By the Name of Almighty Allah who is most Merciful and Compassionate.

Name => Hidayatullah

ID => 16495

Section => "B"

Semester => 2nd

Examination => Mid-Term

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Teacher => Sir M. Shakeel

Subject => Linear Algebra

Q 1. My ID : 16495

Solution:

$$\begin{bmatrix} 1 & ID3 & 3 & 0 & 5 \\ 0 & 1 & -ID\cdot last & 0 & 7 \\ 0 & 0 & 1 & 0 & -6 \\ 0 & 0 & 0 & 1 & ID3 \end{bmatrix}$$

$$\left[\begin{array}{cccc|c} 1 & 4 & 3 & 0 & 5 \\ 0 & 1 & -5 & 0 & 7 \\ 0 & 0 & 1 & 0 & -6 \\ 0 & 0 & 0 & 1 & 4 \end{array}\right]$$

$$\underset{R}{\sim} \left[\begin{array}{cccc|c} 1 & 0 & 23 & 0 & -23 \\ 0 & 1 & -5 & 0 & 7 \\ 0 & 0 & 1 & 0 & -6 \\ 0 & 0 & 0 & 1 & 4 \end{array}\right] \quad R_1 - 4R_2$$

$$\underset{R}{\sim} \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 115 \\ 0 & 1 & 0 & 0 & -23 \\ 0 & 0 & 1 & 0 & -6 \\ 0 & 0 & 0 & 1 & 4 \end{array}\right] \quad \begin{array}{l} R_1 - 23R_3 \\ R_2 + 5R_3 \end{array}$$

$x_1 = 115$

$x_2 = -23$

$x_3 = -6$

$x_4 = 4$

Q 2: Part "A"

Solution: $\begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0 & 2 & -5 & -1 \end{bmatrix}, \begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0 & 0 & 3 & -5 \end{bmatrix}$

(i)

$$\begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0 & 2 & -5 & -1 \end{bmatrix} \quad L.H.S$$

Multiply Row two by "-2" and then add to Row three

$$\begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0 & 2-2 & -5+8 & -1-4 \end{bmatrix} \quad R3-2R2$$

$$\begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0 & 0 & 3 & -5 \end{bmatrix}$$

(ii)

$$\begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0 & 0 & 3 & -5 \end{bmatrix} \quad R.H.S$$

Multiply Row two by "2" and then add to Row one

$$\begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0 & 0+2 & -8+3 & 4-5 \end{bmatrix} \quad R_3 + 2R_2$$

$$\begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0 & 2 & -5 & -1 \end{bmatrix}$$

Q2 : Part "B"

(a). $\begin{bmatrix} e & 0 & 0 & 0 \\ 0 & \pi & 0 & 0 \\ 0 & 0 & \pi & 0 \\ 0 & 0 & 0 & e \end{bmatrix}$ is in echelon form

Answer: Yes, It is in echelon form because of its triangular matrix.

OR

Yes, It is in echelon form because It agrees with all following conditions.

i) All the entries in a column below a leading entry is zero.

(ii) Each leading entry of a row is in a column to the right of the leading entry of the above row

iii) To satisfy the third condition there is no zero-row which should be below all non-zero rows.

b). $\begin{bmatrix} 1 & 0 & \pi \\ 0 & 1 & e \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ is in echelon form

Answer: Yes, It is also in echelon form because of its upper triangular matrix.

OR

It is in reduced echelon form because it is already in echelon form and agrees with given conditions.

i) All the leading entries in non-zero rows are 1.

ii) Each leading 1 is the only non-zero entry in its column.

C). $\begin{bmatrix} 5 & 0 & 0 & 7 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & 4 \end{bmatrix}$ is in reduced row echelon form

Answer: It is not in reduced row echelon form because the diagonals are not identity.
So that's why it is not reduced row echelon form.

D). $\begin{bmatrix} 1 & 0 & 0 & 7 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 4 \end{bmatrix}$ is in reduced row echelon form

Answer: Same as the above answer
It is not in reduced row echelon form because the diagonals.

All the zero rows are below the non-zero rows.

Q3. Part "A"

Answer:

The difference between the row echelon and reduced row echelon form.

Row Echelon Form:

In row echelon form the matrix is not unique which means there are infinite answers when we perform row operation.

For example:

$$\begin{bmatrix} 1 & 5 & -5 & 15 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

OR

$$\begin{bmatrix} 1 & 0 & 2 & 3 & 4 \\ 0 & 1 & 5 & 6 & 7 \\ 0 & 0 & 0 & 1 & 8 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

if a column contains a leading entery then all enteries below that leading entery are zero.

Reduced Row Echelon Form:

A matrix is in reduced row echelon form when it satisfies the following conditions:

i) The leading entery in each non-zero row is 1

ii) Rows with all zero elements if any below rows having a non-zero element.

For example:

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \qquad \begin{bmatrix} 1 & 2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

A B

Q 3. Part "B" ID: 16495

Solution:

$$\begin{bmatrix} 1 & 102 & 8 \\ 2 & 8 & -1 \\ -103 & 0 & 0 \\ 1 & -4 & 10\text{-first-last} \end{bmatrix}$$

$$\underset{\sim}{R} \begin{bmatrix} 1 & 6 & 8 \\ 2 & 8 & -1 \\ -4 & 0 & 0 \\ 1 & -4 & 15 \end{bmatrix}$$

$$\underset{\sim}{R} \begin{bmatrix} 1 & 6 & 8 \\ 0 & -4 & -17 \\ 0 & 24 & 32 \\ 0 & -10 & 7 \end{bmatrix} \quad \begin{matrix} R_2 - 2R_1 \\ R_3 + 4R_1 \\ R_4 - R_1 \end{matrix}$$

$$\underset{\sim}{R} \begin{bmatrix} 1 & 6 & 8 \\ 0 & 1 & \frac{17}{4} \\ 0 & 24 & 32 \\ 0 & -10 & 5 \end{bmatrix} \quad -\frac{1}{4} R_2$$

$$\begin{bmatrix} 1 & 6 & 8 \\ 0 & 1 & 17/4 \\ 0 & 0 & -70 \\ 0 & 0 & 41 \end{bmatrix} \quad \begin{matrix} R_3 - 24R_2 \\ R_4 + 10R_2 \end{matrix}$$

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Thanks My Respectable

Sir....!

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