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Subject:- Linear Algebra

Instructor:-

MID TERM

Question:- 1

$$\left[\begin{array}{cccc|c} 1 & 103 & 3 & 0 & 5 \\ 0 & 1 & -103 & 0 & 7 \\ 0 & 0 & 1 & 0 & -6 \\ 0 & 0 & 0 & 1 & 103 \end{array} \right]$$

Solution:- This augmented matrix can be written as;

$$\left[\begin{array}{cccc|c} 1 & 0 & 3 & 0 & 5 \\ 0 & 1 & -1 & 0 & 7 \\ 0 & 0 & 1 & 0 & -6 \\ 0 & 0 & 0 & 1 & 0 \end{array} \right]$$

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(2)

Now to solve it, we should convert it into reduced echelon form.

$$\begin{array}{l} R_1 - 3R_3 \sim \\ R_2 + R_3 \sim \end{array} \left[\begin{array}{cccc|c} 1 & 0 & 3 & 0 & 5 \\ 0 & 1 & -1 & 0 & 7 \\ 0 & 0 & 1 & 0 & -6 \\ 0 & 0 & 0 & 1 & 0 \end{array} \right]$$

$$\left[\begin{array}{cccc|c} 1-0 & 0-0 & 3-3 & 0-0 & 5+18 \\ 0 & 1+0 & -1+1 & 0+0 & 7-6 \\ 0 & 0 & 1 & 0 & -6 \\ 0 & 0 & 0 & 1 & 0 \end{array} \right]$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 23 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & -6 \\ 0 & 0 & 0 & 1 & 0 \end{array} \right] \text{ Answer!}$$

We can see that it is converted into reduced echelon matrix. Hence, we can say that for solving the given matrix, we have to perform 2 elementary row operation and these operation are, $R_1 - 3R_3$ and $R_2 + R_3$

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(3)

Find the elementary row operation that transform the first matrix into

(a) Second and reverse row operation that transform the second matrix into first.

$$\begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0 & 2 & -5 & -1 \end{bmatrix}, \begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0 & 0 & 3 & -5 \end{bmatrix}$$

Solution:-

$$R_3 - 2R_2 \quad \begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0 & 2 & -5 & -1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0-0 & 2-2 & -5+8 & -1-4 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0 & 0 & 3 & -5 \end{bmatrix}$$

So, the first matrix can be

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(4)

Converted into the second with elementary row operation,

$$\boxed{R_3 - 2R_2}$$

$$\begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0 & 2 & -5 & -1 \end{bmatrix}, \begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0 & 0 & 3 & -5 \end{bmatrix}$$

Solution:-

$$R_3 + 2R_2 \begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0 & 0 & 3 & -5 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0+0 & 0+2 & 3-8 & -5+4 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0 & 2 & -5 & -1 \end{bmatrix}$$

So, the reverse row operation that transforms the second matrix into first is

$$\boxed{R_3 + 2R_2}$$

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Question 2 Part b

$$a. \begin{pmatrix} e & 0 & 0 & 0 \\ 0 & \pi & 0 & 0 \\ 0 & 0 & -\pi & 0 \\ 0 & 0 & 0 & e \end{pmatrix}$$

Statements

A matrix said to be Row echelon form
Multiply one row by a constant and then the result to the other row.

$$b. \begin{pmatrix} 1 & 0 & \pi \\ 0 & 1 & e \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Statement

A matrix said to be in echelon form because there have leading element.

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$$c. \begin{bmatrix} 5 & 0 & 0 & 7 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & 4 \end{bmatrix}$$

Statement:

This is is
echelon form.

$$d. \begin{bmatrix} 1 & 0 & 0 & 7 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 4 \end{bmatrix}$$

Statements-

A matrix is
said to be in echelon
form because here have
leading element 1.

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Question 3

The row echelon form is used to solve the system of linear equation.

a) what is the difference between the row echelon and reduced row echelon form? what is the practical use of reduce row echelon form?

