

(1)

Question 1(a):-

i)  $x_a(t) = 3\cos 100\pi t + 4\sin 200\pi t$

Sol:-

According to sampling theorem

$$F_1 = 100\text{Hz}, F_2 = 200\text{Hz}$$

$$F_s \geq 2f_{\max}$$

$$f = \frac{\omega}{2\pi}$$

So,

$f_2$  is max (greater than  $f_1$ )

$$f_s > 2 \times 100$$

$$f_s = 200\text{Hz}$$

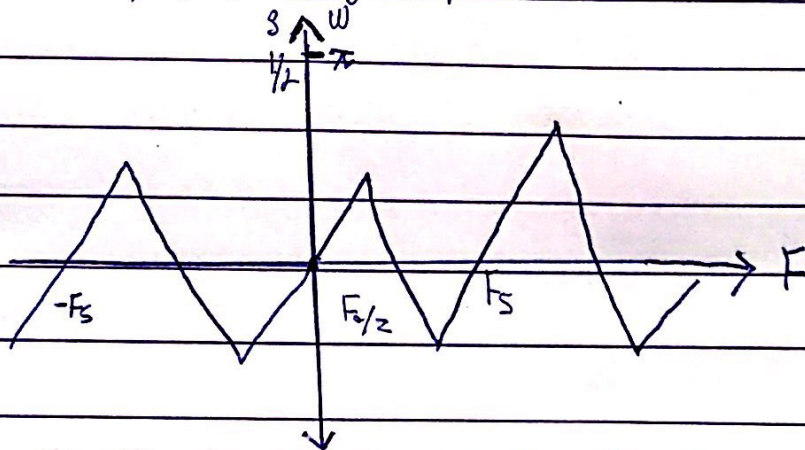
ii) Sol:-

$$f_s = 100\text{Hz}$$

$$f = \frac{100}{2} = 50\text{Hz}$$

This is the max frequency that can be represented uniquely by the sampled signal

$$\begin{aligned} \text{As } x_a[n] &= 3\cos 2\pi \left(\frac{50}{100}\right)n + 4\sin 2\pi \left(\frac{100}{100}\right)n \\ &= 3\cos \pi \left(\frac{5}{10}\right)n + 4\sin 2\pi n \end{aligned}$$



The effect of sampling rate on the newly generated discrete time signal is that. There will be no display phenomenon mean there will be not present unwanted component in Reconstruct of the signal.

iii) Sol<sup>o</sup> - folding frequency =  $\frac{f_s}{2} = \frac{100}{2}$

$f_1 = 50 \text{ Hz}$ ,  $f_2 = 100 \text{ Hz}$  = 50

frequency are either equal or greater than folding frequency. Hence for ideal interpolation we can construct the original signal.

$x_a(t) = 3 \cos 100\pi t + 4 \sin 200\pi t$

since only the frequency components at 100 Hz are present on the sampled signal. The analog signal we can recover or reconstruct is

$y_a(t) = 3 \cos 100\pi t$  Ans.

Question 1(b):-

Consider a discrete time signal which is given

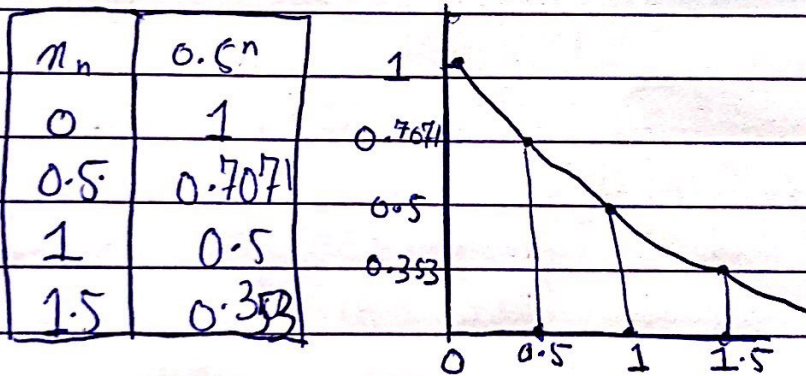
$$x(n) = \begin{cases} 0.5^n, & n \geq 0 \\ 0, & n < 0 \end{cases}$$

This signal is sampled at the rate  $F_s = 2\text{Hz}$

i) Draw the sampled signal

$$f_s = \frac{1}{T} = T = \frac{1}{f_s}$$

$$= \frac{1}{2} = 0.5 \text{ sec.}$$



ii) The samples of the signal ..... signal achieved.

