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Date	25/06/2020
Semester	10^{th}.

Q.No 1(A)

Forier Transform of differentiation Integration of continous time

Let $x(t)$ be a continous-time signal with a fourier transform of $x(j\omega)$

i.e $$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(j\omega)\, e^{j\omega t}\, d\omega$$

Differentiating both side with respect to (t)

$$\frac{dx}{dt}(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} x(j\omega)\, \frac{d}{dt} \{ e^{j\omega t} \}\, d\omega$$

$$\frac{dx}{dt}(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} x(j\omega) \{ e^{j\omega t} . j\omega \}\, d\omega$$

$$\frac{dx}{dt}(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \{ j\omega\, x(j\omega) \}\, e^{j\omega t}\, d\omega$$

$$\Rightarrow F \{ \frac{d}{dt} x(t) \} = j\omega\, x(j\omega)$$

Result:-

We concluded that if a function is differentiated in time domain by $j\omega$ in frequency domain.

Part (B)

1) $x[n] = 2\delta[n] - 4\delta[n-2] + 2\delta[n-3]$

$h[n] = 3\delta[n] + \delta[n-1] + 2\delta[n-2]$

Solution:- Y $H(z)$ $X(z)$

Find $Y[n]$

$x(z) = 2 - 4z^{-2} + 2z^{-3}$

$H(z) = 3 + 1z^{-1} + 2z^{-2}$

Now, $Y(z) = H(z)\,x(z)$

$= (2 - 4z^{-2} + 2z^{-3})(3 + z^{-1} + 2z^{-2})$

$= 2z^{-1} + 4z^{-2} - 12z^{-2} - 4z^{-3} - 8z^{-4} + 6z^{-3} +$

$2z^{-4} + 4z$

$= 6 + 2z^{-1} - 8z^{-2} - 2z^{-3} + 6z^{-4} + 4z^{-5}$

To find $Y[n]$ use the delay property.

$Y[n] = 6\delta[n] + 2\delta[n-1] - 8\delta[n-2] - 2\delta[n-3]$

$+ 6\delta[n-4] + 4\delta[n-5]$

$$f(x) = \begin{cases} -\pi/2 & -\pi \le x \le 0 \\ x/2 & 0 \le x \le \pi \end{cases}$$

Reterive the fourier Series for given function.

Ans 1 (A) Part.

We conclude that a function is differentiated in time domain it is multiplied by jw in frequency domain. From above result we also conclude that if a function is differentiated in time domain it is multiplied with time domain jw in frequency domain. Similarly if a function is integrated in time domain then it is divided by "jw" frequency domain.

We know that differentiation in domain corresponds to multiplication jw in frequency domain. From the property we might respect the multiplication by it in the time domain correspond roughly to differentiation frequency domain. As we know that

$$x(j\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt$$

differentiate both side with respect "$j\omega$"

$$\frac{d}{d\omega} x(j\omega) = \int_{-\infty}^{\infty} -jt\, x(t)\, e^{-j\omega t}\, dt$$

$$d/d\omega\, x(j\omega) = -jt \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt$$

$$\frac{d}{d\omega} x(j\omega) = -jt\, \mathcal{F}\{x(t)\}$$

$$\Rightarrow -jt\, x(t) \xleftrightarrow{\mathcal{F}} d/dt\, x(j\omega)$$

Ans: 2

$$f(x) = \begin{cases} -\pi/2 & -\pi \le x \le 0 \\ \pi/2 & 0 \le x \le x \end{cases}$$

as

$$a_0 = 1/2\pi \int_{-\pi}^{\pi} f(x)\, dx$$

$$= 1/2\pi \left[\int_{-\pi}^{0} f(x)\, dx + \int_{0}^{\pi} f(x)\, dx \right.$$

$$= 1/2\pi \int_{-\pi}^{0} -\pi/2\, dx + \int_{0}^{\pi} x/2\, dx$$

$$= 1/2\pi \left[-\pi/2 \int_{-\pi}^{0} 1\, dx + \pi/2 \int_{0}^{\pi} 1\, dx \right.$$

$$= \frac{1}{2\pi}\left[-\frac{\pi}{2}x\Big|_{-\pi}^{0} + \frac{\pi}{2}\left[\pi - 0\right]\right]$$

$$= \frac{1}{2\pi}\left[\frac{\pi}{2}(\pi) + \frac{\pi}{2}\left[\pi - 0\right]\right]$$

$$= \frac{1}{2\pi}\left[\frac{-\pi^2}{2} + \frac{\pi^2}{2}\right]$$

$$= \frac{1}{2\pi}\left[\frac{0}{2}\right] \Rightarrow (a_0 \Rightarrow 0)$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx\,dx$$

