $E = 29(10)^3$ ksi $= 600\ in^4$ for both members. Use method of virtual work.

Figr



Given data:-

- $\Delta_{C_y} = ?$
- $E = 29 \times 10^3$ ksi
- $I = 600\ in^4$

Sol:-

Finding Reaction

$\zeta + \Sigma M_A = 0$

$-4(10)(5) + C_y(8) = 0$

$\boxed{C_y = 25\ kip}$

$\Sigma F_y = 0 \uparrow^+$

$25 + A_y = 0$

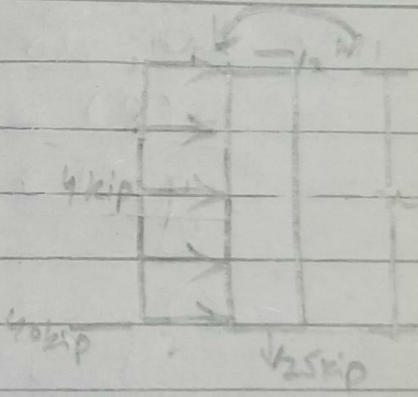
$$\boxed{A_y = -25 \text{ kips}}$$

$\Sigma F_x = 0 \quad \rightarrow^+$

$40 - A_C = 0$

$$\boxed{A_C = 40 \text{ kips}}$$

Taking Section

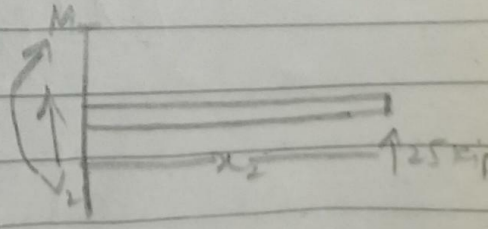


Real moments :-

$\zeta^+ \quad \Sigma M_1 = 0$

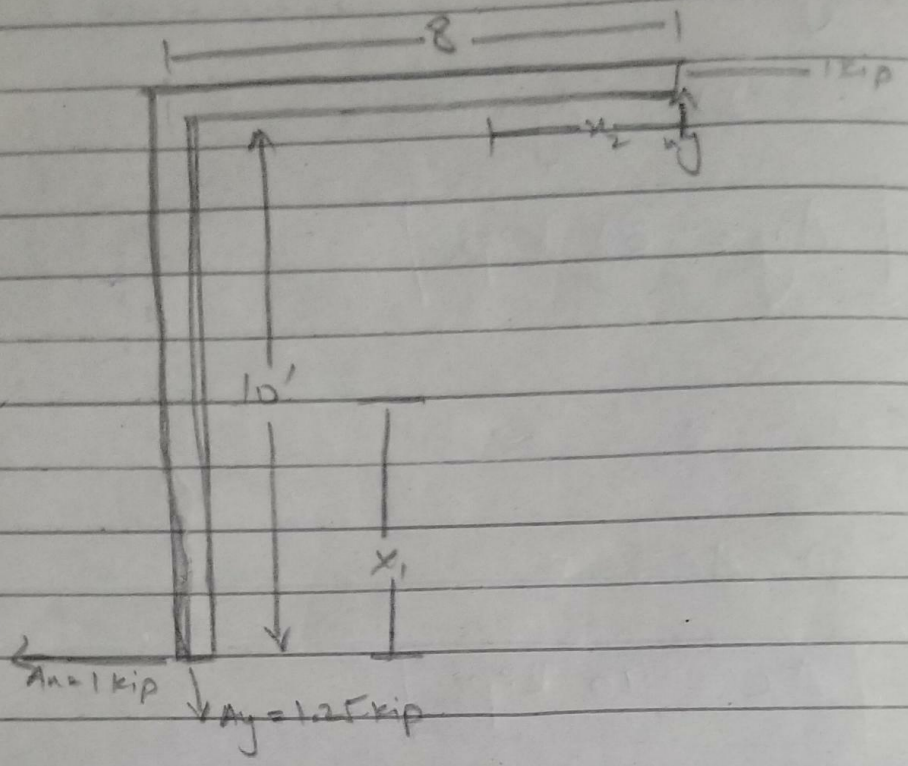
$-40(x_1) + 4x_1(x_1/2) + M_1 = 0$

$$\boxed{M_1 = 40x_1 - 2x_1^2}$$



$-25(x_2) + M_2 = 0$

$$\boxed{M_2 = 25x_2 \text{ kip}}$$



$\circlearrowleft^{+} \Sigma M_A = 0$

$C_y = (8') - 1(10') = 0$

$\boxed{C_y = 1.25 \text{ kips}}$

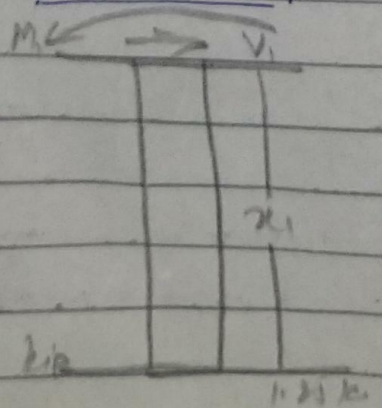
$\Sigma F_y = 0 \uparrow^{+}$

