HASEEB - Ullah

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FINAL PAPER

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Question 2
Part (b)

Find the curve $y = \sqrt{x}$, $0 \leq x \leq 4$

Solution

Given that $y = \sqrt{x}$

$0 \leq x \leq 4$
$a \leq x \leq b$

As

$$V = \int_a^b \pi y^2 \, dx$$

$$V = \int_0^4 \pi (\sqrt{x})^2 \, dx$$

$$V = \pi \int_0^4 x \, dx$$

$$V = \pi \frac{x^2}{2}$$

$$V = \frac{\pi}{2} \left[(4)^2 - 0 \right]$$

$$\boxed{V = 8\pi}$$

Question 2
Part (a)

Illustrate the centre and radius sphere

$x^2+y^2+z^2+3x-4z+1$

Solution:-

$x^2+y^2+z^2+3x-4z+1=0$

$(x^2+3x)+y^2+z^2-4z+1=0$

$\left(x^2+3x+\left(\frac{3}{2}\right)^2\right)+(y-0)^2+\left(z^2-4z+\left(\frac{-b}{2}\right)^2\right)$

$= -1+\left(\frac{3}{2}\right)^2+\left(\frac{-b}{2}\right)^2$

$\left(x+\frac{3}{2}\right)^2+(y-0)^2+(z-2)^2=\frac{21}{4}$

So, (x_0, y_0, z_0) = Centre

$=\left(-\frac{3}{2}, 0, 2\right)$

&

$\boxed{\text{Radius}, \; a=\sqrt{\frac{21}{4}}}$ A

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Question 5
Part (a)

Estimate the angle between A and B
$A = i - 2j - 2k$ and $B = 6i + 3j + 2k$

Solution.

$$A = i - 2j - 2k$$

$$|A| = \sqrt{(1)^2 + (-2)^2 + (-2)^2}$$

$$= \sqrt{1 + 4 + 4}$$

$$= \sqrt{9}$$

$$|A| = 3$$

Now

$$B = 6i + 3j + 2k$$

$$|B| = \sqrt{(6)^2 + (3)^2 + (2)^2}$$

$$= \sqrt{36 + 9 + 4}$$

$$= \sqrt{49}$$

$$|B| = 7$$

So

$$\theta = \cos^{-1}\left(\frac{A \cdot B}{|A||B|}\right)$$

$$\theta = \cos^{-1}\left\{\frac{(i - 2j - 2k) \cdot (6i + 3j + 2k)}{3 \times 7}\right\}$$

$$\theta = \cos^{-1} \frac{\{(1)(6)+(-2)(3)+(-2)(2)\}}{21}$$

$$\theta = \cos^{-1} \frac{(6-6-4)}{21}$$

$$\theta = \cos^{-1}\left(\frac{-4}{21}\right)$$

$$\boxed{\theta = 100.97}$$ A-.

Question 5
Part b

find a sphrical Coordinate equation for the Sphere $x^2+y^2+(z-1)^2=1$

Solution:-

$$x^2+y^2+(z-1)^2 = 1$$

$$(\rho \sin\phi \cos\theta)^2 + (\rho \sin\phi \sin\theta)^2 + (\rho \cos\phi - 1)^2 = 1$$

$$\rho^2 \sin^2\phi \cos^2\theta + \rho^2 \sin^2\phi \sin^2\theta + \rho^2 \cos^2\phi + 1 - 2\rho\cos\phi =$$

$$\rho^2 \sin^2\phi (\cos^2\theta + \sin^2\theta) + \rho^2 \cos^2\phi + 1 - 2\rho \cos\phi = 1$$

$$\rho^2(\sin^2\phi) + \rho^2 \cos^2\phi - 2\rho\cos\phi = 1-1$$

$$\rho^2 \cdot (\sin^2\phi + \cos^2\phi) - 2\rho\cos\phi = 0$$

$f' = 2 \int \cos\theta$

$\boxed{f = 2\cos\varphi}$ Ans.