Ans:-

$$L = 2^n$$

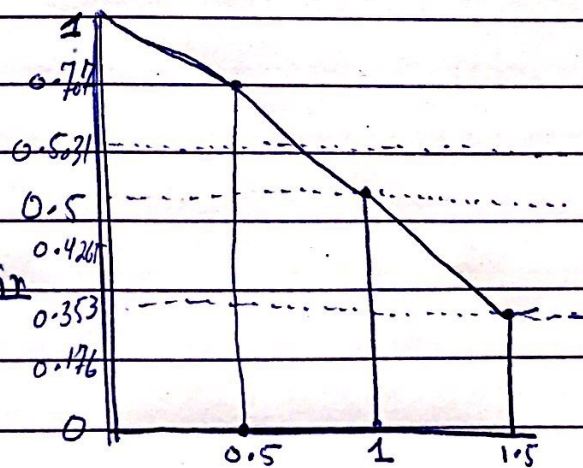
$$n = b + 1 = 3$$

$$L = 2^3 = 8 \text{ levels}$$

$$\text{Resolution} = \frac{x_{\max} - x_{\min}}{L}$$

$$= \frac{1 - 0}{8}$$

$$= 0.125$$



iii)

	Discrete Signal	intention	Reading	error
0	1	1.6	1.0	0.0
1	0.8535	0.8	0.9	-0.1
2	0.707	0.7	0.7	0.0
3	0.6035	0.6	0.6	0.0
4	0.5	0.5	0.5	0.0
5	0.4265	0.4	0.4	0.0
6	0.353	0.3	0.4	-0.1
7	0.1765	0.2	0.1	-0.1

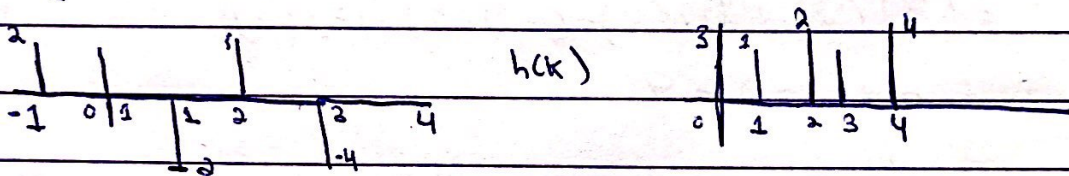
## Question 2 :-

Determine the response of the system

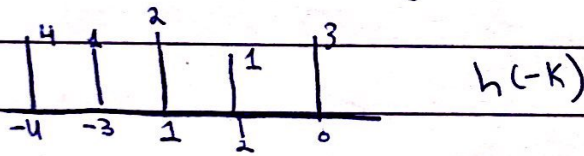
a) Input signal with given impulse response.

$$x[n] = \{2, -2, -4\} \quad h[n] = \{3, 1, 2, 1, 4\}$$

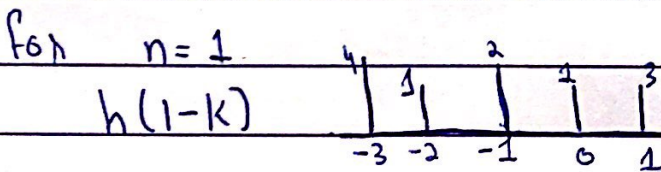
Sol:-  $y[n] = \sum_{k=-\infty}^{\infty} x[k] h[n-k]$



