LINEAR ALGEBRA

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Q 1 :

Consider the Given Below matrix as the Augmented Matrix of A linear System. Explain in your words the next elementary row Opration that Should Be performed in Order To Solve this System. Where ID3 is 3rd digit of your ID and ID-last is the last digit of your ID in

Inverse e.g. if your ID is (16742) Than ID-last = 2

$$\begin{bmatrix} 1 & ID3 & 3 & 0 & 5 \\ 0 & 1 & -ID\text{-}last & 0 & 7 \\ 0 & 0 & 1 & 0 & -6 \\ 0 & 0 & 0 & 1 & ID3 \end{bmatrix}$$

Solution: Q.1.

Given That

$$\begin{bmatrix} 1 & ID3 & 3 & 5 \\ 0 & 1 & -ID\text{-}Last & 7 \\ 0 & 0 & 1 & -6 \\ 0 & 0 & 0 & ID3 \end{bmatrix} \rightarrow (I)$$

Since my ID is 16742

That (I) becomes

$$\begin{bmatrix} 1 & 7 & 3 & 0 & 5 \\ 0 & 1 & -2 & 0 & 7 \\ 0 & 0 & 1 & 0 & -6 \\ 0 & 0 & 0 & 1 & 7 \end{bmatrix}$$

Abviously the above Augmented matrix is in Row echelon form.
The Solution Set can Be Obtained by writing The Matrix in System of equation form.
However to reduced The Matrix in reduced Row echelon form We Whould Suggest The

Next elementary row
Opration as

$R_1 - 7R_3$

X X X X X X X X X

Q2 . B)
Find the elementry
Row opration that
Transform the first
matrix into

Q
B) . Below given are the
Some Matrixes find
Which one is reduced
Row echelon form.
Explain in your own

Words for each of the Selection in Detail

a). $\begin{bmatrix} e & 0 & 0 & 0 \\ 0 & \mathrm{II} & 0 & 0 \\ 0 & 0 & -\mathrm{II} & 0 \\ 0 & 0 & 0 & e \end{bmatrix}$ is in echelon form

B). $\begin{bmatrix} 1 & 0 & \mathrm{II} \\ 0 & 1 & e \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ is in echelon form

C). $\begin{bmatrix} 5 & 0 & 0 & 7 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & 4 \end{bmatrix}$ is in reduced row echelon form.

D). $\begin{bmatrix} 1 & 0 & 0 & 7 \\ 0 & 0 & 0 & 4 \\ 0 & 0 & 1 & 0 \end{bmatrix}$ is in reduced row echelon form

Q2 :: Solution :: B

B).

$$A = \begin{bmatrix} e & 0 & 0 & 0 \\ 0 & \pi & 0 & 0 \\ 0 & 0 & \pi & 0 \\ 0 & 0 & 0 & e \end{bmatrix}$$ is in echelon form.

Obviously

The Given number of Zeros before first Non-zero Entries Encreases row by Row.

So the Given Matrix is in row echelon form.

Furthermore, the first

Non-zero entry in each Non-zero row is Not 1.

So the Given Matrix is Not in reduced Row echelon form.

B).

$$\begin{bmatrix} 1 & 0 & \Pi \\ 0 & 1 & e \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$ is in echelon form.

Explanation:

As the number of Zeros increases row By row before Non-zero Entry of the Non-zero Rows and Non-zero

Rows are

By zero rows.

There fore, The given Matrix is in row Echelon form.

C).
$$\begin{bmatrix} 5 & 0 & 0 & 7 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & 4 \end{bmatrix}$$
is in reduced row echelon form.

Explanation:

Since the first Entry of a Non-zero row(R1) In the Given Matrix is Other Than 1.

So, The Given Matrix

Is Not in reduced Raw echelon form. However As The Number of Zeros in each Non-zero Row increases row by row. The given matrix is in eachelo form.

D). $\begin{bmatrix} 1 & 0 & 0 & 7 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 4 \end{bmatrix}$ is in reduced row echelon form.

Explanation:

The Given Matrix Is Neither in row echelon form. Because Number of Zeros Before The first Non-zero

Entry does not increas
Row by row and
is not in
Reduced row echelon form.

~~Solution~~ Q.No.(2)

A). Second and revers
Row opration That
Transforms The Second
Matrix into first.

$$\begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0 & 2 & -5 & -1 \end{bmatrix},$$

$$\begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0 & 0 & 3 & -5 \end{bmatrix}$$

Q2: Solution. (A)

Let the first Matrix is A and the Second Matrix is B. That

$$A=\begin{bmatrix}1 & 3 & -1 & 5\\0 & 1 & -4 & 2\\0 & 2 & -5 & -1\end{bmatrix},\quad B=\begin{bmatrix}1 & 3 & -1 & 5\\0 & 1 & -4 & 2\\0 & 0 & 3 & -5\end{bmatrix}.$$

Consider

$$A=\begin{bmatrix}1 & 3 & -1 & 5\\0 & 1 & -4 & 2\\0 & 2 & -5 & -1\end{bmatrix}$$

By performing the elementary Row opration R_3-2R_2 We get Matrix B.

