NAME : AHMAD ZAIB

ID NO : 13132

ASSIGMENT : MID SUMMEER

SUBJECT : DIGITAL SIGNAL PROCESSING

SUBMITTED TO : SIR RAFIQ MANSOOR

DATE : 24 AUGUST 2020

(a) consider the following analog signal

$$x_a(t) = 3\cos 100\pi t + 4\sin 200\pi t$$

(i) Determine the minimum sampling rate required to avoid aliasing.

Ans.

According to sampling therem

$f_1 = 100 Hz$ $F_2 = 200 Hz$

$f_s \geq 7max$ $f_0 = \frac{\omega}{2\pi}$

So

f_s is max (greater then f_1)

$f_s > 2 \times 100$

$f_s = 200 Hz$

(ii) suppose that the signal is sampled at the rate $f_s = 100 Hz$ What is the discrete-time signal obtained after sampling rate on the newly generated discrete time signal.

Solution:-

$f_s = 100 Hz$

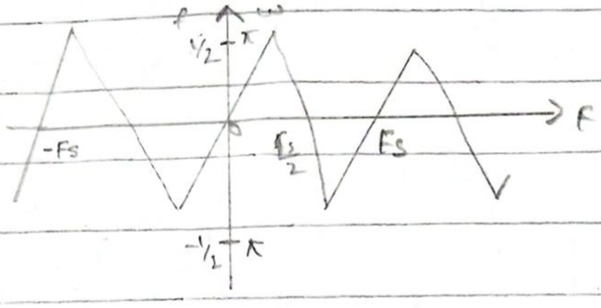
$f = \frac{100}{2} = 50 Hz$

This is the max frequency that can be represented unquely by the sampled signal

As

$$x_a[n] = 3\cos 2\pi\left(\frac{50}{100}\right)n + 4\sin 2\pi\left(\frac{100}{100}\right)n$$

$$= 3\cos\pi\left(\frac{5}{10}\right)n + 4\sin 2\pi n$$



The effect of sampling rate on the newely generated discrete time signal is that.
There will be no disply phenomena mea. There will not present un wanted component in Recontruct of the signal. The reconstruct orginal signal.

(iii) What is the analog signal $y_a(t)$ we can reconstruct from the samples if we use ideal interpolation.

Ans.

$$\text{folding frequency} = \frac{f_1}{2} = \frac{100}{2}$$

$$= 50\,Hz$$

$f_1 = 50\,Hz$ $f_2 = 100\,Hz$

frequency are either equal or greater the folding frequency. Hence for ideal interpolation we can construct the orginal signal.

$$x_a(t) = 3\cos 100\pi + 4\sin 200\pi t.$$

since only the frequency component at 100 Hz are present on the sampled signal the analog signal we can remove or reconstruct is

$$y_a(t) = 3\cos 100\pi t$$ Ans.

(b) consider a discreate time siagnal which is give by

$$x(n) = \begin{cases} 0.5^n & n \geq 0 \\ 0 & n < 0 \end{cases}$$

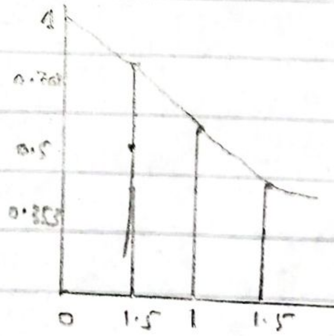
This is signal is sampled at the rate $f_s = 2Hz$

(i) Draw the sampled signal.

$$f_s = \frac{1}{T} = T = 1/f_s$$

$$= \frac{1}{2} = 0.5 sec.$$

X_n	0.5^n
0	1
0.5	0.707
1	0.5
1.5	0.353



(ii) The sample of the signal are intended to carry 3 bits per sample Determine the quantization level and quantization resolution to quantized the sampled signal achived in part.