$\boxed{A_y = 1.25 \text{ kip}}$

$\Sigma F_x = 0 \xrightarrow{+}$

$1 + A_x = 0$

$\boxed{A_x = -1}$



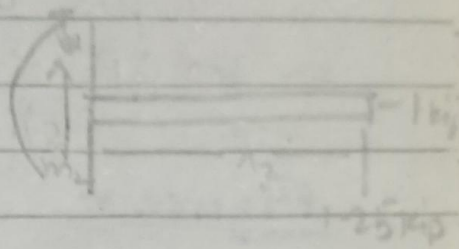
$\zeta^{+} \Sigma M_1 = 0$

$-1(x_1) + m_1 = 0$

$\boxed{m_1 = 1x_1}$

$-m_2 + 1.25x_2 = 0$

$\boxed{m_2 = 1.25x_2}$



Virtual Work Equation:-

$$1.\ \Delta = \int_0^L \frac{M_m}{EI}\, dx$$

$$1K\ \Delta C_x = \int_0^{10} \frac{(40x_1 - 2x_1^2)(1x_1)}{EI}\, dx$$

$$\frac{1}{EI}\left[\int_5^{10}(40x_1^2 - 2x_1^2)\, dx + \int_0^8 (31.25x^2)\right.$$

$$\Delta C_n = \frac{8333.3}{EI} + \frac{5333.3}{EI}$$

$$\Delta C_n = \frac{13666.7}{EI}\ K.ft^2$$

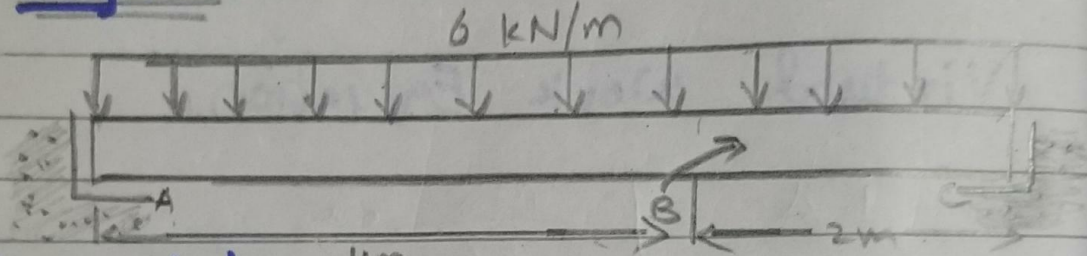
$$13.6667\ \frac{K.ft^2\,(10in)^2\ in^2}{29 \times 10^3\ K/m^2 \times 600\ in}$$

$$\boxed{\Delta C_n = 1.36\,in}$$

Question :- 2.

Determine the slope and displacement at point B. Assume the support at A is a pin and C roller. Take $E = 200\ GPa$, $I = 60(10)^6\ mm^4$. Use Castigliano Theorm.

