Name	Asim Ali
ID	7763
Sec	C
Exam Type	Improvement for GPA (Summer 2020)
Paper	Applied Calculus
Submitted to	Miss Shomaila Mazhar
Department	BS Civil

INU Peshawar

Q No ① The function $g(t)$ is defined by

$$g(t) = 0 \qquad t < 0$$
$$= t^2 \qquad 0 \le t \le 3$$
$$= 2t+3 \qquad 3 < t \le 4$$
$$= 12 \qquad t > 4$$

a. State any point of discontinuity

Sol First of all we check the continuity at $t = 4$

i. When $t=4$, then $g(t) = 2t+3$

$$\Rightarrow g(4) = 2(4)+3$$
$$g(4) = 11, \text{ defined}$$

ii. $$L.H.L = \lim_{t \to 4^-} g(t)$$
$$= \lim_{t \to 4} 2t+3 \qquad \text{for } t < 4$$
$$= 2(4)+3$$
$$= 11$$

$$R.H.L = \lim_{t \to 4^+} g(t)$$

$$= \lim_{t \to 4} 12 \quad \text{for } t > 4$$

$$= 12$$

Since $L.H.L \neq R.H.L$

So the Limit does not exists at $t = 4$

Hence the given function is discontinous at $t = 4$.

ii. $\lim_{t \to 3} g(t)$?

Sol: To find the above limit, for this we will find out R.H.L and L.H.L at $t = 3$

$$L.H.L = \lim_{t \to 3^-} g(t)$$

$$= \lim_{t \to 3} t^2$$

$$= (3)^2$$

$$= 9$$

$$R.H.L = \lim_{t \to 3^+} g(t)$$

$$= \lim_{t \to 3} 2t + 3$$

$$= 2(3) + 3$$

$$= 6 + 3$$

$$= 9$$

Since $L.H.L = R.H.L$, so the Limit exists, Hence

$$\lim_{t \to 3} g(t) = 9 \quad \text{Ans.}$$

QNo 2 i. Find the Maclaurin's Series for

$$y(x) = x^2 + \sin x$$

Sol:

Let $f(x) = x^2 + \sin x$

using Maclaurin's series expansion

$$f(x) = f(0) + x f'(0) + \frac{x^2}{2!} f''(0) + \frac{x^3}{3!} f'''(0) + \frac{x^4}{4!} f^{iv}(0)$$

$$+ \frac{x^5}{5!} f^{(v)}(0) + \frac{x^6}{6!} f^{(vi)}(0) + \cdots \quad (1)$$

As $f(x) = x^2 + \sin x \Rightarrow f(0) = (0)^2 + \sin 0$

$$\Rightarrow \boxed{f(0) = 0}$$

$f'(x) = 2x + \cos x \Rightarrow f'(0) = 2(0) + \cos 0$

$\Rightarrow f'(0) = 0 + 1$ $\quad \because \cos 0 = 1$

$$\Rightarrow \boxed{f'(0) = 1}$$

$f''(x) = 2 + (-\sin x)$

$f''(x) = 2 - \sin x \Rightarrow f''(0) = 2 - \sin 0$

$\because \sin 0 = 0$

$\Rightarrow f''(0) = 2 - 0$

$$\boxed{f''(0) = 2}$$

$f'''(x) = 0 - (\cos x)$

$f'''(x) = -\cos x \Rightarrow f'''(0) = -\cos 0$

$$\boxed{f'''(0) = -1}$$

$f^{iv}(x) = -(-\sin x)$

$f^{iv}(x) = \sin x \Rightarrow f^{iv}(0) = \sin 0$

$$\boxed{f^{iv}(0) = 0}$$

$f^{(v)}(x) = \cos x \Rightarrow f^{(v)}(x) = \cos 0$

$$\boxed{f^{v}(x) = 1}$$

$f^{(vi)}(x) = -\sin x \Rightarrow f^{(vi)}(x) = -\sin 0$

$$\boxed{f^{(vi)}(x) = 0}$$

Putting all of the above values in equation 1 we get

$$f(x) = 0 + x \cdot 1 + \frac{x^2}{2!} \cdot 2 + \frac{x^3}{3!}(-1) + \frac{x^4}{4!}(0) + \frac{x^5}{5!}(1) + \frac{x^6}{6!}(0) + \cdots$$

$$f(x) = x + \frac{x^2}{\cancel{2}\cdot 1}\cancel{2} - \frac{x^3}{3\cdot 2\cdot 1} + \frac{x^5}{5\cdot 4\cdot 3\cdot 2\cdot 1} + \cdots$$

$$f(x) = x + x^2 - \frac{x^3}{3!} + \frac{x^5}{5!} + \cdots$$

Ans.

