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Section A

Semester BS (SE)

Subject Discrete Structure

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Q₁)

SOLUTION:

Let a be the first term and d be the common difference of the arithmetic sequence. Then

$$a_n = a + (n-1)d$$

$$a_3 = a + (3-1)d$$

$$a_8 = a + (8-1)d$$

GIVEN:

$$a_3 = 7 \text{ and } a_8 = 17$$

$$7 = a + 2d \rightarrow (i)$$

$$17 = a + 7d \rightarrow (ii)$$

Subtracting (i) from (ii) we get:

$$10 = 5d$$

$$d = 2$$

Substituting $d = 2$ in (i)

$$7 = a + 2(2)$$

$$\boxed{a = 3}$$

②

$$a_n = a + (n-1)d$$

$$a_n = 3 + (n-1)2$$

The 36th term is:

$$a_{36} = 3 + (36-1)2$$

$$= 3 + 70$$

$$a_{36} = 73$$

Q₂) SOLUTION:

$$f \circ g(x) = ?$$

$$g \circ f(x) = ?$$

$$f(x) = 2x + 3$$

$$g(x) = -x^2 + 5$$

$$\begin{aligned} f \circ g(x) &= f(g(x)) \\ &= f(-x^2 + 5) \\ &= 2(-x^2 + 5) + 5 \end{aligned}$$

$$f \circ g(x) = -2x^2 + 10 + 5$$

$$f \circ g(x) = -2x^2 + 15$$

$$g \circ f(x) = g(f(x))$$

$$= g(2x + 3)$$

$$= -1(2x + 3)^2 + 5$$

$$= -1(4x^2 + 12x + 9) + 5$$

$$= -4x^2 - 12x - 9 + 5$$

$$g \circ f(x) = -4x^2 - 12x - 4$$

(4)

Q₃) SOLUTION:

$$1^2 + 2^2 + 3^2 + \dots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

$$n \geq 1$$

We put (1) is true for $n=1$

L.H.S $P(1) = n^2$

$$P(1) = 1$$

$$\boxed{1 = 1}$$

R.H.S :

$$P(1) = \frac{n(n+1)(2n+1)}{6}$$

put $n=1$

$$P(1) = \frac{1(1+1)(2(1)+1)}{6}$$

$$P(1) = \frac{1(2)(2+1)}{6}$$

$$P(1) = \frac{6}{6} = 1$$

⑤

$$1 = 1$$

Hence L.H.S = R.H.S

Then $P(1)$ is true.



Q4) TYPES OF RELATION:

The different types of relation are the following:

(i) EMPTY RELATION:

An empty relation is one in which there is no relation between any element of a set.

EXAMPLE:

If $A = \{1, 2, 3\}$ then, one of the void relations can be

$$R = \{x, y\}$$

where, $[x - y] = 8$

$$R = \varnothing \subset A \times A$$

(ii) UNIVERSAL RELATION:

A universal relation is one in which every element of set is related to each other.

EXAMPLE:

(7)

$$A = \{a, b, c\}$$

$$R = \{x, y\}$$

where $|x - y| = 0$

for universal relation.

$$R = A \times A$$

iii)

IDENTITY RELATION:

In this relation every element of a set is related to itself only.

EXAMPLE:

$A = \{a, b, c\}$, the identity relation will be $I = \{a, a\}, \{b, b\}, \{c, c\}$.

$$I = \{(a, a), a \in A\}$$

iv)

INVERSE RELATION:

In inverse relation a set has elements which are inverse pairs of another set.

EXAMPLE:

$$A = \{(a, b), (c, d)\}$$

then inverse relation will be

$$R^{-1} = \{(b, a), (d, c)\}$$

$$R^{-1} = \{(b, a) : (a, b) \in R\}$$

v) REFLEXIVE RELATION:

In reflexive relation, every element maps for itself.

EXAMPLE:

$$A = \{1, 2\}$$

$$\text{the } R = \{(1, 1), (2, 2), (1, 2), (2, 1)\}$$

$$(a, a) \in R$$

vi) SYMMETRIC RELATION:

In a symmetric relation, if $a = b$ is true then $b = a$ is also true. A relation R is symmetric only if $(b, a) \in R$ is true when $(a, b) \in R$.