Ans :

$L = 2^n$

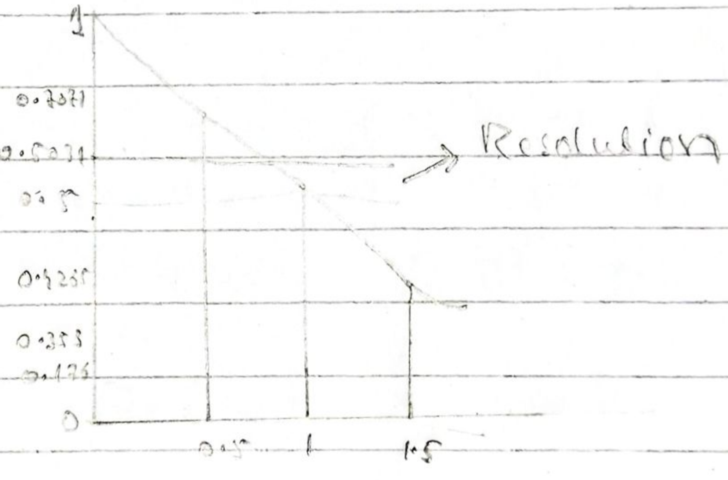
$n = \text{bits} = 3$

$L = 2^3 = 8$ level

$$\text{Resolution} = \frac{x_{max} - x_{min}}{L}$$

$$= \frac{1-0}{8}$$

$$= 0.125$$



	Dist signal	interpolation	Reading	Error
0	1	1.0	1.0	0.0
1	0.8535	0.8	0.9	-0.1
2	0.707	0.7	0.7	0.0
3	0.6035	0.6	0.6	0.0
4	0.5	0.5	0.5	0.0
5	0.4265	0.4	0.4	0.0
6	0.353	0.3	0.4	-0.1
7	0.1765	0.1	0.1	-0.1

Q 2(a) Determine the response of the system to the following input signal with given impulse response.

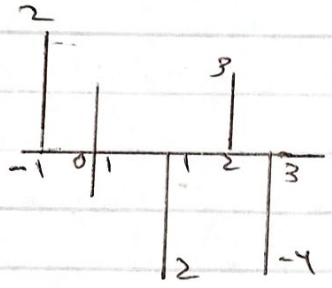
$$x[n] = \{2, \underset{\uparrow}{1}, -2, 3, -4\}$$

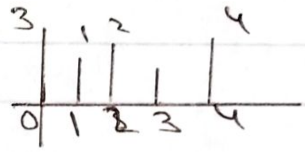
$$h[n] = \{\underset{\uparrow}{3}, 1, 2, 1, 4\}$$

Solution.

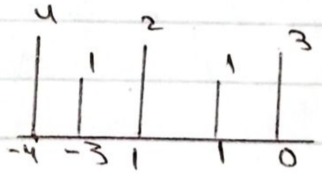
$$y[n] = \sum_{100\ \infty}^{\infty} x[k]\,h[n-k]$$

$n[k]$





$h[-k]$ = folded signal.



$h[-k]$

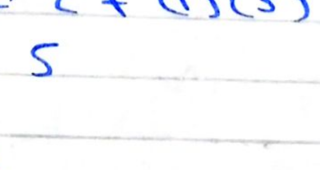
$$y[0] = \sum_{k=1}^{0} x[-1]\,h[-1] - 1[0]\,h(0)$$

$$= 2\times 2 + (1)(3)$$

$$= 5$$

for $n = 1$

$h[1-k]$



Pag-06 ID=13132

$y[1] = \sum_{k=1}^{3} x[n]\,[h(1-k)]$

$= x(-2)h(-1) + x(0)h(0) + x(1)h(1)$

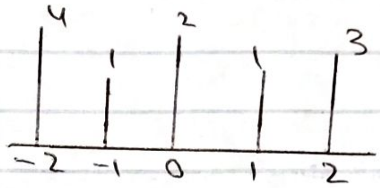
$= (2)(2) + (1)(1) + (3)(-2)$

$= 4+1-6$

$= -1$

$n=2$

$h[2-k]$



$h(2-k)$

$y(2) = \sum_{k=1}^{1} x[n]\,h[2-k]$

$= x(-1)h(-1) + 2(0) + h(0) + x(1)h(1)$

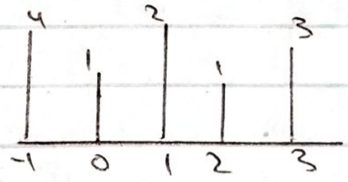
$x(-1)h(-1) + x(0) + h(0) + x(1)(1)$

$= (2)(1) + (1)(2) + (-2)(1) + (3)(3)$

$= 2+2-2+9$

$= 11$

$n=3$



$n(3-k)$

$y(3) = \sum_{k=d}^{3} x[n]\,h(3-k]$

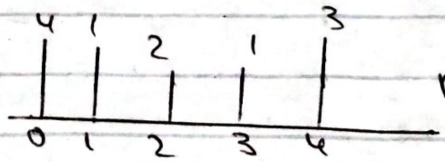
$= x(-1)h(-1) + x(0)h(0) + x(1)h(1) + x(x)\,h(2)$

