

QUESTION: 1:-

$$\frac{dy}{dx} = 2x \quad ; \quad y(0) = 1$$

$$\underline{h = 0.1}$$

SOLUTION:-

$$f(x, y) = 2x$$

$$x_0 = 0, \quad y_0 = 1$$

$$h = 0.1$$

$$x_{n+1} = x_n + h$$

$$\text{put } n=0$$

$$x_1 = x_0 + h$$
$$= 0 + 0.1$$

$$= 0.1$$

$$x_2 = x_1 + h$$
$$= 0.1 + 0.1$$

$$= 0.2$$

$$x_3 = 0.3$$

$$x_4 = 0.4$$

$$x_5 = 0.5$$

1st Iteration :-

"Euler's FORMULA"

$$y_{n+1} = y_n + hf(x_n, y_n)$$

$$n = 0$$

$$y_1 = y_0 + hf(x_0, y_0)$$

$$= 1 + 0.1(0)(1)$$

$$= \boxed{1.1}$$

Modified Euler's formula :

$$y_{n+1} = y_n + \frac{h}{2} [f(x_n, y_n) + f(x_{n+1}, y_{n+1})]$$

$$y_1 = y_0 + \frac{h}{2} [f(x_0, y_0) + f(x_1, y_1)]$$

$$y_1 = 1 + \frac{0.1}{2} [(0) + (1) + (0.1)(1.1)]$$

$$y_1 = 1 + 0.05 [1 + 1.1]$$

$$= 1 + 0.5(2.1)$$

$$= \boxed{1.1}$$

2ND ITERATION :-

$$y_2 = y_1 + hf(x_1 + y_1)$$

$$y_2 = 1.11 + 0.1 [(0.1) + (1.11)]$$

$$y_2 = 1.11 + 0.1 (1.21)$$

$$= 1.11 + 0.121$$

$$= \boxed{1.231}$$

"Modified Euler's formula"

$$y_2 = y_1 + 0.1/2 [x_1 + y_1] + [x_2 + y_2]$$

$$= 1.11 + 0.05 [(0.1) + (1.11) + (0.2) + (1.231)]$$

$$= 1.11 + 0.05 [(0.1) + (1.11) + (0.2) + (1.231)]$$

$$= 1.11 + 0.05 [1.21 + 1.431]$$

$$= 1.11 + 0.05 (2.641)$$

$$= \boxed{1.242}$$

3RD ITERATION :-

$$y_3 = y_2 + hf(x_2 + y_2)$$

$$y_3 = 1.242 + 0.1 (0.2) + (1.242)$$

$$= 1.242 + 0.1442 \Rightarrow 1.3862$$

MODIFIED Euler's METHOD :-

$$y_3 = y_2 + 0.5 \left((x_2, y_2) + (x_3, y_3) \right)$$

$$= 1.242 + 0.05 \left[(0.2) + (1.242) + (0.3) + (1.3862) \right]$$

$$= 1.242 + 0.05 \left[1.442 + 1.6862 \right]$$

$$= \boxed{1.398}$$

4th ITERATION :-

$$y_4 = y_3 + h f(x_3, y_3)$$

$$y_4 = 1.398 + 0.05 (0.3 + 1.398)$$

$$\boxed{y_4 = 1.483}$$

MODIFIED EULER'S METHOD :-

$$y_4 = y_3 + 0.05 \left((x_3, y_3) + (x_4, y_4) \right)$$

$$= 1.398 + 0.05 \left[0.3 + 1.398 \right] + \left[0.4 + 1.483 \right]$$

$$= 1.398 + 0.05 (1.698 + 1.883)$$

$$= \boxed{1.577}$$

5th ITERATION :-

$$\begin{aligned}y_5 &= y_4 + hf(x_4, y_4) \\ &= 1.577 + 0.05(0.4 + 1.577) \\ &= \underline{1.675}\end{aligned}$$

MODIFIED Euler's METHOD :-

$$\begin{aligned}y_3 &= y_4 + 0.05 f(x_4, y_4) + (x_5, y_5) \\ &= 1.577 + 0.05(0.4 + 1.577) + (0.5 + 1.675) \\ &= 1.577 + 0.05 [4.152] \\ &= \boxed{1.785} \quad \text{ANSWER.}\end{aligned}$$



QUESTION: 2:-

" Use the fourth order Runge Kutta method to obtain as of $\frac{dy}{dx} = x^2 - y$

Subject to $y=0$ when $x=0$, for $0 \leq x \leq 0.6$ with $h=0.2$. work throughout to four decimal places.

