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subject Numerical  
Analysis

Semester 4<sup>th</sup>

MID Examination.

Q. NO 1 :-

Find a root of the equation  $x^3 + 3.993 \times 10^{-4} = 0.165x$

use Newton Raphson method with  $x_0 = 0.02$ .

Sol:-

Re-arranging the equation

$$x^3 + 0.165x^3 + 0.00039 = 0.$$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$f'(x) \Rightarrow 3x^2 - 0.33x = 0.$$

$$\Rightarrow x_1 = x_0 \frac{-f(x_0)}{f'(x_0)} \quad * \quad \frac{f(x_0)}{f'(x_0)} = 0.00033$$

$$f'(x_0) = -0.0054$$



$$x_1 = 0.08 - \frac{0.00033}{-0.0054}$$

$$\Rightarrow x_1 = 0.081$$

$$\Rightarrow x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} \quad * \quad \begin{aligned} f(x_1) &= -0.000016 \\ f'(x_1) &= -0.000070 \end{aligned}$$

$$\Rightarrow x_2 = 0.081 - \frac{0.000016}{-0.00070}$$

$$x_2 = 0.058$$

$$\Rightarrow x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} \quad * \quad \begin{aligned} f(x_2) &= 0.000030 \\ f'(x_2) &= 0.00090 \end{aligned}$$

$$= 0.058 - \frac{0.000030}{0.00090}$$

$$\boxed{x_3 = 0.061}$$

the value of

$$\alpha_1 = 0.081$$

$$\alpha_2 = 0.058$$

$$\alpha_3 = 0.061$$

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Q NO 2:-

use the numbers  $x_0 = 2$ ,

$x_1 = 2.75$ ,  $x_2 = 4$  to find

the Lagrange interpolation polynomial for  $f(x) = \frac{1}{x}$  at

$x = 3$ .

Sol:-

$$x_0 = 2 \quad y_0 = 0.5$$

$$x_1 = 2.75 \quad y_1 = 0.36$$

$$x_2 = 4 \quad y_2 = 0.25$$

As we know that

By using formula

$$y = \frac{(x-x_1)(x-x_2)\dots(x-x_n)}{(x_0-x_1)(x_0-x_2)\dots(x_0-x_n)} y_0$$

$x_0 = 2$	$y_0 = 0.5$
$x_1 = 2.75$	$y_1 = 0.36$
$x_2 = 4$	$y_2 = 0.25$



$$y = \frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)} y_0 + \frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)} y_1$$

$$+ \frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)} y_2$$

Putting values.

$$y = \frac{(3-2.75)(3-4)}{(2-2.75)(2-4)} (0.5)$$

$$+ \frac{(3-2)(3-4)}{(2.75-2)(2.75-4)} (0.36)$$

$$+ \frac{(3-2)(3-2.75)}{(4-2)(4-2.75)} (0.25)$$

$$= -0.083 + 0.3084 + 0.025$$

$$= -0.083 + 0.384 + 0.025$$

$$\boxed{y = -0.429}$$



7 SO/8-

$\lambda = 1.0$

$y = 0.7651917$

1.3

$0.6200860$

1.6

$0.4554022$

1.9

$0.2818186$

2.2

$0.1103623$

$x_i$	$f(x_i)$	$f(x_{i-1}, x_i)$	$f(x_{i-2}, x_{i-1}, x_i)$	$f(x_{i-3}, \dots, x_i)$	$f(x_{i-4}, \dots, x_i)$
$x_0$	1.0 0.7651917				
$x_1$	1.3 0.6200860	-0.9837056			
$x_2$	1.6 0.4554022	-0.548946	-0.108734	0.0658785	
$x_3$	1.9 0.2818186	-0.578612	-0.0494433	0.06251255	-0.0028049
$x_4$	2.2 0.1103623	-0.57121	0.006818		



②

$$\begin{aligned} 1) f(x_0, x_1) &= \frac{f(x_1) - f(x_0)}{x_1 - x_0} \\ &= \frac{0.6200866 - 0.7651977}{1.3 - 1} \end{aligned}$$

$$f(x_0, x_1) = -0.4837056$$

②

$$\begin{aligned} f(x_1, x_2) &= \frac{f(x_2) - f(x_1)}{x_2 - x_1} \\ &= \frac{0.4554022 - 0.6200866}{1.6 - 1.3} \end{aligned}$$

$$f(x_1, x_2) = 0.548946$$

③

$$\begin{aligned} f(x_2, x_3) &= \frac{f(x_3) - f(x_2)}{x_3 - x_2} \\ &= \frac{0.2818186 - 0.4554022}{1.9 - 1.6} \end{aligned}$$

$$= 0.578612$$



$$4) f(x_3, x_4) = \frac{f(x_4) - f(x_3)}{x_4 - x_3}$$

$$= \frac{0.1183623 - 0.2818186}{2.02 - 1.09}$$

$$f(x_3, x_4) = -0.571521$$

Second divided difference

$$f(x_0, x_1, x_2) = \frac{f(x_1, x_2) - f(x_0, x_1)}{x_2 - x_0}$$

$$= \frac{-0.548946 - (-0.4831856)}{1.6 - 1}$$

$$f(x_0, x_1, x_2) = -0.108734$$



$$f(x_1, x_2, x_3) = \frac{f(x_2, x_3) - f(x_1, x_2)}{x_3 - x_1}$$

$$= \frac{0.578612 - (-0.548964)}{1.9 - 1.3}$$

$$f(x_1, x_2, x_3) = -0.0494433$$

$$f(x_2, x_3, x_4) = \frac{f(x_3, x_4) - f(x_2, x_3)}{x_4 - x_2}$$

$$= \frac{-0.571621 - (-0.578612)}{2.2 - 1.6}$$

$$f(x_2, x_3, x_4) = 0.006818$$

Third Divided Difference:-

$$f(x_0, x_1, x_2, x_3) = \frac{f(x_1, x_2, x_3) - f(x_0, x_1, x_2)}{x_3 - x_0}$$

$$f(x_0, x_1, x_2, x_3) = \frac{0.0494433 - (-0.108734)}{1.9 - 1}$$



$$f(x_0, x_1, x_2, x_3) = 0.0658785$$

$$f(x_1, x_2, x_3, x_4) = \frac{f(x_2, x_3, x_4) - f(x_1, x_2, x_3)}{x_4 - x_1}$$

$$= \frac{0.006818 - (-0.494433)}{2.2 - 1.3}$$

$$= 0.06251255$$

### Fourth Divided Difference

$$f(x_0, x_1, x_2, x_3, x_4) = \frac{0.06251255 - 0.0658785}{2.2 - 1}$$

$$= -0.0028049$$

$$f(x) = f(x_0) + (x-x_0) f(x_0, x_1) + (x-x_0)(x-x_1) f(x_0, x_1, x_2) + (x-x_0)(x-x_1)(x-x_2) f(x_0, x_1, x_2, x_3) + (x-x_0)(x-x_1)(x-x_2)(x-x_3) f(x_0, x_1, x_2, x_3, x_4)$$



$$\begin{aligned}
&= 0.7651977 + (\gamma - 1) - 0.4837056 + (\gamma - 1) \\
&(\gamma - 1.03) (-0.108734) + (\gamma - 1) (\gamma - 1.03) \\
&(\gamma - 1.06) (0.0065885) + (\gamma - 1) (\gamma - 1.03) \\
&(\gamma - 1.06) (\gamma - 1.09) (-0.00028049)
\end{aligned}$$


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