Mid Exam Summer

Submitted By:

Hamza Riaz (12282)

BS (SE-8) Section: A

Submitted To:

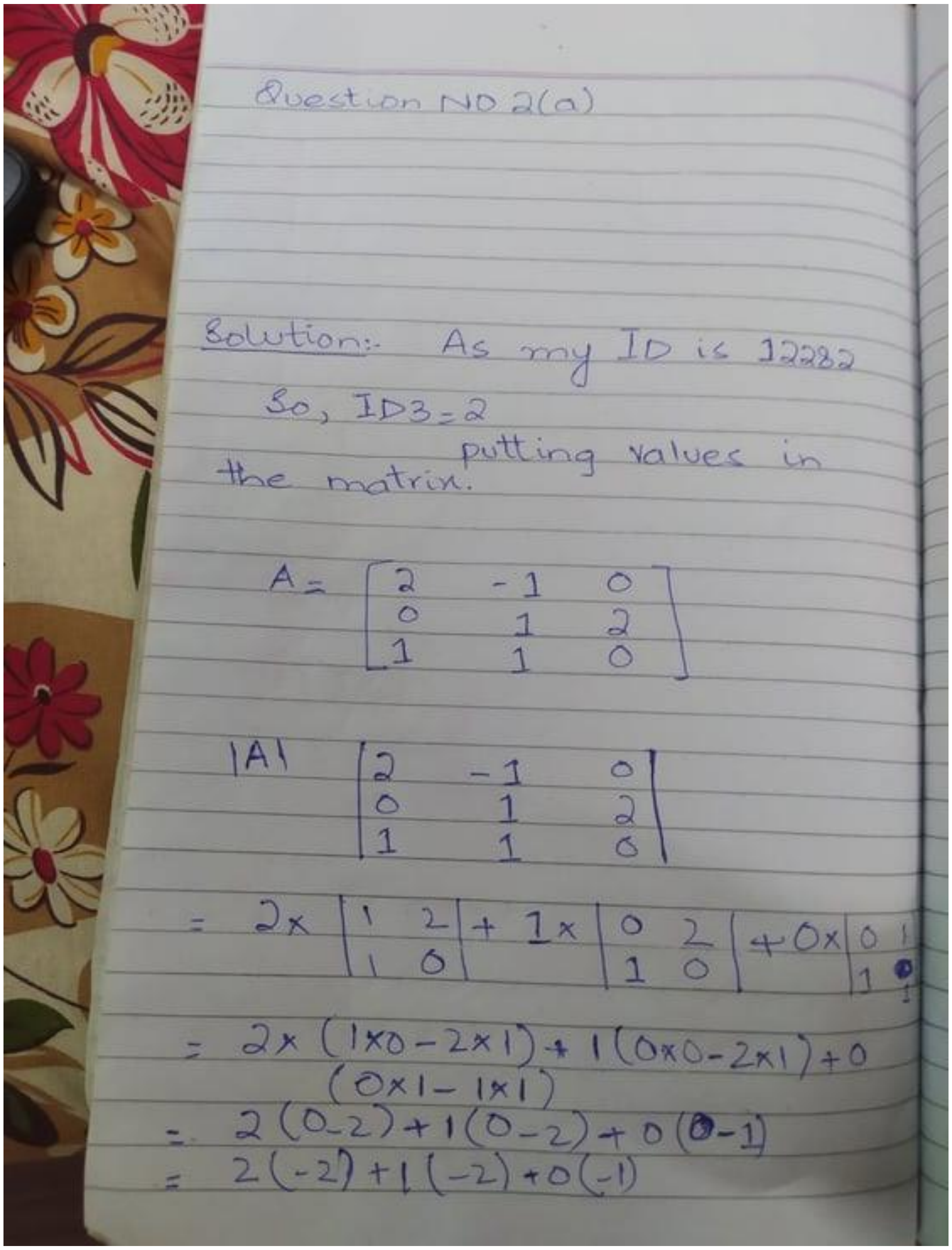
Sir Mansoor

Dated: 21st August 2020

Department of Computer Science,
IQRA National University, Peshawar Pakistan

Question2 part A

Answer:



Question NO 2(a)

Solution:- As my ID is 12282

So, ID3 = 2

putting values in the matrix.

