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Sessional As  
Subject : Probability on statistics  
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Q NO 1:-

Answer:- 1:-

E (1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (1, 7), (1, 8)  
(2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6), (2, 7), (2, 8)  
(3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6), (3, 7), (3, 8)  
(4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6), (4, 7), (4, 8)  
(5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6), (5, 7), (5, 8)  
(6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6), (6, 7), (6, 8)

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(7, 1), (7, 2), (7, 3), (7, 4), (7, 5), (7, 6), (7, 7), (7, 8)

(8, 1), (8, 2), (8, 3), (8, 4), (8, 5), (8, 6), (8, 7), (8, 8) }

Set:

$$A = \{ \text{The sum is 7} \}$$

$$B = \{ \text{The sum is even} \}$$

$$C = \{ \text{The sum is greater than 8} \}$$

$$D = \{ \text{The two dice had two same outcome} \}$$

Now

$$A = \{ (1, 6), (2, 5), (3, 4), (5, 2), (6, 1), (4, 3) \}$$

$$B = \{ (1, 1), (1, 3), (1, 5), (1, 7), (2, 2), (2, 4), (2, 6), (2, 8)$$

$$(3, 1), (3, 3), (3, 5), (3, 7), (4, 2), (4, 4), (4, 6)$$

$$(4, 8), (5, 1), (5, 3), (5, 5), (5, 7), (6, 2)$$

$$(6, 4), (6, 6), (6, 8), (7, 1), (7, 3), (7, 5)$$

$$(7, 7), (8, 2), (8, 4), (8, 6), (8, 8) \}$$

$$C = \{ (1, 8), (2, 7), (2, 8), (3, 6), (3, 7), (3, 8)$$

$$(4, 5), (4, 6), (4, 7), (4, 8), (5, 4), (5, 5)$$

$$(5, 6), (5, 7), (5, 8), (6, 3), (6, 4), (6, 5)$$

$$(6, 6), (6, 7), (6, 8), (7, 2), (7, 3), (7, 4)$$

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$(7,5), (7,6), (7,7), (7,8), (8,1), (8,2),$   
 $(8,3), (8,4), (8,5), (8,6), (8,7), (8,8)$

$D = \{ (1,1), (2,2), (3,3), (4,4), (5,5), (6,6),$   
 $(7,7), (8,8) \}$

$$A \cap B = \{ \} \text{ OR } \emptyset$$

$$A \cap C = \{ \}$$

$$A \cap D = \{ \}$$

$$P(A) = 6/64, \quad P(B) = 32/64$$

$$P(C) = 36/64, \quad P(D) = 8/64$$

$$P(A \cap B) = 0, \quad P(A \cap C) = 0, \quad P(A \cap D) = 0$$

Hence,

$$P(A/B) = \frac{P(A \cap B)}{P(B)} = 0 \times \frac{32}{64}$$

$$P(A/C) = 0$$

$$P(A/D) = \frac{P(A \cap D)}{P(D)} = 0 \times \frac{8}{64}$$

$$P(A/D) = 0$$

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(Q 3):

A and B play a game in which A's probability of winning is  $\frac{2}{3}$  in a series of 8 games. What is the probability that A will win.

1. Exactly 4 games
2. At least 4 games
3. from 3 to 6 games

Given that =

$$p = \frac{2}{3} \quad n = 8$$

$$q = 1 - p$$

$$= 1 - \frac{2}{3}$$

$$q = \frac{1}{3}$$

Let "x" denotes the number of games won by A then

$$\begin{aligned} \text{(i) } P(x=4) &= \binom{8}{4} \left(\frac{2}{3}\right)^4 \left(\frac{1}{3}\right)^4 \\ &= \frac{1120}{6561} \\ &= 0.1707 \end{aligned}$$