$= 2\times4 + (1)(1) + (-2)(2) + 3(1) + (-4)(3)$

$= 4+1-4+3-12$

$= -8$

$n=4$

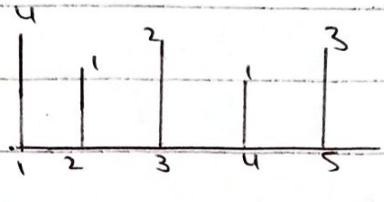


$n(4-k$

$$f(4) = \sum_{K=0}^{4} x(n)h(4-K)$$

$$= x(0)h(0) + x(1)h(1) + x(2)h(2) + x(3)h(3)$$
$$= 4 - 2 + 6 - 4$$
$$= 4$$

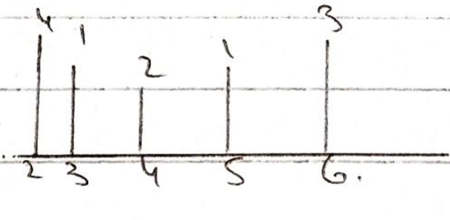
$n = 5$



$$y(5) = \sum_{K=1}^{5} x(n)h(5-K)$$

$$x(1)h(1) + x(2)h(2) + x(3)h(3)$$
$$= (-2)(4) + 3(1) + (-4)(2)$$
$$= -8 + 3 - 8$$
$$= 13.$$

$n = 6$



$$y(6) = \sum_{K=3}^{K=6} x(2)h(2) + x(3)h(3)$$

$$= (3)(4) + (1)(-4)$$

$$= 8.$$

(b) Compute the convolution $y(n)$ of the following signal

$$x[n] = \begin{cases} a^{n+1} & -3 \leq n \leq 5 \\ 0, & \text{else where} \end{cases}$$

$h[n] = \begin{cases} 2^n & 0 \le n \le 4 \\ 0 & \text{else where.} \end{cases}$

Solution:

we have.

$x(n) = x(k) = \{\alpha^{-2}, \alpha^{-1}, 1, \alpha, \alpha^2, \alpha^3, \alpha^4, \alpha^5, \alpha^6, 0, 0 \cdots\}$

$h(n) = h(k) = \{\cdots 0, 1, 2, 4, 8, 16, 0 \cdots\}$

To find $y[n]$:

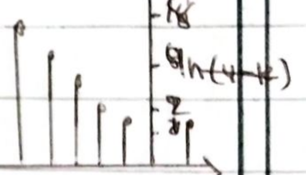
$y(n) = \sum_{k=-\infty}^{\infty} x(k)\, h(n-k)$

for $n = 0$ first to find $h(n-k) = h(0-k)$

So by inverting $h(k)$ we get $h(-k)$

$\Rightarrow h(-k) = \{16, 8, 4, 2, 1\}$ —(2)

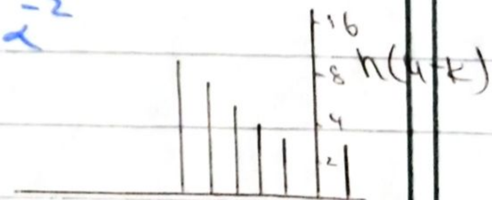
so $y(0) = \sum_{k=-\infty}^{\infty} x(k) \times h(-\overset{\uparrow}{k})$



$y(0) = (\alpha^{-2} \times 8) + (\alpha^{-1} \times 4) + (1 \times 2) + (\alpha \times 1)$

$y(0) = 8\alpha^{-2} + 4\alpha^{-1} + \alpha + 2$

$= \alpha . -2 + 4\alpha^{-2} + 4\alpha^{-2}$



for $n = 1$

$h(1-k) = \{16, 8, 4, 2, 1\}$

So $y(1)$ $(\alpha^{-2} \times 16) + (\alpha^{-2} \times 8) + (1 \times 4) + (\alpha \times 2) + (\alpha^2 \times 1)$