Figure r



Given data:-

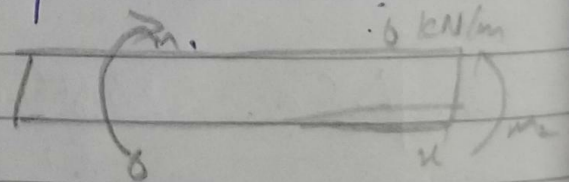
$E = 200\ GPa$

$I = 60 \times 10^6\ mm^4$

Required:-

Slope and displacement =?

Sol:-



$$m_1 - m_2 = \frac{1}{2}(x_2)(6 + x_1)$$

$$m_1 = m' + \frac{6x^2 + x_2^2}{2}$$

$$m' = -m' + 3x^2 + \frac{x_2^2}{2}$$

taking partial derivative with respect to m

$$\frac{\partial m_2}{\partial P} = -x$$

Slop :-

$$m + \tfrac{1}{2} \times (6x_1) = 0$$

$$m = -\tfrac{1}{2} x \quad (6x_2) = 3x^2$$

So

$$\frac{\partial m_1}{2m_1'} = 0$$

$$m_1' - m_2 - \tfrac{1}{2}(x_2)(6 + x_2)$$

$$m_2 = -m' + 6x_2 + x_2^2$$

$$m = -m' + 3x^2 + x^2/2$$

$$\frac{2m_2}{2m_1} = \frac{1}{}$$

$$\Rightarrow \int_0^{x} \frac{-3x^2}{E \cdot I} + \int_0^{x} \left(-2 + 6x^2 + x^2/2\right) dx$$

$$DB = \int_0^{L} \frac{m(\partial m)}{2P} \cdot \frac{dx}{E}$$

$$= \int_0^6 \frac{-3x^2(-x)\,dx}{EI} + \int_0^4 \frac{-3x^2-(x)\,dx}{EI}$$

$$D_B = \frac{-3x^2}{4EI}\Big|_0^6 + \frac{-3x^4}{4EI}\Big|_0^4$$

Put the value of EI and I

$$\frac{-3x^2}{2(200)(6\times10^6)}\Big|_0^6 + \frac{-3x^4}{4(200)(60\times10^6)}\Big|_0^4$$

$$\Rightarrow \frac{-216\ KN\cdot ft^3}{4.8\times60} + \frac{-614.4\ KN\cdot ft^3}{4.8\times60}$$

$$\Rightarrow -4.5\times10^{-9} + (-1.28\times10^{-8})$$

$$\rightarrow D_B = 5.76\times10^{-10}\ inch$$

Displacement

Slope :-

$$m + \tfrac{1}{2}\times(6x_1) = 0$$

$$m = -\tfrac{1}{2}x(6x_2) = 3x^2$$

So

$$\frac{2m_2}{2m_1} = 0$$

$$m_1 - m_2 - \tfrac{1}{2}(x_2)(6 + x_2)$$

$$m_2 = -m' + 6x_2 + x_2^2$$

$$m_2 = -m' + 3x^2 + x_2^2/2$$

$$\frac{2m_2}{2m_1} = 1$$

$$\int_0^6 \frac{-3x^2\,dx}{E \cdot I} + \int_0^{10} \frac{-2 + 6x^2 + x^2}{2}\,dx$$

So

$$\Rightarrow 0 + \left(-x + \frac{6x^3}{3} + \frac{x^3}{6}\right)\Bigg|_0^{10} \left(\frac{1}{EI}\right)$$