(ii)  $P(x \geq 4)$

$$\begin{aligned} &1 - P(x < 4) \\ &= 1 - \sum_{x=0}^3 \binom{8}{x} \left(\frac{2}{3}\right)^x \left(\frac{1}{3}\right)^{8-x} \\ &= 1 - \left[ \left(\frac{1}{3}\right)^8 + 8 \left(\frac{2}{3}\right) \left(\frac{1}{3}\right)^7 + 28 \left(\frac{2}{3}\right)^2 \left(\frac{1}{3}\right)^6 + \right. \\ &\quad \left. P-T-O \right] \end{aligned}$$

$$\begin{aligned}
 & 56 \left( \frac{2}{3} \right)^3 \left( \frac{1}{3} \right)^5 \\
 &= 1 - \frac{1}{6561} [1 + 16 + 112 + 448] \\
 &= 1 - \frac{577}{6561} \\
 &= \frac{6561 - 577}{6561} \\
 &= \frac{5984}{6561} \\
 &= \boxed{0.9121}
 \end{aligned}$$

(ii) (iii)  $P(3 \leq x \leq 6)$

$$\begin{aligned}
 & \sum_{x=3}^6 \binom{8}{x} \left( \frac{2}{3} \right)^x \left( \frac{1}{3} \right)^{8-x} \\
 &= \binom{8}{3} \left( \frac{2}{3} \right)^3 \left( \frac{1}{3} \right)^5 + \binom{8}{4} \left( \frac{2}{3} \right)^4 \left( \frac{1}{3} \right)^4 + \binom{8}{5} \left( \frac{2}{3} \right)^5 \left( \frac{1}{3} \right)^3 + \\
 & \quad \binom{8}{6} \left( \frac{2}{3} \right)^6 \left( \frac{1}{3} \right)^2 \\
 &= \frac{8}{(3)^8} [56 + 140 + 224 + 224] \\
 &= \frac{8 \times 644}{6561} = \frac{5152}{6561} = \boxed{0.7852}
 \end{aligned}$$

(Q4)

Proof:-

Since the  $C_i$ 's form a partition of the sample space we can apply the law of total probability for  $A \cap B$ .

$$P(A \cap B) = \sum_{i=1}^m P(A \cap B | C_i)$$

$$P(A \cap B) = \sum_{i=1}^m P(A | C_i) P(B | C_i) P(C_i)$$

$\therefore$  (A and B are conditionally independent)

$$P(A \cap B) = \sum_{i=1}^m P(A | C_i) P(B) P(C_i)$$

$\therefore$  (B is independent of all  $C_i$ 's)

$$P(A \cap B) = P(B) \sum_{i=1}^m P(A | C_i) P(C_i)$$

$$P(A \cap B) = P(B) P(A)$$

$\therefore$  (law of total probability)

hence A and B are independent.

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(105)

Derive the binomial distribution and find its mean and variance.

Ans

The probability function for a binomial random variable is

$$b(x; n, p) = \binom{n}{x} p^x q^{n-x}$$

$$b(x; n, p) = \binom{n}{x} p^x (1-p)^{n-x} \quad q = 1-p$$

This is the probability of having  $x$  success in a series of  $n$  independent trials when the probability of success in any one of the trials is  $p$  if  $x$  is a random variable with this probability distribution.

$$E(x) = \sum_{x=0}^n x \binom{n}{x} p^x (1-p)^{n-x}$$

$$= \sum_{x=0}^n x \frac{n!}{x!(n-x)!} p^x (1-p)^{n-x}$$

$$= \sum_{x=0}^n \frac{n!}{(x-1)!(n-x)!} p^x (1-p)^{n-x}$$

Since  $x=0$  term vanishes, let  $y = x-1$  and  $m = n-1$  Subbing  $x = y+1$  and  $n = m+1$  into the last sum