$h(-k)$  folded signal



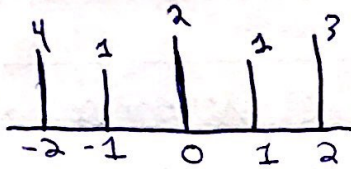
$$\begin{aligned} Y[0] &= \sum_{k=-1}^0 x[-1-k] h[-1-k] \\ &= 2 \times 2 + (1)(3) \\ &= 5 \end{aligned}$$



$$\begin{aligned} Y[1] &= \sum_{k=-1}^1 x[k] h[1-k] \\ &= x(-2)h[-1] + x(0)h(0) + x(1)h(1) \\ &= (2)(2) + (1)(1) + (3)(-2) \\ &= 4 + 1 - 6 \\ &= -1 \end{aligned}$$

$n=2$

$h(2-k)$



$$Y(2) = \sum_{k=-1}^2 m(n) h(2-k)$$

$$m(-1)h(-1) + 2(0) + h(0) + m(1)h(1)$$

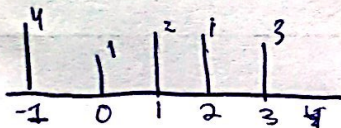
$$n(1)h(-1) + n(0) + h(0) + m(1)(1)$$

$$= (2)(2) + (1)(2) + (-2)(1) + (3)(3)$$

$$= 2 + 2 - 2 + 9$$

$$= 11$$

$n=3$



$$Y(3) = \sum_{k=0}^3 m(n) h(3-k)$$

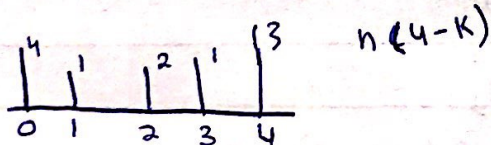
$$\Rightarrow m(-1)h(-1) + n(0)h(0) + m(1)h(1) + m(n)h(2)$$

$$= 2 \times 4 + (1)(1) + (-2)(2) + (3)(1) + (-4)(3)$$

$$= 4 + 1 - 4 + 3 - 12$$

$$= -8$$

$n=4$



$$f(4) = \sum_{k=0}^4 m(n) h(4-k)$$

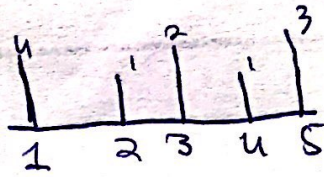
$$= m(0)h(0) + n(1)h(4) + m(2)h(2) + n(3)h(3)$$

$$= 4 - 2 + 6 - 4$$

$$= 4$$

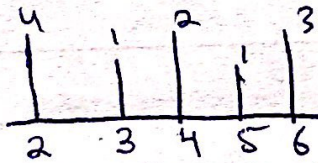
③ Qu. 20

$$n = 5$$



$$\begin{aligned}y(5) &= \sum_{k=1}^5 m(k) h(5-k) \\&= m(1)h(4) + m(2)h(3) + m(3)h(2) \\&= (-2)(4) + 3(1) + 3(-4)(2) \\&= -8 + 3 - 8 \\&= -13\end{aligned}$$

$$n = 6$$



$$\begin{aligned}y(6) &= \sum_{k=2}^6 m(k) h(6-k) \\&= (3)(4) + (1)(-4) \\&= 8\end{aligned}$$

## Q2 (B)

Sol

We have

$$u(m) = u(k) = \{ \alpha^{-2}, \alpha^{-1}, 1, \alpha, \alpha^2, \alpha^4, \alpha^5, \alpha \}$$

$$h_z(m) = h(k) = \{ \dots, 0, 1, 2, 4, 8, 16, 0, \dots \}$$

To find  $y(n)$

$$y(m) = \sum_{k=0}^{\infty} u(k) h(n-k)$$

for  $n=0$  first to find  $h(n-k) = h(-k)$

so by inverting  $h(k)$  we get  $h(-k)$

$$\Rightarrow h(-k) = \{ 16, 8, 4, 2, 1 \} \quad \text{--- (2)}$$

$$\text{So } y(0) = \sum_{k=0}^{\infty} u(k) \times h(-k)$$

$$y(0) = (\alpha^{-2} \times 16) + (\alpha^{-1} \times 8) + (1 \times 4) + (\alpha \times 2) + (\alpha^2 \times 1)$$