$$= \frac{1}{\pi}\int_{-\pi}^{0} f(x)\cos nx\,dx + \int_{0}^{\pi} f(x)\cos nx\,dx$$

$$= \frac{1}{\pi}\int_{-\pi}^{0} -\frac{\pi}{2}\cos nx\,dx + \int_{0}^{\pi}\frac{\pi}{2}\cos nx\,dx$$

$$= \frac{1}{\pi}\ \frac{-\pi}{2}\,\frac{\sin nx}{n}\Big|_{-\pi}^{0} + \frac{\pi}{2}\,\frac{\sin nx}{n}\Big|_{0}^{\pi}$$

$$= \frac{1}{\pi n}\left[\frac{-\pi}{2}\sin n(0) - \sin n(0-\pi)\right]$$

$$+ \frac{\pi}{2}\left[\sin n(\pi) - \sin n(0)\right]$$

$$= \frac{1}{n\pi}\left[\frac{-\pi}{2}(0) + \frac{\pi}{2}(0)\right]$$

$$= \frac{1}{n\pi}(0)$$

$(a_n = 0)$

Now, $b_n = 1/\pi \int_{-\pi}^{\pi} \sin nx\,dx$

$= 1/\pi \left[\int_{-\pi}^{0} f(x) \sin nx\,dx + \int_0^{\pi} f(x) \sin nx\,dx \right]$

$= 1/\pi \left[\int_{-\pi}^{0} -\pi/2 \sin nx\,dx + \int_0^{\pi} \pi/2 \sin nx\,dx \right]$

$= 1/\pi \left[-\pi/2 \int_{-\pi}^{0} \sin nx\,dx + \pi/2 \int_0^{\pi} \sin nx\,dx \right]$

$= 1/\pi \left[-\pi/2 \, \frac{-\cos nx}{n} \Big|_{-\pi}^{0} + \pi/2 \, \frac{-\cos nx}{n} \Big|_0^{\pi} \right]$

$= 1/n\pi \left[-\pi/2 \left[-\cos n(0) + \cos n(-\pi) \right] + \pi/2 \right.$

$\left. \left[-\cos n(\pi) + \cos n(0) \right] \right]$

$= 1/n\pi \left[-\pi/2 \left[-1 + \cos n(-\pi) \right] + \pi/2 \left[-\cos n\pi + \cos n(0) \right] \right]$

$\pi/2 = 1/n\pi \left[-1 \left[-1 + \cos n(-\pi) \right) + 1 \left[-\cos nx + 1 \right] \right]$

$= 1/2n \left[1 - \cos n\pi - \cos n\pi + 1 \right]$

$= 1/2n \left[2 - 2\cos n\pi \right]$

Now,

$$b_n = \begin{cases} 0 & \text{if } n \text{ is even} \\ 4/2n & \text{if } n \text{ is add} \end{cases}$$

$$\left\{ b_n = 4/2n \right\}$$

$f(x) = a_0 + a_1 \cos x + a_2 \cos 2x + a_3 \cos 3x + ---$

$+ b_1 \sin x + b_2 \sin 2x + b_3 \sin 3x + -----$

$f(x) = (0) + (0) \cos x + 0 \cos(2x) + 0 \cos 3x + ----$

$= 4/2 \sin x + (0) \sin^2 x + 4/3(2) \sin 3x + ----$

$\{ 4/2 \sin x + 4/6 \sin 3x + ---- \}$

Ans:- 5

$x(t) = e^{-at} \quad u(t)$

Solution:- CTFT (Continous time Fourier Transform)

if a signal $x(t)$ is given by

$$X(j\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt$$

As the given signal is multiplied by a step function hence limit $0 \leq t \leq \infty$

$$X(j\omega) = \int_0^\infty e^{-at}(1)\, e^{-j\omega t}\, dt$$

$$= \int_0^\infty e^{-(a+j\omega)t}\, dt$$

$$= \frac{e^{-(a+j\omega)t}}{-(a+j\omega)} \Big|_0^\infty$$

$$= \frac{-1}{a+j\omega}\left[e^{-\infty} - e^{0}\right]$$

$$= \frac{-1}{a+j\omega}\left[0-1\right]$$

$$X(j\omega) = 1/a+j\omega$$

Plot:- Since, this fourier transform is a complex valued

So to plot it is a function of "ω" we express $X(j\omega)$ in terms of its megnitude and phase.

i.e $|X(j\omega)| = 1/\sqrt{a^2+\omega^2}$ => megnitude

$\angle X(j\omega) = -\tan^{-1}(\omega/a)$

Note:- We have taken -ve sign with

to Phase because, whenever the whole vector in denomenator the angle is negative because always expression is ~~numer~~ numenator.