$$A = \begin{bmatrix} 2 & -1 & 0 \\ 0 & 1 & 2 \\ 1 & 1 & 0 \end{bmatrix}$$

$$|A| \begin{vmatrix} 2 & -1 & 0 \\ 0 & 1 & 2 \\ 1 & 1 & 0 \end{vmatrix}$$

$$= 2\times \begin{vmatrix} 1 & 2 \\ 1 & 0 \end{vmatrix} + 1\times \begin{vmatrix} 0 & 2 \\ 1 & 0 \end{vmatrix} + 0\times \begin{vmatrix} 0 & 1 \\ 1 & 1 \end{vmatrix}$$

$$= 2\times(1\times0-2\times1)+1(0\times0-2\times1)+0(0\times1-1\times1)$$

$$= 2(0-2)+1(0-2)+0(0-1)$$

$$= 2(-2)+1(-2)+0(-1)$$

$= -4-2+0$
$= -6$

Now taking adj:-

$$\text{Adj}(A) = \text{Adj}\begin{bmatrix} 2 & -1 & 0 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \\ 1 & 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} +\begin{vmatrix} 1 & 2 \\ 1 & 0 \end{vmatrix} & -\begin{vmatrix} 0 & 2 \\ 1 & 0 \end{vmatrix} & +\begin{vmatrix} 0 & 1 \\ 1 & 1 \end{vmatrix} \\ -\begin{vmatrix} -1 & 0 \\ 1 & 0 \end{vmatrix} & +\begin{vmatrix} 2 & 0 \\ 1 & 0 \end{vmatrix} & -\begin{vmatrix} 2 & -1 \\ 1 & 1 \end{vmatrix} \\ +\begin{vmatrix} -1 & 0 \\ 1 & 2 \end{vmatrix} & -\begin{vmatrix} 2 & 0 \\ 0 & 2 \end{vmatrix} & +\begin{vmatrix} 2 & -1 \\ 0 & 1 \end{vmatrix} \end{bmatrix}$$

$$= \begin{bmatrix} +(1\times0-2\times1) & -(0\times0-2\times1) & +(0\times1-1\times1) \\ -(-1\times0-0\times1) & +(2\times0-0\times1) & -(2\times1-(-1)\times1) \\ +(-1\times2-0\times1) & -(2\times2-0\times0) & +(2\times1-(-1) \end{bmatrix}$$

$$= \begin{bmatrix} +(0-2) & -(0-2) & +(0-1) \\ -(0+0) & +(0\times0) & -(2+1) \\ +(-2+0) & -(4+0) & +(2+0) \end{bmatrix}$$

12282

in

$\times\begin{vmatrix} 0 & 1 \\ 1 & 1 \end{vmatrix}$

-0

$$= \begin{bmatrix} -2 & 2 & -1 \\ 0 & 0 & -3 \\ -2 & -4 & 2 \end{bmatrix}^t$$

$$= \begin{bmatrix} -2 & 0 & -2 \\ 2 & 0 & -4 \\ -1 & -3 & 2 \end{bmatrix}$$

Now taking $A^{-1} = \frac{1}{|A|} \times Adj(A)$

$$= \left(1/-6\right) \begin{bmatrix} -2 & 0 & -2 \\ 2 & 0 & -4 \\ -1 & -3 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 0.3333 & 0 & 0.3333 \\ -0.3333 & 0 & 0.6667 \\ 0.1667 & 0.5 & -0.3333 \end{bmatrix}$$

Question 2 Part B

Answer

Question # 2 (b)

Find an echelon form for the below matrix using row operations

$$\begin{bmatrix} 1 & ID3 & 8 \\ 2 & ID4 & -1 \\ -3 & 0 & 0 \\ 1 & -ID3 & 16 \end{bmatrix}$$

Solution :-

My ID = 12280

So, ID3 = 2 and ID4 = 8

Putting values in the given matrix.