$$y(0) = 8\alpha^{-2} + 4\alpha^{-1} + 4 + 2$$

$$= \alpha^{-2} + 4\alpha^{-2} + 4\alpha^{-2} + 4\alpha^{-2}$$

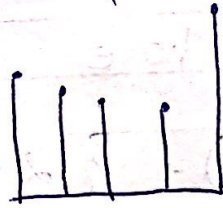
$$\text{for } n=1 \quad h(1-k) = \{ 16, 8, 4, 2, 1 \}$$



so  $y(x) = (x^{-2} \times 16) + (x^{-2} \times 8) + (1 \times 4) + (x^2) + (x^2)$

$$y(x) = 16x^{-2} + 8x^{-2} + 4 + dx + 2x^2$$

$$= x^2 + 2x + 4 + 8x^{-1} + 16x^{-2}$$

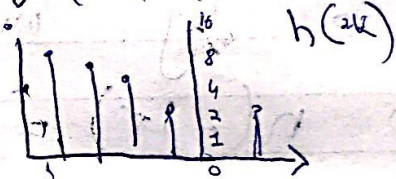


Now for  $n=2$

$$h(2-k) = \{16, 8, 4, 2, 7\}$$

$$y(x) = \{ (x^{-1} \times 16) + (1 \times 8) + (x \times 4) + (x^2 \times 2) + (x^3 \times 1) \}$$

$$= 16x^{-1} + 8 + 4x + 2x^2 + x^3$$



Now

$n=3$

$$h(3-k) = \{16, 8, 4, 2, 1\}$$

$$y(x) = (1 \times 16)(x \times 8) + (x^2 \times 4) + (x^3 \times 2) +$$

$$= 16 + 8x + 4x^2 + 2x^3 + x^4$$



Now

$$h(4-k) = \{16, 8, 4, 2, 1\}$$

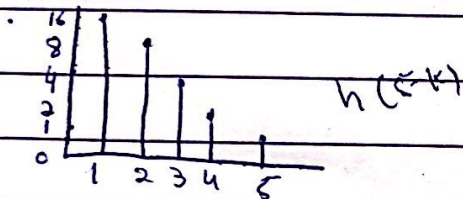
$$y(x) = (x \times 16) + (x^2 \times 8) + (x^3 \times 4) + (x^4 \times 2) + (x^5 \times 1)$$

$$16x + 8x^2 + 4x^3 + 2x^4 + x^5$$

$$h(5-k) = \{0, 16, 8, 4, 2, 1\}$$

$$y(5) = (\alpha^1 \times 0) + (\alpha^2 \times 16) + (\alpha^3 \times 8) + (\alpha^4 \times 4) + (\alpha^5 \times 2) + (\alpha^6 \times 1)$$

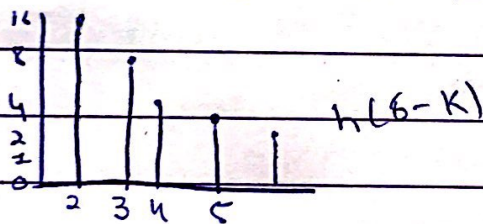
$$= 16\alpha^2 + 8\alpha^3 + 4\alpha^4 + 2\alpha^5 + \alpha^6$$



Similarly if we calculate for rest of the values of n up till there are any common values we get

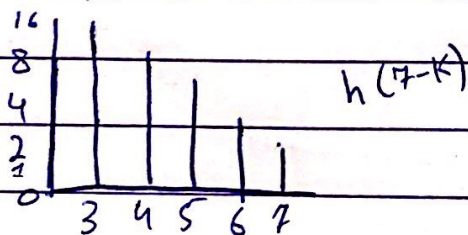
$$y(6) = 0 + 0 + 16\alpha^3 + 8\alpha^4 + 4\alpha^5 + 2\alpha^6$$