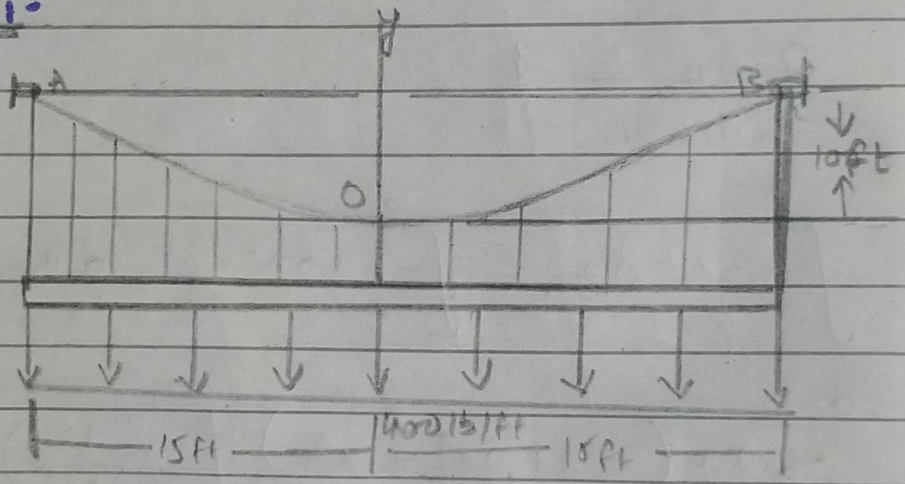
$$\frac{1}{(200)(60 \times 10^6)} \left(-x + \frac{6x^3}{3} + \frac{x^3}{6}\right)\Bigg|_0^{10}$$

$$\Rightarrow \quad a = 4.125 \times 10^{-7} \text{ Inch}$$

Question: 3

The cable is subjected to the uniform loading. If the slope of cable at point O is zero determine the equation of the curve and force in the cable at O and B

Fig:-



Solution:-

From Eq: 5-9

$$y = \frac{h}{L^2} x^2 \Rightarrow \frac{10}{(15)^2} x^2$$

$$\boxed{y = 0.044 x^2}$$

Now we know that

F

$$T_o = F_H = \frac{w_o L^2}{2h}$$

$$= \frac{400(15)^2}{2(10)}$$

$T_o \Rightarrow 4500\,lb \Rightarrow 4.5K$

Now

From Eq 5-10

$$T_B = T_{max} = \sqrt{(FU)^2 + (WOL)^2}$$

$$\Rightarrow \sqrt{(4500)^2 + [(400)(15)]^2}$$

$$\sqrt{20250000 + (400 \times 15)^2}$$

So

$\Rightarrow 7500\,lb \div$ ing by 1000

$\Rightarrow 7500\,lb \Rightarrow 7.50\,K$

OR

From Eq 5-11

$$T_B \Rightarrow T_{max} = WOL\sqrt{1+\left(\frac{L}{2h}\right)^2}$$

Putting Values

$$\Rightarrow 400(15)\sqrt{1+\left(\frac{15}{2(10)}\right)^2}$$

$$\Rightarrow 6000(1.25)$$

$T_{max} \Rightarrow 7500\,lb \Rightarrow 7.50\,K$

$T_{max} \Rightarrow 7.5K$

Question 4.

The three-higed spandrel arch is subjected to the uniform load of 30 kN/m. Determine the internal moment in the arch at point D.