Given data :-

$$y=0, \quad x=0, \quad h=0.2 \quad = \quad 0 \leq x \leq 0.6$$

$$y_{n+1} = y_n + k$$

" Ist ITERATION :-

$$x=0$$

$$y_1 = y_0 + k, \quad k = \frac{1}{4} (k_1 + 2k_2 + 2k_3 + k_4)$$

$$k_1 = hf(x_n, y_n)$$

$$k_2 = h(x_0^2 - x_0 - y_0)$$

$$k_1 = 0.2(0^2 - 0 - 0)$$

$$k_2 = 0$$

$$k_2 = hf(x_n + h/2, y_n + h/2)$$

$$= 0.2f(x_0 + h/2, y_0 + h/2)$$

$$= 0.2f\left(0 + \frac{0.2}{2}, 0 + \frac{0.2}{2}\right)$$

$$= 0.2f(0.1, 0.1)$$

$$= 0.2f(0.1^2 + 0.1 - 0.1)$$

$$k_2 = 0.0020$$

$$k_3 = hf\left(x_n + h/2, y_n + \frac{k_2}{2}\right)$$

$$= 0.2f\left(0 + \frac{0.2}{2}, 0 + \frac{0.002}{2}\right)$$

$$= 0.2f(0.1, 0.001)$$

$$= 0.2(0.1^2 + 0.1 - 0.001)$$

$$k_3 = 0.0218$$

$$k_4 = hf(x_n + h, y_n + k_3)$$

$$= 0.2f(0 + 0.2, 0 + 0.0218)$$

$$= 0.2f(0.2, 0.0218)$$

$$= 0.2 (0.2^2 + 0.2 - 0.0218)$$

$$k_y = 0.0436$$

$$\Rightarrow k = \frac{1}{6} (0 + 2(0.002) + 2(0.0218) + 0.0436)$$

$$k = 0.0152$$

$$y_1 = 0 + 0.0152$$

$$y_1 = 0.0152$$



QUESTION : 3 :-

ANSWER :-

GIVEN DATA :-

$$a = 0, b = 10, n = 10$$

$$h = \frac{b-a}{n} = \frac{10-0}{10} = 1$$

Solution :-

x	0	1	2	3	4	5	6	7	8	9	10
$f(x)$	10.1	17.2	24.4	29.2	34.6	41.2	50.9	57.8	60.3	61.2	62.1

"Using formula"

$$f(x) dx = h \left[\frac{1}{2} (f(x_0) + f(x_n)) + f(x_1) + f(x_2) + \dots + f(x_{n-1}) \right]$$

$$\Rightarrow \frac{1}{2} [10.1 + 2(17.2 + 24.4 + 29.2 + 34.6 + 41.2 + 50.9 + 57.8 + 60.3 + 61.2) + 62.1]$$

$$\Rightarrow = \boxed{412.9} \quad \underline{\underline{\text{ANSWER}}}$$



Q4) $\int_2^3 \ln(x^3 + 1) dx$

use 10 strips -

SOLUTION:-

$$n = 10$$

$$h = \frac{3-2}{10} = 0.1$$

x	x_0	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9
	1	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9
$f(x)$	0.693	0.846	1.003	1.162	1.320	1.476	1.628	1.777	1.922	2.062

Now,

"using formula"

$$\int_a^b f(x) dx = \frac{h}{3} \left[f(x_0) + 4(f(x_1) + f(x_3) + \dots) + 2 \left[f(x_{1/2} + \dots) + f(x_n) \right] \right]$$

$$= 0.1/3 \left[0.693 + 4(0.846 + 1.162 + 1.476 + 1.777) + 2(1.003 + 1.320 + 1.628 + 1.922) + 2.062 \right]$$

$$\Rightarrow = \boxed{1.184 \text{ Ans.}}$$