$$= \begin{bmatrix} 1 & 2 & 8 \\ 2 & 8 & -1 \\ -3 & 0 & 0 \\ 1 & -2 & 16 \end{bmatrix}$$

$$= \begin{bmatrix} -3 & 0 & 0 \\ 2 & 8 & -1 \\ 1 & 2 & 8 \\ 1 & -2 & 16 \end{bmatrix}$$ ∴ Interchanging rows. $R_1 \leftrightarrow R_3$

$$= \begin{bmatrix} -3 & 0 & 0 \\ 0 & 8 & -1 \\ 1 & 2 & 8 \\ 1 & -2 & 16 \end{bmatrix}$$ ∴ $R_2 \leftarrow R_2 + 0.6667 \times R_1$

$$= \begin{bmatrix} -3 & 0 & 0 \\ 0 & 8 & -1 \\ 0 & 2 & 8 \\ 1 & -2 & 16 \end{bmatrix} \quad \therefore R_3 \leftarrow R_3 + 0.3333 \times R_1$$

$$= \begin{bmatrix} -3 & 0 & 0 \\ 0 & 8 & -1 \\ 0 & 2 & 8 \\ 0 & -2 & 16 \end{bmatrix} \quad \therefore R_4 \leftarrow R_4 + 0.3333 \times R_1$$

$$= \begin{bmatrix} -3 & 0 & 0 \\ 0 & 8 & -1 \\ 0 & 0 & 8.25 \\ 0 & -2 & 16 \end{bmatrix} \quad \therefore R_3 \leftarrow R_3 - 0.25 \times R_2$$

$$= \begin{bmatrix} -3 & 0 & 0 \\ 0 & 8 & -1 \\ 0 & 0 & 8.25 \\ 0 & 0 & 15.75 \end{bmatrix} \quad \therefore R_4 \leftarrow R_4 + 0.25 \times R_2$$

$$= \begin{bmatrix} -3 & 0 & 0 \\ 0 & 8 & -1 \\ 0 & 0 & 15.75 \\ 0 & 0 & 8.25 \end{bmatrix} \quad \therefore \text{Interchanging rows } R_3 \leftrightarrow R_4$$

$$= \begin{bmatrix} -3 & 0 & 0 \\ 0 & 8 & 0 \\ 0 & 0 & 15.75 \\ 0 & 0 & 8.25 \end{bmatrix} \quad \therefore R_2 \leftarrow R_2 + 0.0635 \times R_3$$

$$= \begin{bmatrix} -3 & 0 & 0 \\ 0 & 8 & 0 \\ 0 & 0 & 15.75 \\ 0 & 0 & 0 \end{bmatrix} \quad \therefore R_4 \leftarrow R_4 - 0.5238 \times R_3$$

Question 3

Answer

Find Matrix Eigenvalues ...

$$\begin{bmatrix} 2 & -6 & 2 \\ -6 & 2 & -4 \\ 2 & -4 & 8 \end{bmatrix}$$

Solution:
$|A - \lambda I| = 0$

$$\begin{vmatrix} (2-\lambda) & -6 & 2 \\ -6 & (2-\lambda) & -4 \\ 2 & -4 & (8-\lambda) \end{vmatrix} = 0$$

$$\therefore (2-\lambda)((2-\lambda)\times(8-\lambda) - (-4)\times(-4)) - (-6)((-6)\times(8-\lambda) - (-4)\times 2) + 2((-6)\times(-4) - (2-\lambda)\times 2) = 0$$

$$\therefore (2-\lambda)\left(\left(16 - 10\lambda + \lambda^2\right) - 16\right) + 6((-48 + 6\lambda) - (-8)) + 2(24 - (4 - 2\lambda)) = 0$$

$$\therefore (2-\lambda)\left(-10\lambda + \lambda^2\right) + 6(-40 + 6\lambda) + 2(20 + 2\lambda) = 0$$

$$\therefore \left(-20\lambda + 12\lambda^2 - \lambda^3\right) + (-240 + 36\lambda) + (40 + 4\lambda) = 0$$

$$\therefore \left(-\lambda^3 + 12\lambda^2 + 20\lambda - 200\right) = 0$$

$$\therefore -\left(\lambda^3 - 12\lambda^2 - 20\lambda + 200\right) = 0$$

$$\therefore \left(\lambda^3 - 12\lambda^2 - 20\lambda + 200\right) = 0$$

Roots can be found using newton raphson method

− Newton Raphson method for $x^3 - 12x^2 - 20x + 200 = 0$

Here $x^3 - 12x^2 - 20x + 200 = 0$

Let $f(x) = x^3 - 12x^2 - 20x + 200$

$\therefore f'(x) = 3x^2 - 24x - 20$

$x_0 = 3$

1^{st} iteration :

$f(x_0) = f(3) = 3^3 - 12 \times 3^2 - 20 \times 3 + 200 = 59$

$f'(x_0) = f'(3) = 3 \times 3^2 - 24 \times 3 - 20 = -65$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$x_1 = 3 - \frac{59}{-65}$$

