(1)

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Subject Statistics & probiblity

program BS(CS)

Department Computer Sci

Semester 4^{th}

Date 23/6/20

Q NO #1

S = {(1,1), (1,2), (1,3), (1,4), (1,5), (1,6), (2,1), (2,2), (2,3), (2,4), (2,5), (2,6), (3,1), (3,2), (3,3), (3,4), (3,5), (3,6), (4,1), (4,2), (4,3), (4,4), (4,5), (4,6), (5,1), (5,2), (5,3), (5,4), (5,5), (5,6), (6,1), (6,2), (6,3), (6,4), (6,5), (6,6)}

let

A = {The Sum is 7}

B = {The Sum is even}

C = {The Sum is greater than 8}

D = {The two dise had the Same outcomes}

Now

A = {(1,6) (2,5) (3,4) (4,3) (5,2) (6,1)}

B = {(1,1), (1,3), (1,5), (2,2), (2,4), (2,6), (3,1), (3,3), (3,5), (4,2), (4,4), (4,6), (5,1), (5,3), (5,5), (6,2), (6,4), (6,6)

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$C = \{(3,6), (4,5), (4,6), (5,4), (5,5), (5,6)$
$(6,3), (6,4), (6,5), (6,6)\}$

$D = \{(1,1), (2,2), (3,3), (4,4), (5,5), (6,6)\}$

$A \cap B = \{\ \}$ OR ϕ

$A \cap C = \{\ \}$ OR ϕ

$A \cap D = \{\ \}$ OR ϕ

$p(A) = 6/36$, $p(B) = 18/36$, $p(c) = 10/36$

$p(D) = 6/36$

Hence

$\cancel{p(A)(B)}\ p(A/B) = \frac{p(A \cap B)}{p(B)} = 0 \times 18/36$

$p(A|B) = 0$

$p(A|C) = \frac{p(A \cap C)}{p(C)} = 0 \times 10/36$

$p(A|C) = 0$

$p(A|D) = \frac{p(A \cap D)}{p(D)} = 0 \times 6/36$

$p(A|D) = 0$

Q NO #9

When we are rolling two dice, there are 36 different combination Counting these up there are 15 possibilities less than 7: (1,1),(1,2),(1,3),(1,4),(1,5),(2,1)(2,2),(2,3),(2,4),(3,1),(3,2),(3,3),(4,1),(4,2),(5,1) The probibility of getting less than a 7 is

$$15/36 = 15/12$$

There are 6 possible combination of getting a 7: (1,6),(2,5),(3,4),(4,3),(5,2),(6,1) which give a probiblity of

$$6/36 = 1/6$$

This mean that 21 possibities account for getting less than or evual to 7, so there are 15 remaining possiblities of getting more than 7. This is the same as the probiblity of getting less than 7 so the probiblity must be 5/12 as well. In calculating this we must assume that each combination is evualy likely to roll as any other & therefore the dice are fair or else the calculations don't work.

Q NO # 3

A & B play a game in which A's probability of winning is 2/3. In Series of 8 games what is the probability ~~of winning is 2/3~~ that A will win.

1 Exactly 4 games
2 At least 4 games
3 From 3 to 6 games

Sol Given $p = 2/3$ $n = 8$

$$q = 1 - p$$
$$= 1 - 2/3$$
$$q = 1/3$$

I) $p(X = 4)$

$$\binom{8}{4}\left(\frac{2}{3}\right)^4\left(\frac{1}{3}\right)^4$$

$$= \frac{1120}{6561}$$
$$= 0.1707$$

II) $p(x \geq 4)$

$$= 1 - p(x < 4)$$
$$= 1 - \sum_{x=0}^{3}\binom{8}{x}\left(\frac{2}{3}\right)^x\left(\frac{1}{3}\right)^{8-x}$$
$$= 1 - \left[\left(\frac{1}{3}\right)^8 + 8\left(\frac{2}{3}\right)\left(\frac{1}{3}\right)^7 + 28\left(\frac{2}{3}\right)^2\left(\frac{1}{3}\right)^6 + 56\left(\frac{2}{3}\right)^3\left(\frac{1}{3}\right)^5\right]$$

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$= 1 - \frac{1}{6561}[1+16+112+448]$

$= 1 - \frac{577}{6561}$

$= \frac{6561 - 577}{6561}$

$= \frac{5984}{6561}$

$= 0.9121$

III) $P(3 \le x \le 6)$

$\sum_{x=3}^{6} \binom{8}{x}\left(\frac{2}{3}\right)^{x}\left(\frac{1}{3}\right)^{8-x}$

$= \binom{8}{3}\left(\frac{2}{3}\right)^{3}\left(\frac{1}{3}\right)^{5} + \binom{8}{4}\left(\frac{2}{3}\right)^{4}\left(\frac{1}{3}\right)^{4} + \binom{8}{5}\left(\frac{2}{3}\right)^{5}\left(\frac{1}{3}\right)^{3}$