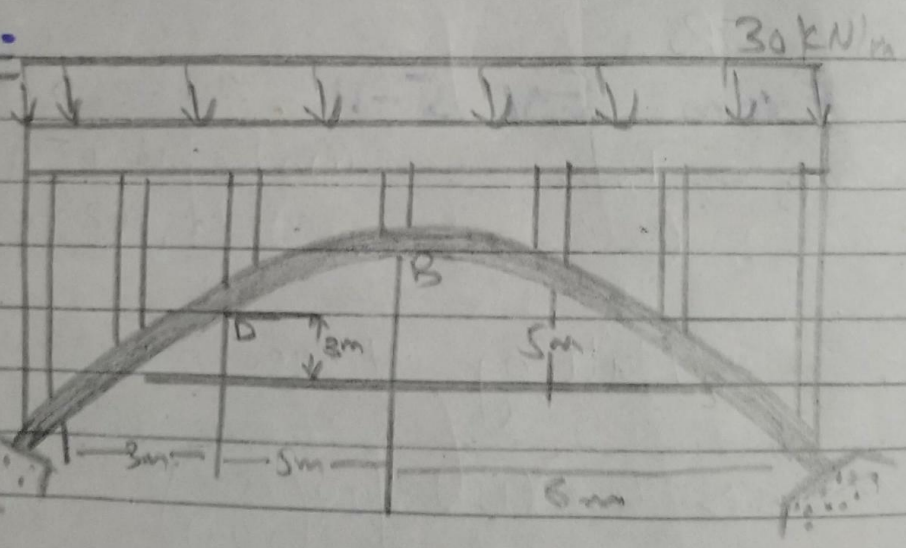
Given data:-

Uniform load ⇒ 30 kN/m

Required:-

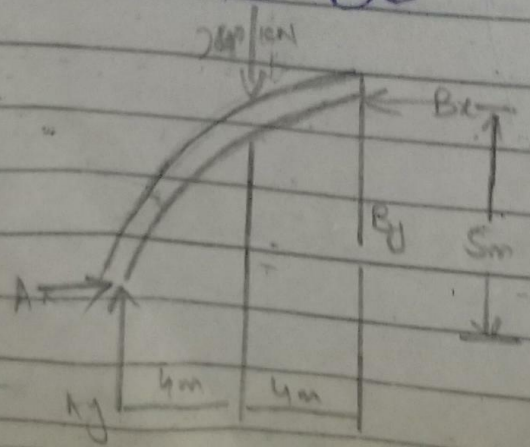
Internal moment at D =?

Fig :-

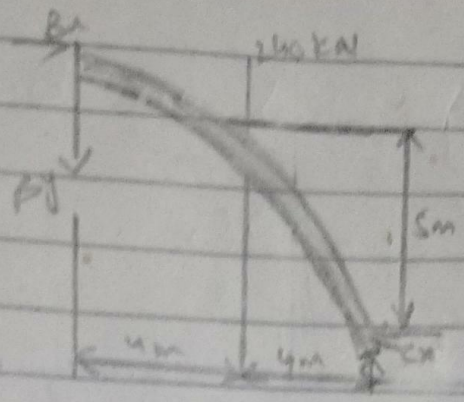


Sol :-

. Dividing in-to two members AB and BC



Member AB



Member BC

AB :-

$$\circlearrowleft + \Sigma M_A = 0 \quad Bx(5) + By(8) - 240(4) = 0 \quad -(a)$$

BC :-

$$\circlearrowleft \Sigma M_C = 0 \quad -Bx(5) + By(8) + 240(4) = 0 \quad -(b)$$

Adding eqv a and b

$$\begin{array}{r} Bx(5) + By(8) - 240(4) = 0 \\ -Bx(5) + By(8) + 240(4) = 0 \\ \hline 0 + 2By(8) + 0 = 0 \end{array}$$

$$2By(8) = 0$$

$$\boxed{By = 0 \text{ KN}}$$

Putting the value of by in eqv (b)

So

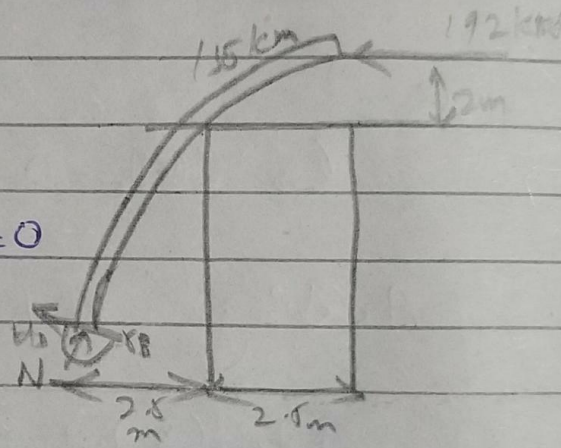
$$-Bx(5) + By(8) + 960 = 0$$
$$-Bx(5) + 0(8) + 960 = 0$$
$$Bx5 \Rightarrow 960$$

Dividing 5 on b.s

$$\frac{B_x \cancel{5}}{\cancel{5}} = \frac{960}{5}$$

$$\boxed{B_x = 192 \text{ KN}}$$

Now at Segment DB



$\curvearrowright^{+} \quad \Sigma M_D = 0$

$\Rightarrow 192(2) - 150(2.5) - M_D = 0$

$\Rightarrow 384 - 375 - M_D = 0$

$9 - M_D = 0$

$$\boxed{M_D = 9 \text{ KN.M}}$$

Member DB