$x_1 = 3.9077$

2^{nd} iteration :

$f(x_1) = f(3.9077) = 3.9077^3 - 12 \times 3.9077^2 - 20 \times 3.9077 + 200 = -1.7239$

$f'(x_1) = f'(3.9077) = 3 \times 3.9077^2 - 24 \times 3.9077 - 20 = -67.9744$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$x_2 = 3.9077 - \frac{-1.7239}{-67.9744}$$

$x_2 = 3.8823$

3^{rd} iteration :

$f(x_2) = f(3.8823) = 3.8823^3 - 12 \times 3.8823^2 - 20 \times 3.8823 + 200 = -0.0002$

$f'(x_2) = f'(3.8823) = 3 \times 3.8823^2 - 24 \times 3.8823 - 20 = -67.9585$

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$$

$$x_3 = 3.8823 - \frac{-0.0002}{-67.9585}$$

$x_3 = 3.8823$

Approximate root of the equation $x^3 - 12x^2 - 20x + 200 = 0$ using Newton Raphson mehtod is 3.8823

n	x_0	$f(x_0)$	$f'(x_0)$	x_1	Update
1	3	59	-65	3.9077	$x_0 = x_1$
2	3.9077	-1.7239	-67.9744	3.8823	$x_0 = x_1$
3	3.8823	-0.0002	-67.9585	3.8823	$x_0 = x_1$

Now, using long division $\frac{x^3 - 12x^2 - 20x + 200}{x - 3.8823} = x^2 - 8.1177x - 51.5155$

Now, $x^2 - 8.1177x - 51.5155 = 0$

$\therefore x = -4.1867$ and $x = 12.3044$

$\therefore$ The eigenvalues of the matrix A are given by $\lambda = -4.1867, 3.8823, 12.3044$

1. Eigenvectors for $\lambda = -4.1867$

1. Eigenvectors for $\lambda = -4.1867$

$$A - \lambda I = \begin{bmatrix} 2 & -6 & 2 \\ -6 & 2 & -4 \\ 2 & -4 & 8 \end{bmatrix} + 4.1867 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & -6 & 2 \\ -6 & 2 & -4 \\ 2 & -4 & 8 \end{bmatrix} + \begin{bmatrix} 4.1867 & 0 & 0 \\ 0 & 4.1867 & 0 \\ 0 & 0 & 4.1867 \end{bmatrix}$$

$$= \begin{bmatrix} 6.1867 & -6 & 2 \\ -6 & 6.1867 & -4 \\ 2 & -4 & 12.1867 \end{bmatrix}$$

Now, reduce this matrix

$R_1 \leftarrow R_1 \div 6.1867$

$$= \begin{bmatrix} 1 & -0.9698 & 0.3233 \\ -6 & 6.1867 & -4 \\ 2 & -4 & 12.1867 \end{bmatrix}$$

$R_2 \leftarrow R_2 + 6 \times R_1$

$$= \begin{bmatrix} 1 & -0.9698 & 0.3233 \\ 0 & 0.3679 & -2.0604 \\ 2 & -4 & 12.1867 \end{bmatrix}$$

$\Rightarrow x_1 + 0.8554x_3 = 0, x_2 - 0.6017x_3 = 0$

$\Rightarrow x_1 = -0.8554x_3, x_2 = 0.6017x_3$

$\therefore$ eigenvectors corresponding to the eigenvalue $\lambda = 3.8823$ is

$$v = \begin{bmatrix} -0.8554x_3 \\ 0.6017x_3 \\ x_3 \end{bmatrix}$$

Let $x_3 = 1$

$$v_2 = \begin{bmatrix} -0.8554 \\ 0.6017 \\ 1 \end{bmatrix}$$

+ 3. Eigenvectors for $\lambda = 12.3044$

$$v_3 = \begin{bmatrix} 0.6356 \\ -0.7583 \\ 1 \end{bmatrix}$$

$R_3 \leftarrow R_3 - 2 \times R_1$

$$= \begin{bmatrix} 1 & -0.9698 & 0.3233 \\ 0 & 0.3679 & -2.0604 \\ 0 & -2.0604 & 11.5402 \end{bmatrix}$$

interchanging rows $R_2 \leftrightarrow R_3$

$$= \begin{bmatrix} 1 & -0.9698 & 0.3233 \\ 0 & -2.0604 & 11.5402 \\ 0 & 0.3679 & -2.0604 \end{bmatrix}$$