The echelon form of a matrix isn't unique, which means there are infinite answers possible when you perform row reduction.

Row echelon reduced form is at the other end of the spectrum; it is unique which mean row-reduction

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on a matrix will produce the same answer no matter how you perform the row operation.

For example

~~the~~ echelon form

1. The first non-zero element in each row (column), called the leading entry (54)

2. Each leading entry is in a column (row) to the right of the leading entry in the previous row (column).

3. Rows (column) with all zero elements, if any, are below (after) the row (column) having a non-zero element

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$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \rightarrow \text{echelon form}$$

Reduce echelon form:-

A matrix is said to be in reduce row (column) echelon form when it satisfies the following condition,

1. The matrix satisfies condition for a row (column) echelon form

2. The leading entry in each row (column) is the only non-zero entry in its column row.

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$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \rightarrow \text{Reduce echelon form.}$$

Question 3

Part b

$$\begin{bmatrix} 1 & 102 & 8 \\ 2 & 8 & -1 \\ -103 & 0 & 0 \\ 1 & -4 & 10 - \text{first} - \text{last} \end{bmatrix}$$

I put my 1st at the matrix

now this matrix solution is that;

$$\begin{bmatrix} 1 & 6 & 8 \\ 2 & 8 & -1 \\ -10 & 0 & 0 \\ 1 & -4 & 11 \end{bmatrix}$$

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$$\begin{bmatrix} 1 & 6 & 8 \\ 2 & 8 & -1 \\ 0 & 0 & 0 \\ 1 & -4 & 11 \end{bmatrix} \quad \begin{matrix} * \\ 2R_1 - R_2 \end{matrix}$$

$$\begin{bmatrix} 1 & 6 & 8 \\ 0 & -4 & -17 \\ 0 & 0 & 0 \\ 1 & -4 & 11 \end{bmatrix} \quad R_3 \leftrightarrow R_2$$

$$\begin{bmatrix} 1 & 6 & 8 \\ 0 & -4 & -17 \\ 1 & -4 & 11 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 6 & 8 \\ 0 & -4 & -17 \\ 1 & -4 & 11 \\ 0 & 0 & 0 \end{bmatrix} \quad R_3 - R_1$$

$$\begin{bmatrix} 1 & 6 & 8 \\ 0 & -4 & -17 \\ 0 & -10 & 3 \\ 0 & 0 & 0 \end{bmatrix}$$

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$$\begin{bmatrix} 1 & 6 & 8 \\ 0 & -4 & -17 \\ 0-0 & -16(-4) & 3-(-17) \\ 0 & 0 & 0 \end{bmatrix} \left. \begin{array}{l} \\ \\ \\ \end{array} \right\} R_3 - R_2$$

$$\begin{bmatrix} 1 & 6 & 8 \\ 0 & -4 & -7 \\ 0 & -6 & 20 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 6 & 8 \\ 0 & -4 & -7 \\ 0 & -6 & 20 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 6 & 8 \\ 0 & -4 & -7 \\ 0+1 & -6+6 & 20+8 \\ 0 & 0 & 0 \end{bmatrix} \left. \begin{array}{l} \\ \\ \\ \end{array} \right\} R_3 + R_1$$

$$\begin{bmatrix} 1 & 6 & 8 \\ 0 & -4 & -7 \\ 1 & 0 & 18 \\ 0 & 0 & 0 \end{bmatrix}$$

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$$\begin{bmatrix} 1 & 6 & 8 \\ 0 & -4 & -7 \\ 0x1 & 0x-4 & 28x-7 \\ 0 & 0 & 0 \end{bmatrix} \quad R_3 \times R_2$$

$$\begin{bmatrix} 1 & 6 & 8 \\ 0 & -4 & -7 \\ 0 & 0 & 196 \\ 0 & 0 & 0 \end{bmatrix} \quad \text{Answer!}$$