EXAMPLE:

$$A = \{1, 2\}$$

$$R = \{(1, 2), (2, 1)\}$$

$$aRb \Rightarrow bRa, \forall a, b \in A.$$

⑨

vii)

TRANSITIVE RELATION:

For transitive relation, if $(x, y) \in R$, $(y, z) \in R$ then $(x, z) \in R$.

$$aRb \text{ and } bRc \Rightarrow aRc \quad \forall a, b, c \in A.$$

viii)

EQUIVALANCE RELATION:

If a relation is reflexive, symmetric and transitive at the same time it is known as equivalence relation.



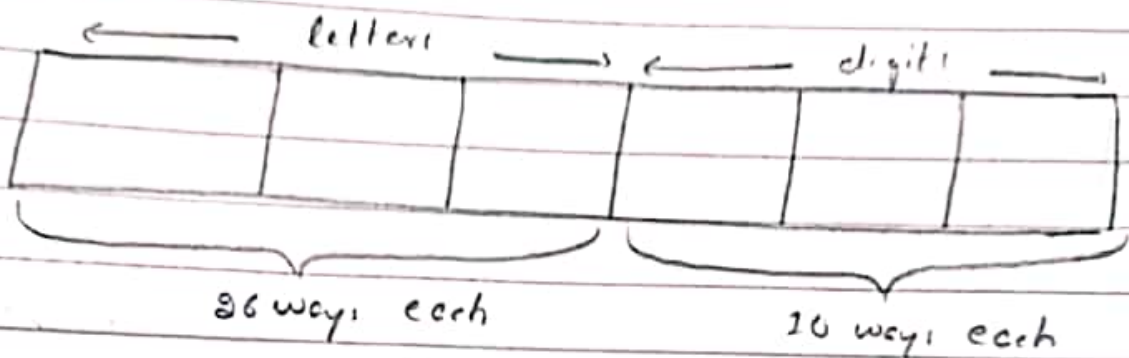
Q5)

SOLUTION:

(10)

1)

Each of the three letters can be written in 26 different ways and each of the three digits can be written in 10 different ways



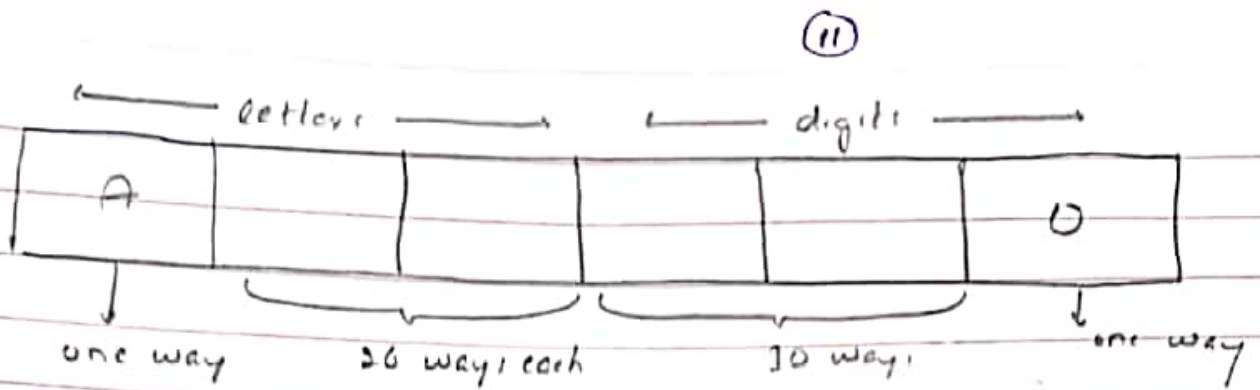
Hence by product rule

$$26 \times 26 \times 26 \times 10 \times 10 = 17,576,000$$

different license plates possible.

17,576,000

The first and last place can be filled in one way only, while each second and third place can be filled in 26 ways and each fourth and fifth place can be filled in 10 ways.



$$1 \times 26 \times 26 \times 10 \times 10 \times 1 =$$

67600

3) Number of license plates begin with PQR are

$$1 \times 1 \times 1 \times 10 \times 10 \times 10$$

1000

