Summer Mid term 2020

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Sec:- B

Subject:- Applied Calculus

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Date:- 21st August, 2020

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Question # 1

a.) State any point of discontinuity

The function $g(t)$ is defined by

$$g(t) = \begin{cases} 0 & t < 0 \\ t^2 & 0 \le t \le 3 \\ 2t+3 & 3 < t \le 4 \\ 12 & t > 4 \end{cases}$$

Sol:-

To check possibility of the discontinuity of the function is at $t = 0$ & 4.

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First at $t=0$

$$g(t) = t^2$$

$$g(0) = 0^2 = 0$$

For R.H.L :-

$$\lim g(1+h) = \lim_{h\to 0} (1+h)^2$$

$$= \lim_{h\to 0} 1+h^2+2h$$

Apply limit :-

$$= 1 + 0^2 + 2(0)$$

$$= 1$$

For L.H.L :-

$$\lim_{h\to 0} g(1-h) = 2t+3$$

$$= \lim_{h\to 0} 2(1-h)+3$$

$$= \lim_{h \to 0} 2 - 2h + 3$$

Apply limit:-

$$= 2 - 2(0) + 3$$

$$= 5$$

$$R.H.L \neq L.H.L = g(t) = 5$$

Now at $t = 4$

$$g(4) = 2(4) + 3$$

$$g(4) = 8 + 3$$

$$g(4) = 11$$

For R.H.L :-

$$\lim_{h \to 0} g(1+h) = \lim_{h \to 0} 2(1+h) + 3$$

$$= \lim_{h \to 0} 2 + 2h + 3$$

Apply limit:-

$= 2 + 2(0) + 3$

$= 5$

For L.H.L :-

$$\lim_{h \to 0} g(1-h) = 12$$

$g(4) = R.H.L \neq L.H.L$

Point of discontinuity is at

$t = 4.$

b.) Find, if they exist

i.) $\lim_{t \to 3} g$

Sol:-

$$\lim_{t \to 3} g(t)$$

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$= \lim_{t \to 3} (t)^2$

Applying limit,

$= (3)^2$

$= 9$

$\lim_{t \to 3} g(t) = 9$

Ans

Question # 2

Find the Maclaurin's series for

$$Y(x) = x^2 + \sin x$$

Sol:-

$$Y(x) = x^2 + \sin x$$

By Maclaurin's Series expansion, We have

$$f(x) = y(x) = f(0) + x f'(0) + \frac{x^2 f''(0)}{2!} + \frac{x^3 f'''(0)}{3!} + \cdots\cdots$$

Now,

$$f(x) = y(x) = x^2 + \sin x$$

$$f'(x) = 2x + \cos x$$

$$f''(x) = x - \sin x$$

$$f'''(x) = -\cos x$$

Now,

$$f(0) = (0)^2 + \sin(0) = 0$$

$$f'(0) = 2(0) + \cos(0) = 1$$

$$f''(0) = 0 - \sin(0) = 0$$

$$f'''(0) = -\cos(0) = -1$$

Hence,

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$$f(x) = y(x) = 0 + x(1) + \frac{x^2(0)}{2!} + \frac{x^3(-1)}{3!} + \dots\dots$$

$$f(x) = y(x) = 0 + x + 0 - \frac{x^3}{3!} + \dots\dots$$

$$f(x) = y(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \frac{x^9}{9!} + \dots\dots$$

is the required Maclaurin's Expansion,

Question #3

(a.) Find y'' given

$$1+xy = x^2+y^2$$

Sol:-

$$1+xy = x^2+y^2$$

Diff w.r.t (x)

$$0+x\frac{dy}{dx}+y(1)=2x+2y\frac{dy}{dx}$$

$$x\frac{dy}{dx}-2y\frac{dy}{dx}=2x-y$$

$$(x-2y)\frac{dy}{dx}=2x-y$$

$$\frac{dy}{dx}=\frac{2x-y}{x-2y} \rightarrow (i)$$

Again diff w.r.t (x)

$$\frac{d^2y}{dx^2} = \frac{(x-2y)\frac{d}{dx}(2x-y) - (2x-y)\frac{d}{dx}(x-2y)}{(x-2y)^2}$$

$$\frac{d^2y}{dx^2} = \frac{(x-2y)\left(2(1)-\frac{dy}{dx}\right) - (2x-y)\left(1-2\frac{dy}{dx}\right)}{(x-2y)^2}$$

$$\frac{d^2y}{dx^2} = \frac{(x-2y)\left(2-\frac{2x-y)}{x-2y}\right) - (2x-y)\left(1-2\left(\frac{2x-y}{x-2y}\right)\right)}{(x-2y)^2}$$

$$\frac{d^2y}{dx^2} = \frac{(x-2y)\left(\frac{2(x-2y)-(2x-y)}{x-2y}\right) - (2x-y)\left(\frac{x-2y-2(2x-y)}{x-2y}\right)}{(x-2y)^2}$$

$$\frac{d^2y}{dx^2} = \frac{(x-2y)(\cancel{2x}-4y-\cancel{2x}+y) - (2x-y)(x-\cancel{2y}-4x+\cancel{2y})}{(x-2y)(x-2y)^2}$$

$$\frac{d^2y}{dx^2} = \frac{(x-2y)(-3y)-(2x-y)(-3x)}{(x-2y)^3}$$

$$\frac{d^2y}{dx^2} = \frac{-3xy+6y^2+6x^2-3xy}{(x-2y)^3}$$

$$\frac{d^2y}{dx^2} = \frac{6x^2+6y^2-6xy}{(x-2y)^3}$$

Ans

b.) Find y' by using logarithmic differentiation

$$Y = x^3 (1+x^7)^9 e^{6x}$$

Sol:-

$$Y = x^3 (1+x)^9 \; e^{6x}$$

take ln on both sides

$$\ln Y = \ln (x^3 (1+x)^9 \cancel{\ast} \cdot e^{6x})$$

$$\ln Y = \ln x^3 + \ln (1+x)^9 + \ln e^{6x}$$

$$\ln Y = 3 \ln x + 9 \ln (1+x) + \ln e^{6x}$$

Now,

Diff w.r.t x $\quad \frac{d}{dx}(1+x)$

$$\frac{1}{y}\frac{dy}{dx} = 3 \times \frac{1}{x} + 9 \cdot \frac{1}{1+x} + \frac{1}{e^{6x}}$$

$$\cdot \frac{d}{dx} 6x$$

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$$\frac{dy}{dx} = y\left(\frac{3}{x} + \frac{9}{1+x}(0\times1) + \frac{1}{\cancel{e^{6x}}} \times \cancel{e^{6x}} \cdot \frac{d}{dx}6x\right)$$

$$\frac{dy}{dx} = y\left(\frac{3}{x} + \frac{9}{1+x} + 6(1)\right)$$

$$\frac{dy}{dx} = x^3(1+x)^9 \cdot e^{6x}\left(\frac{3}{x} + \frac{9}{1+x} + 6\right)$$

Ans