Q No ③ i. Find y'' given

$$1 + xy = x^2 + y^2$$

Sol
=:

$$1 + xy = x^2 + y^2$$

using Implicit differentiation;
Differentiate w.r.t x both sides

$$\frac{d}{dx}(1+xy) = \frac{d}{dx}(x^2+y^2)$$

$$\frac{d}{dx}(1) + \frac{d}{dx}(xy) = \frac{d}{dx}x^2 + \frac{d}{dx}y^2$$

$$0 + x\frac{d}{dx}(y) + y\frac{d}{dx}(x) = 2x + 2y\frac{dy}{dx}$$

$$x\frac{dy}{dx} + y \cdot 1 = 2x + 2y\frac{dy}{dx}$$

$$x\frac{dy}{dx} + y = 2x + 2y\frac{dy}{dx}$$

$$x\frac{dy}{dx} - 2y\frac{dy}{dx} = 2x - y$$

$$(x-2y)\frac{dy}{dx} = 2x - y$$

$$\frac{dy}{dx} = \frac{2x-y}{x-2y}$$

or $$y' = \frac{2x-y}{x-2y} \longrightarrow ①$$

Again differentiate w.r.t "x"

②

$$\frac{d}{dx}(y') = \frac{d}{dx}\left(\frac{2x-y}{x-2y}\right)$$

using quotient rule;

$$y'' = \frac{(x-2y)\frac{d}{dx}(2x-y) - (2x-y)\frac{d}{dx}(x-2y)}{(x-2y)^2}$$

$$y'' = \frac{(x-2y)(2-y') - (2x-y)[1-2y']}{(x-2y)^2}$$

$$y'' = \frac{(2x - xy' - 4y + 2yy') - (2x - 4xy' - y + 2yy')}{(x-2y)^2}$$

$$y'' = \frac{\cancel{2x} - xy' - 4y + \cancel{2yy'} - \cancel{2x} + 4xy' + y - \cancel{2yy'}}{(x-2y)^2}$$

$$y'' = \frac{-xy' + 4xy' - 4y + y}{(x-2y)^2}$$

$$y'' = \frac{3xy' - 3y}{(x-2y)^2}$$

$$y'' = \frac{3x\left(\frac{2x-y}{x-2y}\right) - 3y}{(x-2y)^2}$$

$$y'' = \frac{\frac{3x(2x-y)}{x-2y} - 3y}{(x-2y)^2}$$

$$y'' = \frac{3x(2x-y) - 3y(x-2y)}{(x-2y)^3}, \text{ using}$$

$$\div \frac{\frac{a}{b} - c}{d} = \frac{a-bc}{bd}$$

$$y'' = \frac{6x^2 - 3xy - 3xy + 6y^2}{(x-2y)^3}$$

$$y'' = \frac{6x^2 - 6xy + 6y^2}{(x-2y)^3}$$

$$y'' = \frac{6(x^2 - xy + y^2)}{(x-2y)^3}$$ ~~Ans~~.

AS $1 + xy = x^2 + y^2$

$\Rightarrow 1 = x^2 - xy + y^2$

$$y'' = \frac{6(1)}{(x-2y)^3}$$

$$y'' = \frac{6}{(x-2y)^3}$$ Ans.

QNo(3) ii. Find y' by using logarithmic differentiation

$$y = x^3(1+x)^9 e^{6x}$$

Sol:

$$y = x^3 (1+x)^9 . e^{6x}$$

Taking natural log on B/s

$$\ln y = \ln\left[x^3 (1+x)^9 e^{6x} \right]$$

using $\ln(a.b.c) = \ln a + \ln b + \ln c$

$$\ln y = \ln x^3 + \ln(1+x)^9 + \ln e^{6x}$$

$$\ln y = 3\ln x + 9\ln(1+x) + 6x\ln e \quad \because \text{using } \ln x^n = n\ln x$$

$$\ln y = 3\ln x + 9\ln(1+x) + 6x(1) \quad \because \ln e = 1$$

$$\ln y = 3\ln x + 9\ln(1+x) + 6x$$

Differentiate w.r.t "x"

$$\frac{d}{dx}\ln y = \frac{d}{dx}(3\ln x) + \frac{d}{dx}9\ln(1+x) + \frac{d}{dx}(6x)$$

$$\frac{1}{y}\frac{d}{dx}(y) = 3.\frac{1}{x} + 9\frac{1}{1+x}(0+1) + 6(1)$$

$$\frac{1}{y}.y' = \frac{3}{x} + \frac{9}{1+x} + 6$$

Multiplying "y" on B/s

$$y' = y\left[\frac{3}{x} + \frac{9}{1+x} + 6\right]$$

$$y' = x^3(1+x)^9 e^{6x}\left[\frac{3}{x} + \frac{9}{1+x} + 6\right]$$

"OR"
$$y' = x^3 (1+x)^9 e^{6x} \left[\frac{3(1+x) + 9x + 6x(1+x)}{x(1+x)} \right]$$

$$y' = x^3 (1+x)^9 e^{6x} \left[\frac{3+3x+9x+6x+6x^2}{x(1+x)} \right]$$

$$y' = x^3 (1+x)^9 e^{6x} \left[\frac{6x^2+18x+3}{x(1+x)} \right]$$

$$y' = x^{\cancel{3}} (1+x)^{\cancel{9}} e^{6x} \left[\frac{3(2x^2+6x+1)}{\cancel{x}(\cancel{1+x})} \right]$$

$$y' = x^2 (1+x)^8 e^{6x}\, 3(2x^2+6x+1)$$ Ans.