$y(1) = 16\alpha^{-2} + 8\alpha^{-1} + 4 + 2\alpha + \alpha^2$

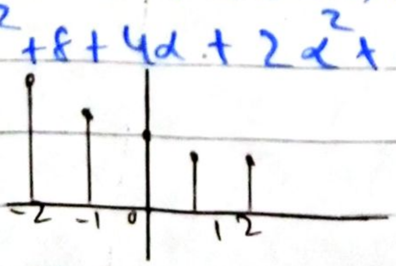
$= \alpha^2 + 2\alpha + 4 + 8\alpha^{-1} + 16\alpha^{-2}$

Now for $n = 2$

$h(2-k) = \{16\ 8\ 4\ 2, 1\}$

$y(2) = \{\alpha^{-1} \times 16) + (1 \times 8) + (\alpha \times 4) + (\alpha^2 \times 2)(\alpha^3 \times 1)$

$= 16\alpha^{-2} + 8 + 4\alpha + 2\alpha^2 + \alpha^3$

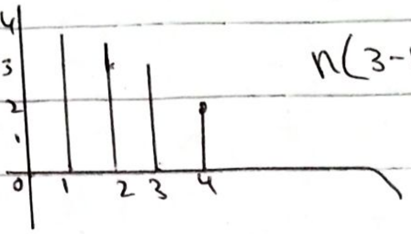


similarly for $n=3$

$h(3-k) = \{16, 8, 4, 2, 1\}$

$y(3) = (1\times 16) + (\alpha \times 8) + (\alpha^2 \times 4) + (\alpha^3 \times 2) + (\alpha^4 \times 1)$

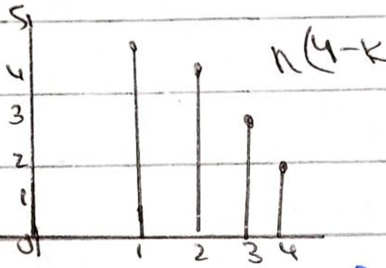
$= 16 + 8\alpha + 4\alpha^2 + 2\alpha^3 + \alpha^4$



Now $h(4-k) = \{16, 8, 4, 2, 1\}$

$y(4) = (\alpha \times 16) + (\alpha^2 \times 8) + (\alpha^3 \times 4) + (\alpha^4 \times 2) + (\alpha^5 \times 1)$

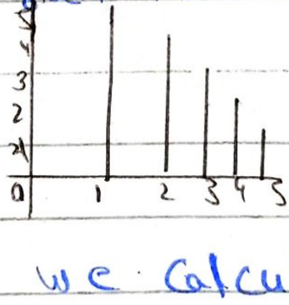
$= 16\alpha + 8\alpha^2 + 4\alpha^3 + 2\alpha^4 + \alpha^5.$



$h(5-k) = \{0, 16, 8, 4, 2, 1\}$

$y(5) = \alpha \times 10 + (\alpha^2 \times 16) + (\alpha^3 \times 8) + (\alpha^4 \times 4) + (\alpha^5 \times 2)$ $(\alpha^6 \times 1$

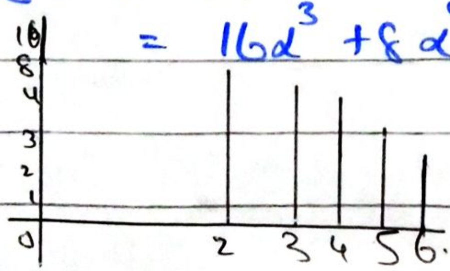
$= 16\alpha^2 + 8\alpha^3 + 4\alpha^4 + 2\alpha^5 + \alpha^6$



similarly if we calculate for rest of the values of n until there are any common values we get

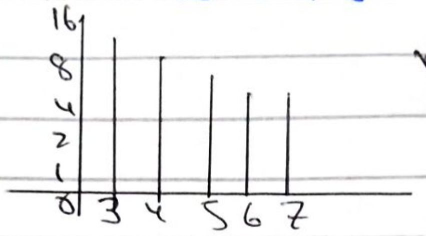
$y(6) = 0 + 0 + 16\alpha^3 + 8\alpha^4 + 4\alpha^5 + 2\alpha^6$

