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Q1

Ans

Given data :-

$E = 29,000\ Ksi$, $I = 150\ in^4$

δ_{st} = deflection due to 7725 lb Static load
Beam is pulled ½" downwards

Required:

Natural time period of System Develope and Soln. Equation of motion.
Draw graph To Show The variation of displacement With Time and The variation of equavalent Static forces With time

Sol

The general E.O.M for SDOF System is

$$Ku + C\dot{u} + m\ddot{u} = P(t)$$

In our Case System is undamped (C=0) undergoing free vibration $P(t)=0$

Hence general EOM become

$$Ku + m\ddot{u} = 0 \quad \text{------} (1)$$

$$K = 3EI/L^3$$

$$= \frac{3 \times 29000\ k/in^2 \times 150\ in^4}{(10 \times 12\ in)^3}$$

$$\boxed{= 7.55\ K/in}$$

In order to eliminate the chances of mistake during calculation, it is more appropriate to use fundamental units like lb, ft sec or kg, m, sec.

$$K = 7.55\,K/in = 90625\ lb/ft$$

$$m = \frac{7725\ sec^2}{32.2\ ft} = 240\ slug$$

$$\omega_n = \sqrt{K/m} = \sqrt{96025/240}$$

$$\boxed{W_n = 20\ rad/sec}$$

$$T_n = 2\pi/W_n = 2\pi/20$$

$$T_n = \frac{2 \times 3.14}{20} = \boxed{0.314\ sec}$$

Subsituting the corresponding values in eq ①

$$90625u + 240\ddot{u} = 0$$

where "K" is in lb/ft and "m" is in lb sec/ft²

General Solution to EOM for undamped free vibration is

$$u(t) = u(0)\cos(w_n t) + \dot{u}(0)/w_n \sin(w_n t)$$

$$u(0) = 1/2'' = 1/24 \, ft \quad \text{and} \quad \dot{u}(0) = 0$$

$$u(t) = (1/24) \times \cos(20t) + 0$$

$$= 1/24 \times \cos(20 \cdot t)$$

Equivalent Static force at any time "t" is

$$f_s(t) = K \cdot u(t) = \frac{90625 \times \cos(20t)}{24}$$

$$f_s(t) = 3776 \, (\cos 20t)$$

Amplitude of dynamic displacement, u_0 for undamped free vibration is

$$u_0 = \sqrt{[(u(0))^2 + (\dot{u}(0)/w_n)^2]}$$

$$= \sqrt{((1/24)^2 + 0)} \qquad = 1/24 \, ft$$

Amplitude of equivalent Static forces f_{so}

$$K u_0 = 90625 \times 1/24$$

$$\boxed{= 3776 \, lb}$$

Q2 Problem

Given data

$E = 29000$ Ki
$I = 150$ in^4
$\delta_{st} = 7725$ lb

Take $\zeta_o = 2.5\%$

Sol

E.O.M for damped free vibration

$$Ku + C\dot{u} + m\ddot{u} = 0 \quad \text{---} \; ①$$

it is know from problem No 1 that;
$K = 90625$ lb/ft and $m = 240$ lb·sec^2/ft

$$C = \zeta_o \times 2 m w_n$$

$$C = 0.025 \times 2(240)(20)$$

$$C = 0.025 \times 9600$$

$$\boxed{C = 240 \text{ lb·sec/ft}}$$

By subsituting values of K, C and m in eq ①

$$Ku + C\dot{u} + m\ddot{u} = 0$$
$$90625u + 240\dot{u} + 240\ddot{u} = 0$$

Solution The E.O.M for damped free vibration is;

$$u(t) = e^{-\zeta_0 \omega_n t}\left[u(0)(0)(\omega_D t) + \frac{1}{\omega_D}\left(u(0)\,\zeta_0\omega_n\right]\cdot \sin(\omega_D t)\right)$$

$$\Rightarrow \omega_D = \sqrt{\frac{K}{m}} = \sqrt{\frac{90625}{240}}$$

$$\boxed{\omega_D = 19.43 \text{ rad/Sec}}$$

$$\rightarrow u(t) = e^{-0.025\times 19.43 t}\left[\frac{1}{24}\times\cos(19.43t) + \frac{1}{19.43}\times\left[0 + \frac{1}{24}\times 0.025\times 19.43\times \sin(19.43t)\right.\right.$$