$$E(x) = \sum_{y=0}^m \frac{(m+1)!}{y!(m-y)!} p^{y+1} (1-p)^{m-y}$$

$$= (m+1)p \sum_{y=0}^m \frac{m!}{y!(m-y)!} p^y (1-p)^{m-y}$$

$$= np \sum_{y=0}^m \frac{m!}{y!(m-y)!} p^y (1-p)^{m-y}$$

By binomial theorem  $(a+b)^m = \sum_{y=0}^m \frac{m!}{y!(m-y)!} a^y b^{m-y}$

Set  $a=p$  and  $b=1-p$

$$\sum_{y=0}^m \frac{m!}{y!(m-y)!} p^y (1-p)^{m-y}$$

$$= \sum_{y=0}^m \frac{m!}{y!(m-y)!} p^y (1-p)^{m-y}$$

$$= (a+b)^m$$

$$= (p+1-p)^m$$

$$= (p+1-p)^m$$

$$= 1$$

So that

$$\boxed{E(x) = np}$$

Similarly but this time using  $y = x-2$  and  $m = n-2$

$$E(x(x-1)) = \sum_{x=0}^n x(x-1) \binom{n}{x} p^x (1-p)^{n-x}$$

$$= \sum_{x=0}^n x(x-1) \frac{n!}{x!(n-x)!} p^x (1-p)^{n-x}$$

$$= \sum_{x=2}^n \frac{n!}{(x-2)!(n-x)!} p^x (1-p)^{n-x}$$

$$= n(n-1)p^2 \sum_{x=2}^n \frac{(n-2)!}{(x-2)!(n-x)!} p^{x-2} (1-p)^{n-x}$$

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$$\begin{aligned} &= n(n-1)p^2 \sum_{j=0}^m \frac{m!}{j!(m-j)!} p^j (1-p)^{m-j} \\ &= n(n-1)p^2 (1p + (1-p))^m \\ &= n(n-1)p^2 \end{aligned}$$

So the variance of  $x$  is

$$\begin{aligned} E(x^2) - E(x)^2 &= E(x(x-1)) + E(x) - E(x)^2 \\ &= n(n-1)p^2 + np - (np)^2 \\ &= \boxed{np(1-p)} \end{aligned}$$

Q 6 Ans

Binomial Distribution:

of repeated independent trials each may experiment consist  
trial having two possible outcomes

eg The two possible outcomes of a trial may  
head or tail. Success and failure. True  
and false etc.

The formula of binomial distribution is:

$$P(X=x) f(x) = \binom{n}{x} p^x q^{n-x}$$

P.T.O

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### Binomial frequency Distribution :-

If the binomial probability distribution is multiplied by  $N$ , then the number of experiments or sets the resulting distribution is known as the binomial frequency distribution. The formula of binomial frequency distribution is:

$$N \binom{n}{x} p^x q^{n-x}$$

(1) 2

**Ans:-** When we are rolling two dice there are 36 different combinations. Counting these up there are 15 possibilities less than 7:  $(1,1), (1,2), (1,3), (1,4), (1,5), (2,1), (2,2), (2,3), (2,4), (3,1), (3,2), (3,3), (4,1), (4,2), (5,1), (5,2)$ . The probability of getting less than 7 is  $\frac{15}{36} = \frac{5}{12}$ .

There are 6 possible combinations of getting a 7:  $(1,6), (2,5), (3,4), (4,3), (5,2), (6,1)$  which gives a probability of  $\frac{6}{36} = \frac{1}{6}$ .

This means that 21 possibilities account for getting less than or equal to 7. So there are 15 remaining possibilities.

(11)

possible of getting more than 7.  
This is the same as the probability  
of getting less than 7, so the  
probability must be  $\frac{5}{12}$  as well.  
must assume that each combination  
is equally likely to all as any other  
one, therefore the dice are fair or  
else the calculations don't

work.

(Q7)

Coefficient of variation  
for Data set A:-

$$CV = \frac{S}{\mu} \times 100$$

$$CV = \frac{3}{45} \times 100$$

$$CV = 6.7$$

for Data Set B:-

$$CV = \frac{11}{60} \times 100$$

$$CV = 18.3$$

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for Data Set C:-

$$CV = \frac{5}{50} \times 100$$

$$CV = 10$$

for Data Set D:-

$$CV = \frac{15}{25} \times 100$$

$$CV = 60$$

The END