That is:

$$\underset{\sim}{R}\begin{bmatrix}1 & 3 & -1 & 5\\0 & 1 & -4 & 2\\0-0(2) & 2-2(1) & -5-2(-4) & -1-2(2)\end{bmatrix} R_3-2R_2$$

$$= \begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0 & 0 & 3 & -5 \end{bmatrix}$$

= B (The Second Matrix)

Now by performing the Reverse elementary Row Opration $R_3 + 2R_2$ on Second Matrix B. We get the first One A That is

$$\overset{R}{\sim} \begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0+2(0) & 0+2(1) & 3+2(-4) & -5+2(2) \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0 & 2 & -5 & -1 \end{bmatrix} = A$$ (The first Matrix)

Q3:: The row echelon form Is Used to Solve The System of linear equations.

a) What is The Defference B/w The row echelon And reduced row echelon form? What is The practical Use of reduced Row echelon form? Give one Example.

Solution :: A)

(A):: Row echelon form Is also Known as

Gauss elemination method

This method is very helpful in Solving a System of equation.

Row echelon form

Gives values of Unknown Variable involved in the Given System of equation. By Doing So we get value of one variable in last Raw in by back Subtitution (putting the values of Obtained variable in back-ward Direction.)

We get the values of Remaining variables.

Forexample. Row echelon form yeilds.

$a_1x_1 + b_2x_2 + c_1x_3 = d_1 \longrightarrow (i)$

$b_2x_2 + c_2x_3 = d_2 \longrightarrow (ii)$

$c_3x_3 = d_3 \longrightarrow (iii)$

following system of equation. from equation (iii) we Get value of x_3. By putting value of x_3 in eq (ii). We get x_2 and finally We Obtained Value of x_1 from eq (iii).

B) → Reduced row echelon form.

Reduced row echelon form is also Known as Gauss-Jordon Method is A technique used to Solve A System of equation. Unlike row echelon form, Reduced row echelon form Gives Solution Set of Variable Simultaneously. That is reduced row Echelon form yield in the following System.

$x_1 + 0x_2 + 0x_3 = d_1$

$0x_1 + 0x_2 + 0x_3 = d_2$

In Augmented Matrix form We would have

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & d_1 \\ 0 & 1 & 0 & d_2 \\ 0 & 0 & 1 & d_3 \end{array}\right]$$

Major practical Use of reduced row echelon form Is to Solve a System of Simultaneous equation. For example a System of equation as given as Below.

$$x + y + 2z = 1$$

$$2x + y + 8z = 12$$

$$3x + 5y + 4z = -3$$

Having Augmented Matrix as

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & 1 \\ 2 & -1 & 8 & 12 \\ 3 & 5 & 4 & -3 \end{array}\right] \longrightarrow (\mathrm{I})$$

And By Correcting in Reduced row echelon form Using elementary row Opration, we get

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 1 \end{array}\right]$$ That is

$\{(1, -2, 1)\}$ As Solution Set

Q3 :- B) :-

(B) :- Find an echelon form for the Below Matrix Using Row Operations. Wher ID2 is 2nd digit in your ID e.g. if your ID is (12345) ID2=2, ID3=3

IDfirst_last is the first And last digit of your ID i.e 15

Solution :: (B)

(B) :- **Given That**

$$\begin{bmatrix} 1 & ID2 & 8 \\ 2 & 8 & -1 \\ ID3 & 0 & 0 \\ 1 & -4 & ID_{first\_last} \end{bmatrix} \longrightarrow (I)$$

Suppose ID is 12345 Than Matrix

(I) becomes :-

$$\begin{bmatrix} 1 & 2 & 8 \\ 2 & 8 & -1 \\ -3 & 0 & 0 \\ 1 & -4 & 15 \end{bmatrix}$$

$$= \underset{\sim}{R} \begin{bmatrix} 1 & 2 & 8 \\ 0 & 4 & -17 \\ -3 & 0 & 0 \\ 1 & -4 & 15 \end{bmatrix} \quad R_2 - 2R_1$$

$$= \underset{\sim}{R} \begin{bmatrix} 1 & 2 & 8 \\ 0 & 4 & -17 \\ 0 & 6 & 24 \\ 1 & -4 & 15 \end{bmatrix} \quad R_3 + 3R_1$$

$$= \underset{\sim}{R} \begin{bmatrix} 1 & 2 & 8 \\ 0 & 4 & -17 \\ 0 & 6 & 24 \\ 0 & -6 & 7 \end{bmatrix} \quad R_4 - R_1$$

$$= \underset{\sim}{R} \begin{bmatrix} 1 & 2 & 8 \\ 0 & 4 & -17 \\ 0 & 1 & 4 \\ 0 & -6 & 7 \end{bmatrix} \quad \frac{1}{6} R_3$$

$$= \underset{R}{\sim} \begin{bmatrix} 1 & 2 & 8 \\ 0 & 1 & 4 \\ 0 & 4 & -17 \\ 0 & -6 & 7 \end{bmatrix} \quad R_2 \longleftrightarrow R_3$$

$$= \underset{R}{\sim} \begin{bmatrix} 1 & 2 & 8 \\ 0 & 1 & 4 \\ 0 & 0 & -33 \\ 0 & -6 & 7 \end{bmatrix} \quad R_3 - 4R_2$$

$$= \underset{R}{\sim} \begin{bmatrix} 1 & 2 & 8 \\ 0 & 1 & 4 \\ 0 & 0 & -33 \\ 0 & 0 & 33 \end{bmatrix} \quad R_4 - R_2$$

$$= \underset{R}{\sim} \begin{bmatrix} 1 & 2 & 8 \\ 0 & 1 & 4 \\ 0 & 0 & -33 \\ 0 & 0 & 0 \end{bmatrix} \quad R_4 + R_3$$

The end