Q#1:

Apply Both Eulers method & the improved Euler method to the solution of $\frac{dy}{dx} = 2x$, $y(0) = 1$

For $0 \leq x \leq 0.5$ using $h = 0.1$. Compare your answers with the analytic solution. work throughout to three decimal places.

Solution:-

By Euler's Method:-

Given Data:

$y(0) = 1$, $h = 0.1$, $x_0 = 0$

By formula.

$$y_{n+1} = y_n + hf(x_n, y_n)$$

$$y_{n+1} = y_n + h\,[2x_n]$$

1st Iteration:-

$n = 0$

$y_1 = y_0 + h\,(2x_0)$

$y_1 = 1 + 0.1\,(2(0))$

$y_1 = 1 + 0.1$

$\boxed{y_1 = 1.0}$

$\rightarrow x_{n+1} = x_n + h$

$x_1 = x_0 + h$

$x_1 = 0 + 0.1 \Rightarrow \boxed{x_1 = 0.1}$

2nd Iteration:-

$n = 1$

$y_2 = y_1 + h\,(2x_1)$

$y_2 = 1.0 + 0.1\,(2(0.1))$

$\boxed{y_2 = 1.02}$

$x_{n+1} = x_n + h$

$x_2 = x_1 + h$

$x_2 = 0.1 + 0.1$

$\boxed{x_2 = 0.2}$

3rd Iteration:

$n = 2$

$y_3 = y_2 + h\,(2x_2)$

$y_3 = 1.02 + 0.1\,(2\,(0.2))$

$\boxed{y_3 = 1.06}$

$x_{n+1} = x_n + h$

$x_3 = x_2 + 0.1$

$x_3 = 0.2 + 0.1$

$\boxed{x_3 = 0.3}$

b) By Modified Eular method:-

$\frac{dy}{dx} = 2x$, Given data , $y_0 = 1$, $x_0 = 0$, $h = 0.1$

Formula:

$y^*_{n+1} = y_n + h\,[f(x_n)]$

$y^*_{n+1} = y_n + h\,(2x_n) \longrightarrow (1)$

$y_{n+1} = y_n + \frac{h}{2}\left[f(x_n, y_n) + f(x_{n+1}, y^*_{n+1})\right]$

$= y_n + \frac{h}{2}\left[2x_n + 2x_n\right]$

$= y_n + \frac{h}{2}\left[4x_n\right]$

Ist Iteration:-

$n = 0$

$x_{n+1} = x_n + h$

$x_1 = x_0 + h$

$x_1 = 0 + 0.1$

$\boxed{x_1 = 0.1}$

$y_1 = y_0 + \frac{h}{2}(4x_0)$

$y_1 = 1 + \frac{0.1}{2}(4(0))$

$\boxed{y_1 = 1}$

2nd Iteration:-

$n=1$

$x_2 = x_1 + h$

$x_2 = 0.1 + 0.1$

$\boxed{x_2 = 0.2}$

$y_2 = y_1 + \frac{h}{2}(4x_1)$

$y_2 = 1 + \frac{0.1}{2}(4(0.1))$

$\boxed{y_2 = 1.02}$

3rd Iteration:-

$n=2$

$x_3 = x_2 + h$

$x_3 = 0.2 + 0.1$

$\boxed{x_3 = 0.3}$

$y_3 = y_2 + \frac{h}{2}(4x_2)$

$= 1.02 + \frac{0.1}{2}(4(0.2))$

$\boxed{y_3 = 1.06}$

Q#2:

use The fourth-order Runge Kutta method to obtain a solution of

$$\frac{dy}{du} = x^2 + u - y$$

subject to $y=0$ when, $x=0$, for $0 \le x \le 0.6$ with $h=0.2$. work throughout to four decimal places.

Given Data:-

$y=0$, $u=0$, $h=0.2$ $0 \le u \le 0.6$

$y_{n+1} = y_n + k$

I^{st} Iteration:- $n=0$

$y_1 = y_0 + k$, $k = 1/6 (k_1 + 2k_2 + 2k_3 + k_4)$

$k_1 = hf(x_n, y_n)$

$k_1 = h(x_0^2 - x_0 - y_0)$

$k_1 = 0.2(0^2 - 0 - 0)$

$\boxed{k_1 = 0}$

$k_2 = hf\left(x_n + \frac{h}{2}, y_n + \frac{h}{2}\right)$

$= 0.2 f(x_0 + h/2, y_0 + h/2)$

$= 0.2 f\left(0 + \frac{0.2}{2}, \frac{0 + 0.2}{2}\right)$

$= 0.2 f(0.1, 0.1)$

$= 0.2(0.1^2 + 0.1 - 0.1)$

$\boxed{k_2 = 0.0020}$

$k_3 = hf\left(x_n + \frac{h}{2}, y_n + \frac{k_2}{2}\right)$

$= 0.2 f\left(0 + \frac{0.2}{2}, 0 + \frac{0.002}{2}\right)$

$= 0.2 f(0.1, 0.001)$

$= 0.2(0.1^2 + 0.1 - 0.001)$

$\boxed{k_3 = 0.0218}$

$k_4 = hf(x_n + h, y_n + k_3)$

$= 0.2 f(0 + 0.2, 0 + 0.0218)$

$= 0.2 f(0.2, 0.0218)$

$= 0.2(0.2^2 + 0.2 - 0.0218)$

$\boxed{k_4 = 0.0436}$

$k = \frac{1}{6}(0 + 2(0.002) + 2(0.0218) + 0.0436)$

~~k = 0.0152~~ $\boxed{k = 0.0152}$

Q#3

Given Data:-

$a = 0, \; b = 10, \; n = 10$

$$h = \frac{b-a}{n} = \frac{10-0}{10} = 1$$

Sol:-

x	0	1	2	3	4	5	6	7	8	9	10
$f(x_0)$	10.1	17.2	24.4	29.2	34.6	41.2	50.9	57.8	60.3	61.2	62.1

using formula.

$$f(x)\,dx = \frac{h}{2}\left[f(x_0) + 2\left(f(x_1) + f(x_2) + f(x_3) \ldots f(x_9)\right) + f(x_{10})\right]$$

$$= \frac{1}{2}\left[10.1 + 2\left(17.2 + 24.4 + 29.2 + 34.6 + 41.2 + 50.9 + 57.8 + 60.3 + 61.2\right) + 62.1\right]$$

$$= \boxed{412.9 \text{ Ans}}$$

Q #4:

$$\int_2^3 \ln(u^3+1)\,du$$

use 10 strips

Sol:. $n=10$

$h = \frac{3-2}{10} = 0.1$

x	x_0 1	x_1 1.1	x_2 1.2	x_3 1.3	x_4 1.4	x_5 1.5	x_6 1.6	x_7 1.7	x_8 1.8	x_9 1.9
$f(u)$	0.693	0.846	1.003	1.162	1.320	1.476	1.628	1.777	1.922	2.062

Now using formula.

$$\int_a^b f(u)\,du = \frac{h}{3}\left[f(u_0)+4\left(f(u_1)+f(u_3)\cdots\cdots\right]+2\left[f(u_2\cdots\cdots)+f(u_n)\right]\right.$$

$$= \frac{0.1}{3}\left[0.693+4(0.846+1.162+1.476+1.777)+2(1.003+1.320+1.628+1.922)+2.062\right]$$

$$\boxed{= 1.184 \text{ Ans}}$$