$$= 16\alpha^3 + 8\alpha^4 + 4\alpha^5 + 2\alpha^6$$



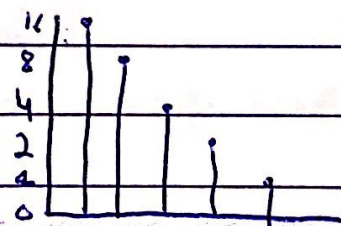
$$y(7) = 0 + 0 + 0 + 16\alpha^4 + 8\alpha^5 + 4\alpha^6$$

$$= 16\alpha^4 + 8\alpha^5 + 4\alpha^6$$



$$y(8) = 0 + 0 + 0 + 0 + 16\alpha^5 + 8\alpha^6$$

$$\Rightarrow 16\alpha^5 + 8\alpha^6$$



Q NO 3

(i)

Sol As we know that

Z - transform

$$x(z) = \sum_{n=0}^{\infty} \left(\frac{1}{4}\right)^n z^{-n} + \sum_{n=0}^{\infty} \left(\frac{1}{3}\right)^n z^{-n-1}$$

Using Geometric Series

$$\Rightarrow \frac{1}{1 - \frac{1}{4}z^{-1}} + \sum_{n=0}^{\infty} \left(\frac{1}{3}\right)^n z^{-n-1}$$

$$\Rightarrow \frac{1}{1 - \frac{1}{4}z^{-1}} + \frac{1}{1 - \frac{1}{3}z^{-1}}$$

$$\Rightarrow \frac{1}{1 - \frac{1}{4}z^{-1}} + \frac{1}{1 - \frac{1}{3}z^{-1}}$$

$$\Rightarrow \frac{1}{\left(1 - \frac{1}{4}z^{-1}\right) \left(1 - \frac{1}{3}z^{-1}\right)}$$

$$\frac{1 - \frac{1}{3}z + 1 - \frac{1}{4}z^2}{\left(1 - \frac{1}{4}z\right) \left(1 - \frac{1}{3}z\right)}$$

$$\frac{1 - \frac{1}{4}z^2}{\left(1 - \frac{1}{4}z\right) \left(1 - \frac{1}{3}z\right)}$$

$$\frac{1 - \frac{1}{3}z + 1 - \frac{1}{4}z' - 1 + \frac{1}{3} + \frac{1}{4}z + \frac{1}{12}}{(1 - \frac{1}{4}z') (1 - \frac{1}{3}z)}$$

$$1 - \frac{1}{12}$$

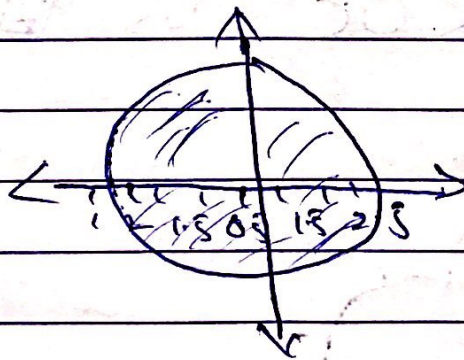
$$1 - \frac{1}{4}z^2 (1 - \frac{1}{3}z)$$

$$\frac{13}{12}$$

$$(1 - \frac{1}{4}z^2) (1 - \frac{1}{3}z)$$

Hence the ROC is  $\frac{1}{4} < |z| < 3$

The Sketch is under



Q3 (ii)

Sol

Using the Z transform Pair eq

$$\text{i.e. } u(n) = \alpha^{|n|} \longleftrightarrow u(z) = \frac{1}{1-\alpha z^{-1}} \quad (1)$$

By putting values

$$u_2(z) = \sum_{n=0}^{\infty} \left(\frac{1}{2}\right) z^{n-1} - \sum_{n=0}^{\infty} 2^n z^{-n}$$

$$\frac{1}{1-\frac{1}{2}z^{-1}} - \frac{1}{1-2z^{-1}}$$

$$= \frac{-\frac{1}{2}z^{-1}}{2}$$

$$\frac{1-\frac{1}{2}z^{-1}}{1-2z^{-1}}$$

As seen The ROC is  $|z| > 2$

The sketch are