$= 16\alpha^3 + 8\alpha^4 + 4\alpha^5 + 2\alpha^6$

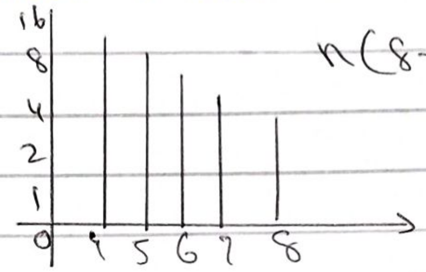


Pg = 10 ID = 13132

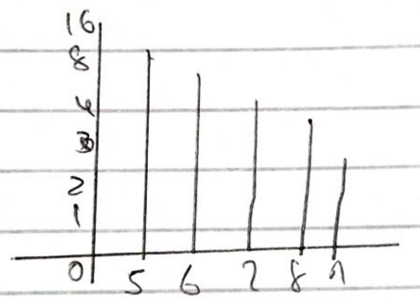
$$y(7) = 0+0+0+16\alpha^4+8\alpha^5+4\alpha^6$$
$$= 16\alpha^4+8\alpha^5+4\alpha^6$$



$$y(8) = 0+0+0+0+16\alpha^5+8\alpha^6$$
$$= 16\alpha^5+8\alpha^6$$



$$y(9) = 0+0+0+0+16\alpha^6$$
$$= 16\alpha^6$$



Q3 Determine the Z-transform of the following signal and also sketch its region of convergence (ROC)

(1) $x(n) = \begin{cases} (1/4)^n, & n \geq 0 \\ (1/3)^{-n}, & n < 0 \end{cases}$

Solution:

As we know that

Z-transform

$$X(z) = \sum_{n=0}^{\infty} (1/4)^n z + \sum_{n=\infty}^{0} (1/3)^n z^{n-1}$$

using geomatric seres

$$\Rightarrow \frac{1}{1-1/4\,z^{-3}} + \sum_{n=0}^{\infty} (1/3)^n z^n - 1$$

$$\Rightarrow \frac{1}{1-\frac{1}{4}z^{-3}} + \frac{1}{1-\frac{1}{3}} - 1$$

$$\Rightarrow \frac{1-\frac{1}{1/4}z^{-1} + 1 - \frac{1}{1} - 1}{\left(1-\frac{1}{4}z^{-1}\right)\left(1-\frac{1}{3}z^{-3}\right)^{-1}}$$

$$= \frac{1-\frac{1}{3}z + 1 - \frac{1}{4}z^{1} - \left(1-\frac{1}{4}z^{-2}\right)\left(1-\frac{1}{3}z\right)}{\left(1-\frac{1}{4}z^{-3}\right)\left(1-1/3\,z\right)}$$

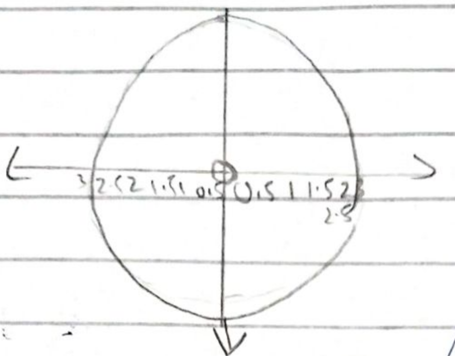
$$= \frac{1-1/3\,z + 1 - 1/4\,z - 1 + 1/3 + 1/4\,z + 1/1\,z}{\left(1-1/4\,z^{1}\right)\left(1-1/3\,z\right)}$$

$$= \frac{1-\frac{1}{12}}{\left(1-\frac{1}{4}z^{1}\right)\left(1-\frac{1}{3}z\right)}$$

$$= \frac{13/12}{\left(1-\frac{1}{4}z^{-1}\right)\left(1-\frac{1}{3}z\right)}$$

Hence the Roc is $\frac{1}{4} < |z| < 3$

The sketch is under:



(ii) $x[n] = \begin{cases} (1/2)^n - 3^n, & n \geq 0 \\ 0 & \text{elsewhere} \end{cases}$

Solution:

using the z-transform pair eq

i.e $x[n] = \alpha^n u[n] \longleftrightarrow X(z) = \frac{1}{1-\alpha z^{-1}}$ → eq (B)

Putting values.

$$X_2[z] = \sum_{n=0}^{\infty} (1/2)^n z^{-n} - \sum_{n=0}^{\infty} 2^n$$

$$= \frac{1}{1 - 1/2 z^{-1}} - \frac{1}{1-2z^{-1}}$$

$$= \frac{-5/2\, z^{-1}}{(1-\frac{1}{2}z^{-1})(1-2z^{-1})}$$

As seen the ROC is $|z| > 2$
The sketch are