$R_2 \leftarrow R_2 \div -2.0604$

$$= \begin{bmatrix} 1 & -0.9698 & 0.3233 \\ 0 & 1 & -5.601 \\ 0 & 0.3679 & -2.0604 \end{bmatrix}$$

$R_1 \leftarrow R_1 + 0.9698 \times R_2$

$$= \begin{bmatrix} 1 & 0 & -5.1087 \\ 0 & 1 & -5.601 \\ 0 & 0.3679 & -2.0604 \end{bmatrix}$$

$R_3 \leftarrow R_3 - 0.3679 \times R_2$

$$= \begin{bmatrix} 1 & 0 & -5.1087 \\ 0 & 1 & -5.601 \\ 0 & 0 & 0 \end{bmatrix}$$

The system associated with the eigenvalue $\lambda = -4.1867$

$$(A+4.1867I)\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & -5.1087 \\ 0 & 1 & -5.601 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$\Rightarrow x_1 - 5.1087x_3 = 0, x_2 - 5.601x_3 = 0$

$\Rightarrow x_1 = 5.1087x_3, x_2 = 5.601x_3$

$\therefore$ eigenvectors corresponding to the eigenvalue $\lambda = -4.1867$ is

$$v = \begin{bmatrix} 5.1087x_3 \\ 5.601x_3 \\ x_3 \end{bmatrix}$$

Let $x_3 = 1$

$$v_1 = \begin{bmatrix} 5.1087 \\ 5.601 \\ 1 \end{bmatrix}$$

2. Eigenvectors for $\lambda = 3.8823$

$$A - \lambda I = \begin{bmatrix} 2 & -6 & 2 \\ -6 & 2 & -4 \\ 2 & -4 & 8 \end{bmatrix} - 3.8823 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & -6 & 2 \\ -6 & 2 & -4 \\ 2 & -4 & 8 \end{bmatrix} - \begin{bmatrix} 3.8823 & 0 & 0 \\ 0 & 3.8823 & 0 \\ 0 & 0 & 3.8823 \end{bmatrix}$$

$$= \begin{bmatrix} -1.8823 & -6 & 2 \\ -6 & -1.8823 & -4 \\ 2 & -4 & 4.1177 \end{bmatrix}$$

Now, reduce this matrix
interchanging rows $R_1 \leftrightarrow R_2$

$$= \begin{bmatrix} -6 & -1.8823 & -4 \\ -1.8823 & -6 & 2 \\ 2 & -4 & 4.1177 \end{bmatrix}$$

$R_1 \leftarrow R_1 \div -6$

$$= \begin{bmatrix} 1 & 0.3137 & 0.6667 \\ -1.8823 & -6 & 2 \\ 2 & -4 & 4.1177 \end{bmatrix}$$

$R_2 \leftarrow R_2 + 1.8823 \times R_1$

$$= \begin{bmatrix} 1 & 0.3137 & 0.6667 \\ 0 & -5.4095 & 3.2549 \\ 2 & -4 & 4.1177 \end{bmatrix}$$

$R_3 \leftarrow R_3 - 2 \times R_1$

$$= \begin{bmatrix} 1 & 0.3137 & 0.6667 \\ 0 & -5.4095 & 3.2549 \\ 0 & -4.6274 & 2.7843 \end{bmatrix}$$

$R_2 \leftarrow R_2 \div -5.4095$

$$= \begin{bmatrix} 1 & 0.3137 & 0.6667 \\ 0 & 1 & -0.6017 \\ 0 & -4.6274 & 2.7843 \end{bmatrix}$$

$R_1 \leftarrow R_1 - 0.3137 \times R_2$

$$= \begin{bmatrix} 1 & 0 & 0.8554 \\ 0 & 1 & -0.6017 \\ 0 & -4.6274 & 2.7843 \end{bmatrix}$$

$R_3 \leftarrow R_3 + 4.6274 \times R_2$

$$= \begin{bmatrix} 1 & 0 & 0.8554 \\ 0 & 1 & -0.6017 \\ 0 & 0 & 0 \end{bmatrix}$$

The system associated with the eigenvalue $\lambda = 3.8823$

$$(A - 3.8823I)\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0.8554 \\ 0 & 1 & -0.6017 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$\Rightarrow x_1 + 0.8554x_3 = 0, x_2 - 0.6017x_3 = 0$