e.g: $1\angle 0° / 26\angle 36° = 1/26 \angle -36°$

Ans:- 4

$$A = \begin{bmatrix} -2 & 1 \\ 1 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 2 \end{bmatrix}, \quad D = [0]$$

$$G(s) = C(SI - A)^{-1} B + D$$

$$= \begin{bmatrix} 1 & 2 \end{bmatrix} \left[s \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} -2 & -1 \\ 1 & 0 \end{bmatrix} \right]^{-1} \begin{bmatrix} 1 \\ 0 \end{bmatrix} + [0]$$

$$= \begin{bmatrix} 1 & 2 \end{bmatrix} \left[\begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} -2 & -1 \\ 1 & 0 \end{bmatrix} \right]^{-1} \begin{bmatrix} 1 \\ 0 \end{bmatrix} + [0]$$

$$= \begin{bmatrix} 1 & 2 \end{bmatrix} \begin{bmatrix} S+2 & 1 \\ 0-1 & S-0 \end{bmatrix}^{-1} \begin{bmatrix} 1 \\ 0 \end{bmatrix} + [0]$$

$$= \begin{bmatrix} 1 & 2 \end{bmatrix} \begin{bmatrix} S+2 & 1 \\ -1 & S \end{bmatrix}^{-1} \begin{bmatrix} 1 \\ 0 \end{bmatrix} + [0]$$

$$G(s) = \begin{bmatrix} 1 & 2 \end{bmatrix} \frac{1}{s^2+2s+1} \begin{bmatrix} s & -1 \\ 1 & s+2 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$= \frac{1}{s^2+2s+1} \begin{bmatrix} 1 & 2 \end{bmatrix} \begin{bmatrix} s \\ 1 \end{bmatrix}$$

$$= \frac{1}{s^2+2s+1} \begin{bmatrix} s & 2 \end{bmatrix}$$

Ans:- 3 $\dfrac{2z^2+2z}{z^2+2z-3}$

$$x(z) = \frac{2z^2+2z}{(z-1)(z+3)}$$

$$\frac{2z^2+2z}{\cancel{z^2+2z}} = \frac{A}{(z-1)} + \frac{B}{z+3} \quad (A)$$

$$2z^2+2 = A(z+3)+B(z-1) \longrightarrow (B)$$

Put $z = -3$ in eq (B)

$$2(-3)^2+2(-3) = A(-3+3)+B(-3-1)$$

$$2(9)-6 = B(-4)$$

$$18-6 = BB(-4)$$

$$B = 12/4$$

$$B = -4$$

Now put $z = 1$ in eq (B)

$$2(1)^2+2 = A(1+3)+B(\cancel{1}-1)$$

$$2+2 = A(4)$$

$$4 = A(4) \qquad 4/4$$

$\boxed{A = 1}$

Now Put value of A and B in eq (A)

$$\frac{2Z^2+2}{(Z-1)(Z+3)} = \frac{1}{(Z-1)} - \frac{-3}{Z+3}$$

$$X(Z) = \frac{Z}{Z-1} - 3\frac{Z}{Z+3}$$

Inverse Z - Transform

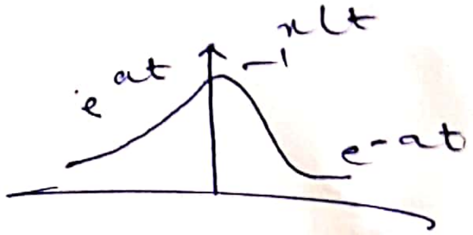
~~$x[n] = {}^{1}u[n] - 3(-3)^{k}$~~

$$x[n] = u[3] + 1[-1]^{k}$$

Ans 5 The fourier transform of the given function $x(t)$ is given by

$$x(j\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt$$

$$x(j\omega) = \int_{-\infty}^{\infty} e^{-a(t)}\, e^{-j\omega t}\, dt$$



Note

$$e^{-a(t)} = \begin{cases} e^{-at} & \text{for } t \geq 0 \\ e^{-a(-t)} = e^{at} & \text{for } t < 0 \end{cases}$$

$$\therefore x(j\omega) = \int_{-\infty}^{0} e^{at}\, e^{-j\omega t}\, dt + \int_{0}^{\infty} e^{-at} \cdot e^{-j\omega t}\, dt$$

$$x(j\omega) = \int_{-\infty}^{0} e^{(a-j\omega)t}\, dt + \int_{0}^{\infty} e^{-(a+j\omega)t}\, dt$$

$$= \frac{e^{(a-j\omega)t}}{a-j\omega}\Bigg|_{-\infty}^{0} + \frac{e^{(a+j\omega)t}}{-(a+j\omega)}\Bigg|_{0}^{\infty}$$

$$= \frac{1}{(a-(j\omega)}\left[e^{0} - e^{-\infty}\right] - 1/(a+j\omega)\left[e^{-\infty} - e^{0}\right]$$