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Q1 a: The sum of two numbers is K. Find the minimum value of the sum of their cubes.

Solution:

Let x and y = the numbers

z = sum of their cubes.

$K = x + y$

$y = K - x$

$z = x^3 + y^3$

$z = x^3 + (K-x)^3$

$dz/dx = 3x^2 + 3(K-x)^2(-1) = 0$

$x2 - (K^2 - 2Kx + x^2) = 0$

$x = 1/2K$

$Y = K - 1/2K$

$Y = 1/2K$

$z = (1/2K)^3 + (1/2K)^3$

$\left(z = 1/4K^3 \right)$ A

Answer

Q1B: Solution.

Let x and y = the numbers

$x+y=2 \longrightarrow$ Equation (1)

$1+y^2=0 \quad y'=-1$

$z=x^3+y^2 \longrightarrow$ Equation (2)

$dz/dx = 3x^2+2yy'=0$

$3x^2+2y(-1)=0$

$y=3/2\ x^2$

from Equation (1)

$x+3/2\ x^2=2$

$2x+3x^2=4$

$3x^2+2x-4=0$

$x=0.8685$ and -1.5352

Use

$x=0.86085$

$y=32(0.86852)$

$y=1.1315$

$z=0.86853+1.13152$

$\left(z=1.9357A\right)$

Answer

Q2(a) Solution:

The linear approximation is give by the equation.

$$f(x) \approx L(x)$$
$$= f(a) + f'(a)(x-a).$$

we just need to plug in the known values and calculation the value of $f(3.5)$:

$$L(x) = f(3) + f'(3)(x-3)$$

$$= 12 - 2(x-3) = 18 - 2x.$$

Then

$$f(3.5) \approx 18 - 2 \cdot 3.5 = 11.$$ Answer

Q 2(B) Solution: Let $f(x) = \sqrt[3]{x}$. The linear approximation at the Point $a = 8$ is given by

$$f(x) \approx L(x)$$
$$= f(8) + f'(8)(x-8).$$

Find the derivative:

$$f'(x) = (\sqrt[3]{x})' = \frac{1}{3}x^{-\frac{2}{3}} = \frac{1}{3\sqrt[3]{x^2}}$$

Compute the value of derivative at

$$a = 8.$$

$$f'(8) = \frac{1}{3\sqrt[3]{8^2}} = \frac{1}{12}$$

Substituting this, we get the function

$L(x)$ in the ~~from~~ form

$$f(x) = L(x) = 2 + \frac{1}{12}(x-8)$$

$$= \frac{x}{12} + \frac{4}{3}$$

Hence;

$$\sqrt[3]{9} = L(9) = \frac{9}{12} + \frac{4}{3} = \frac{9+16}{12}$$

$$= \frac{25}{12}$$ (Ans)

Q(3) Solution: $2xy - 9x2 + (2y + x2 + 1)dy/dx = 0$

$$\frac{dy}{dx} = 2xy - 9x^2 + (2y + x^2 + 1) = 0$$

$$\frac{dy}{dx} = (x^2 + 2y + 1) - 9x^2 + 2yx = 0$$

$$(x^2 + 2y + 1)$$

$$2yx$$

$$\frac{dy}{dx} = x^2 + 2y + 1 - 9x^2 + 2yx = 0$$

$$\frac{dy}{dx} = -8x^2 + 2yx + 2y + 1 = 0$$

$$X = \frac{-2y \pm \sqrt{(2y)^2 - 4.(-8)y}}{2.(-8)}$$

$$x = \frac{-2y \pm \sqrt{2^2y^2 - (-32)y}}{-16}$$

$$-2y \pm \sqrt{2^2y^2 - (-32)y}$$

$$2^2 y^2$$

$$(-32)y$$

$$-16$$

$$X = \frac{-2y \pm \sqrt{4y^2 + 32y}}{-6}$$

$$4y^2$$

$32y$

$$x = \frac{-2y + \sqrt{4y^2+32y}}{-16} \text{ or } x = \frac{-2y \sqrt{4y^2+32y}}{-16}$$

$$x = \frac{-2y \pm \sqrt{4y^2+32y}}{-16}$$

$4y^2$

$32y$

$$x = \frac{-2y + \sqrt{4y^2+3y}}{-16} \text{ or } x = \frac{-2y \sqrt{4y^2+32y}}{-16}$$

$$x = -\frac{-2y \sqrt{4y^2+32y}}{16} \text{ or } x = -\frac{-2y \sqrt{4y^2+32y}}{16}$$

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