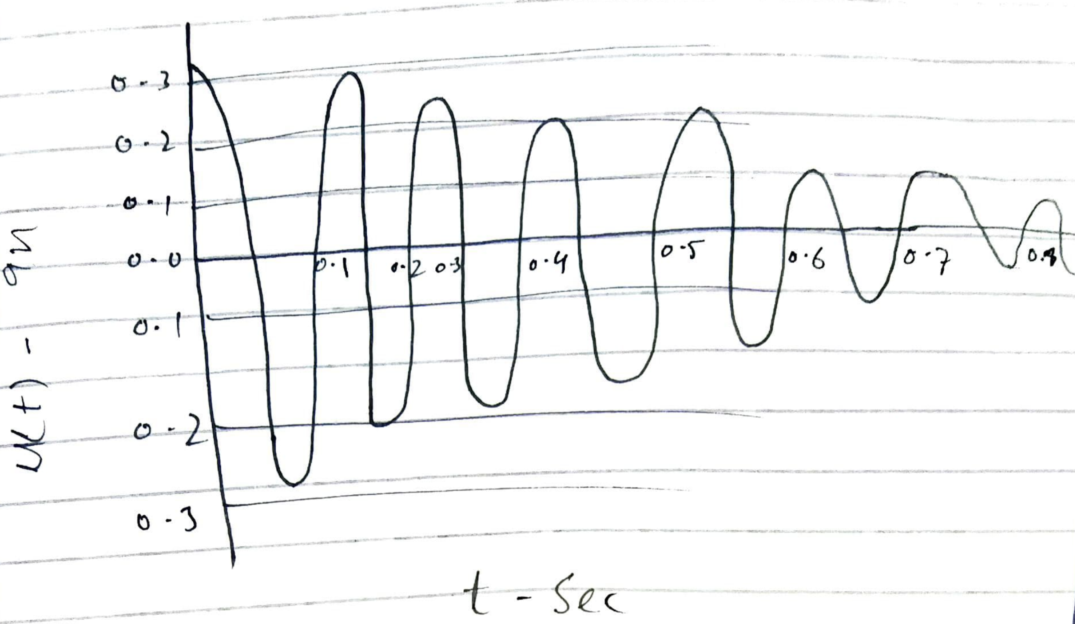
$$u(t) = e^{-0.48675}\left[0.041667\times\cos(19.43t) + 0.05136\times 0.02028 \sin(19.43t)\right.$$

$$u(t) = e^{-0.48675}\left[0.041667\times\cos(19.43) + 0.001041 \sin(19.43t)\right)$$

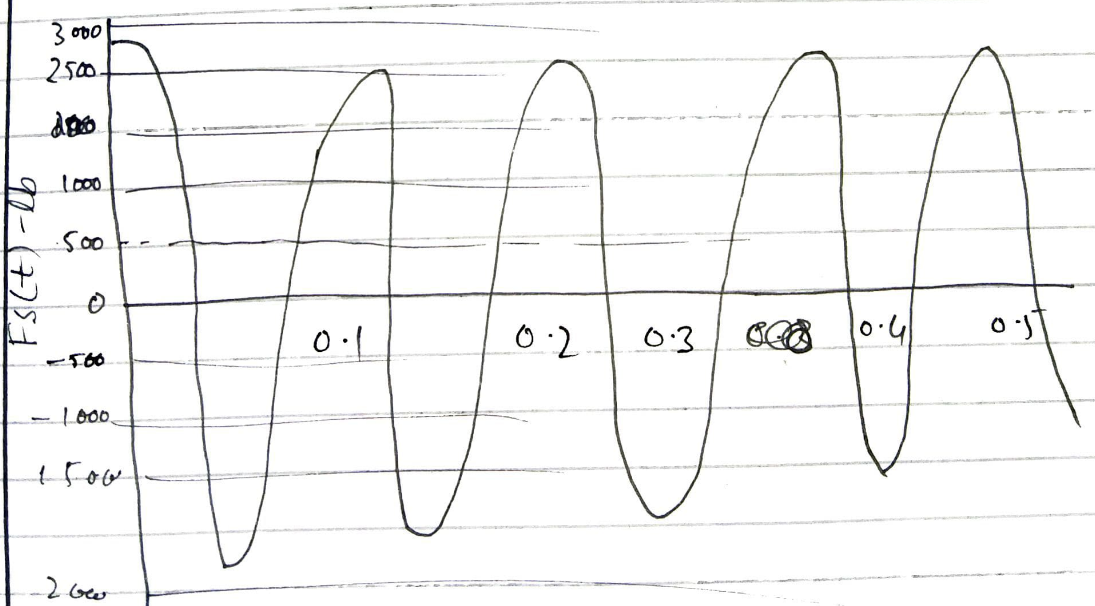
$$\Rightarrow f_s(t) = K\cdot u(t) = 90625\times u(t)$$

$$\boxed{f_s(t) = e^{-0.48675}\left(3776\cos(19.43) + 94.34\sin(19.43t)\right)}$$