$+ \binom{8}{6}\left(\frac{2}{3}\right)^{6}\left(\frac{1}{3}\right)^{2}$

$= \frac{8}{(3)^8}[56 + 140 + 224 + 224]$

$= \frac{8 \times 644}{6561} = \frac{5152}{6561}$

$= 0.7852$

Q NO#4

* proof
* Since the C_i's from a partition of the Sample Space, we can apply the law of total probability for $A \cap B$

$$= P(A \cap B) = \sum_{i=1}^{M} P(A \cap B / C_i)\, P(C_i)$$

$$P(A \cap B) = \sum_{i=1}^{M} P(A/C_i)\, P(B/C_i)\, P(C_i)$$

(A & B are conditionally independent)

$$P(A \cap B) = \sum_{i=1}^{M} P(A/C_i)\, P(B)\, P(C_i)$$

. (B is independent of all C_i's)

$$P(A \cap B) = P(B) \sum_{i=1}^{M} P(A/C_i)\, P(C_i)$$

$$P(A \cap B) = P(B)\, P(A)$$

* (law of total probibuty)

Hence A & B are independent.

Q NO # 5

The probiblity function for a binomial random variable is

$$b(x; n, p) = \binom{n}{x} p^x (1-p)^{n-x}$$

This is the probability of having x success in a series of n independent trials when the probibility of sucesses in any of one trials is p. if x is a random variable with this probiblity distribution.

$$E(x) = \sum_{x=0}^{n} x \binom{n}{x} p^x (1-p)^{n-x}$$

$$\sum_{x=0}^{n} x \frac{n!}{x!(n-x)!} p^x (1-p)^{n-x}$$

$$\sum_{x=1}^{n} \frac{n!}{(x-1)(n-x)!} p^x (1-p)^{n-x}$$

Since the $x=0$ term vanishes let $y = x-1$ & $m = n-1$ subbing $x = y+1$, & $n = m+1$

[illegible]

into the last sum (& using the fact that the limits $x=1$ & $x=n$ correspond to $y=0$ & $y = n-1 = m$, respectively)

$$= E(x) = \sum_{y=0}^{m} \frac{(m+1)!}{y!(m-y)!} p^y (1-p)^{m-y}$$

$$= (m+1)p \sum_{y=0}^{m} \frac{m!}{y!(m-y)!} p^y (1-p)^{m-y}$$

The binomial theorem says that

$$(a+b)^m = \sum_{y=0}^{m} \frac{m!}{y!(m-y)!} a^y b^{m-y}$$

Setting $a = p$ & $b = 1-p$

$$\sum_{y=0}^{m} \frac{m!}{y!(m-y)!} p^y (1-p)^{m-y} = \sum_{y=0}^{m} \frac{m!}{y!(m-y)!} a^y b^{m-y} = (a+b)^m$$

$$= (p+1-p)^m = 1$$

So that

$$\boxed{E(x) = np}$$

Similarly but this time using $y = x-2$ &

$m = n-2$

$$E(x(x-1)) = \sum_{x=0}^{n} x(x-1)\binom{n}{x} p^x (1-p)^{n-x}$$

$$\sum_{x=0}^{n} x(x-1) \frac{n!}{x!(n-x)!} p^x (1-p)^{n-x}$$

$$\sum_{x=2}^{n} \frac{n!}{(x-2)!(n-x)!} p^x (1-p)^{n-x}$$

$$= n(n-1)p^2 \sum_{x=2}^{n} \frac{(n-2)!}{(x-2)!(n-x)!} p^{x-2} (1-p)^{n-x}$$

$$= n(n-1)p^2 \sum_{y=0}^{m} \frac{m!}{y!(m-y)!} p^y (1-p)^{m-y}$$

$$= n(n-1)p^2 (p + (1-p))^m$$

$$= n(n-1)p^2$$

So the varience of x is

$$E(x^2) - E(x)^2 = E(x(x-1)) + E(x) - E(x)^2 =$$

$$n(n-1)p^2 + np - (np)^2$$

$$= \boxed{np(1-p)}$$

Q NO#6

Differentiate between Bi-nomial frequency distribution & Bi-nomial distribution with the help of formulas?

* ## Bi-nomial frequency distribution:-

If the binomial probability distribution is multiplied by N the number of experiments or sets, the resulting bi-nomial distribution is known as the bi-nomial frequency distribution.

$$N\binom{n}{x}p^x q^{n-x}$$

* ## Bi-Nomial distribution:-

A binomial distribution is a discrete distribution which occurs when a variable has

* only two possible outcomes e.g head/tail
* A fixed number of independent trials
* A constant probibility of success for each trial (p)
* Many experiments consist of repeated independent trials each trials having two possible out comes e.g head/tail. etc.

$$P(x=x) \quad f(x) = {}^{n}C_x\, p^x\, q^{n-x}$$

Q NO# 7

* Co-effecient of variation

* For Data Set A:-

$$CV = \frac{\sigma}{\mu} \times 100$$

$$CV = 3/45 \times 100$$

$$\boxed{CV = 6.7}$$

For data set B

$$CV = 11/60 \times 100$$

$$\boxed{CV = 18.3}$$

For Data Set C

$$CV = 5/50 \times 100$$

$$\boxed{CV = 10}$$

For Data Set D

$$CV = 15/25 \times 100$$

$$\boxed{CV = 60}$$