Question 4

Find the area of region the graph and x-axis

$y = -x^2 + 5x - 4 \qquad [0, 2]$

Solution

Given that

$y = -x^2 + 5x - 4$

and

$[a, b] = [0, 2]$

As

$a = 0$

$b = 2$

So

Area under graph will be

$A = \int_0^2 f(x)\, dx$

putting value

$= \int_0^2 (-x^2 + 5x - 4)\, dx$

By Solving integration we will get.

$$A = \left(-\frac{x^3}{3} + \frac{5x^2}{2} - 4x \right)\Big|^{2}$$

$$A = \left(-\frac{1}{3}(2)^3 + \frac{5}{2}(2)^2 - 4(2) \right) - (0)$$

$$A = \left(-\frac{1}{3}(8) + \frac{5}{2}(4) - 8 \right)$$

$$A = -\frac{8}{3} + \frac{20}{2} - 8$$

$$A = \frac{-8}{3} + \frac{20}{2} - \frac{8}{1}$$

$$A = \frac{2\times -8 + 3\times 20 - 6\times 8}{6}$$

$$A = \frac{60 - 64}{6}$$

$$A = \frac{4}{6}$$

$$A = \frac{2}{3}$$

$$\boxed{A = 0.666} \quad \text{Ans.}$$

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Question 1:-

Part (b)

Estimate by using Substitution method.

$$\int_0^1 x^2(1+x^4)^3\,dx$$

Solution:-

let

$$t = 1+x^4$$

$$\frac{dt}{dx} = 4x^3$$

or

$$dt = 4x^3\,dx$$

$$\frac{dt}{4} = x^3\,dx$$

So

$$= \frac{1}{4}\int_0^1 t^3\,dt$$

$$= \frac{1}{4}\left(\frac{t^4}{4}\right)\Big|_0^1$$

$$= \frac{1}{16}\cdot(1^4 - 0^1)$$

$= \frac{1}{16}(1)$

$\boxed{= \frac{1}{16}}$ Ans.

Question 1:-
Part (a)

Estimate $\int \theta \sqrt[4]{1-\theta^2}\, d\theta$

Solution:-

let

$1-\theta^2 = u$

$\frac{d}{d\theta}(1-\theta^2) = \frac{d}{d\theta} u$

$-2\theta = \frac{du}{d\theta}$

$\theta\, d\theta = -\frac{1}{2} du$

Now $= \int (u)^{\frac{1}{4}} * \left(-\frac{1}{2}\right) du$

$\therefore \frac{1}{4}$

$= -\frac{1}{2} \int u^{\frac{1}{4}} du$

$\cdot \frac{5}{4}$

$= -\frac{1}{2} \cdot \frac{4}{5} u^{5/4} + c$

$= -\frac{2}{5} u^{5/4} + c$

By back Substitution

$\boxed{= -\frac{2}{5} (1-\theta^2)^{5/4} + c}$

Question 3:-

Illustrate the vector proje A B.

$A = 2i - 4j + \sqrt{5} k$
$B = -2i + 4j - \sqrt{5} k$

Solution:-

By dot product

$B \cdot A = (-2i + 4j - \sqrt{5} k) \cdot (2i - 4j + \sqrt{5} k)$

$= ((-2)(2) + (4)(-4) + (-\sqrt{5})(\sqrt{5}))$

$= -4 - 16 + \sqrt{-5 \times 5}$

$= -4 - 16 - \sqrt{25}$

$= -4 - 16 - 5$

$= -4 - 16 - 5$

$\boxed{\Rightarrow B \cdot A = -25}$

Now

$A \cdot A = (2i - 4j + \sqrt{5}\,k) \cdot (2i - 4j + \sqrt{5}\,k)$

$= \left((2)(2) + (-4)(-4) + (\sqrt{5})(\sqrt{5})\right)$

$= 4 + 16 + \sqrt{5 \times 5}$

$= 4 + 16 + \sqrt{25}$

$= 4 + 16 + 5$

$\boxed{A \cdot A = 25}$

So

$\text{Proj}_A B = \left(\dfrac{B \cdot A}{A \cdot A}\right) A$

putting value.

$= \left(\dfrac{-25}{25}\right)(2i - 4j + \sqrt{5}\,k)$

$= (-1)(2i - 4j + \sqrt{5}\,k)$

$\boxed{\text{Proj}_A B = -2i + 4j - \sqrt{5}\,k}$ ☆

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