Damped free vibration



Damped free ~~graph~~ vibration



Q3 Problem 3

Given data

- Force = 60 Kips
- $u_1 = \frac{7725}{1000} = 7.725$ in
- $j = 7$ (cycles)
- Completed = 3.57 Sec
- $u_{j+1} = 2.286$ cm $= 0.9$ in
- ignore The verticle vibration

Required

(a) Damping ratios
(b) Natural period of undamped Vibration
(c) Stiffness of Structures
(d) Weight of tank
(e) Damping Co-efficient
(f) Number of Cycles to reduced the displacement amplitude to 0.5".

Solution

"a" ζ = Damping ratio = ?

As;

$$j = \frac{1}{2\pi\zeta} \ln\left[\frac{u_1}{u_{j+1}}\right]$$

By putting values;

$$7 = \frac{1}{2(3.14)\zeta} \ln\left[\frac{7.8725}{0.9}\right]$$

$$\zeta(7 \times 2 \times 3.14) = 2.149$$

$$\zeta(43.96) = 2.149$$

$$\zeta = 2.149/43.96$$

$$\zeta = 0.0489$$

$$\boxed{\zeta = 4.89\%}$$

"b" $T_n = ?$

As "Seven" cycles are Completed in "3.57" Sec.

Thus Time required to Complete one cycle $= 7/3.57 = 1.96$ Sec

$$\boxed{T_D = 1.96 \text{ Sec}}$$

Now $\omega_D = \omega_n\sqrt{(1 - G^2)}$

As;

$$T_D = T_n / \sqrt{1 - G^2}$$

$$\Rightarrow T_n = T_D\left(\sqrt{1 - G^2}\right)$$

$$= 1.96\left(\sqrt{1 - (0.0462)^2}\right)$$

$$\boxed{T_n = 1.957 \text{ Sec}}$$ "Natural period of undumped Vibration"

"c" Stiffness of Structure $K = ?$

As $K = \frac{F \cdot \cos\theta}{2}$

$K = \frac{60 \cdot \cos(60)}{2}$ $\left(\begin{matrix} F = 60\text{ Kips} \\ \theta = 60^\circ \end{matrix}\right)$

$= 15K/in$

$\boxed{K = 18000\ lb/ft}$

"d" 'Weight of Tank"; "$W = ?$"

$W_n = \sqrt{K/m} = \sqrt{\frac{K}{(w/g)}} = \sqrt{\frac{K \cdot g}{w}}$

$\Rightarrow W_n^2 = \frac{K \cdot g}{w} = \left(w = Kg/W_n^2\right)$

By putting values of $W_n = 2\pi/T_n$

$W = K \cdot g / \left(\frac{4\pi^2}{T_n^2}\right) = Kg\left(\frac{T_n^2}{4\pi^2}\right)$

$W = \frac{18000\ lb}{ft} \cdot \frac{32.2\ ft}{sec}\left(\frac{(1.957)^2}{4(3.14)^2}\right)$

$W = 56284.75\ lb = \boxed{56.284\ Klb}$

"e" **Damping Co-efficient ; $C = ?$**

It is Know that ; $\zeta = \dfrac{C}{2 m W_n}$

$$\Rightarrow C = \zeta (2 m W_n) = \zeta \left(2m \left(\frac{2\pi}{T_n}\right)\right)$$

By Putting values

$$C = 0.0489 \frac{\left(2 \left(\frac{56284}{32.2}\right)\right)\left(2(3.14)\right)}{1.957}$$

$$\boxed{C = 518.286 \ lb \cdot sec / ft}$$

"g" **No of cycles To reduce displacement altitude from 7.725 in to 0.5 in"**

$J = ?$

$$j = \frac{1}{2\pi\zeta} \ln \left(\frac{u_1}{u_{j+1}}\right)$$

$$= \frac{1}{2(3.14)(0.0489)} \ln \left[\frac{7.725}{0.9}\right]$$

$$= 7.003$$

$$\boxed{j = 7 \text{ cycles}}$$