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SUBJECT	Advanced Structural Analysis
PROGRAMME	M.S (STRUCURE ENGINNERING)

ANSWER SHEET

Q No 1:

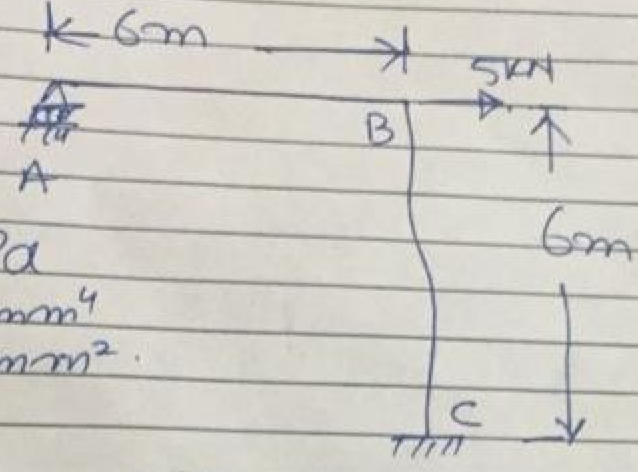
For the frame shown, use the stiffness method to:

- Determine the deflection and rotation at B.
- Determine all the reactions at supports.
- Draw the quantitative shear and bending moment diagrams.

E = 200 GPa, I = 60*(106) mm4, A = 600 mm2

ANSWER:

Q.No.1:



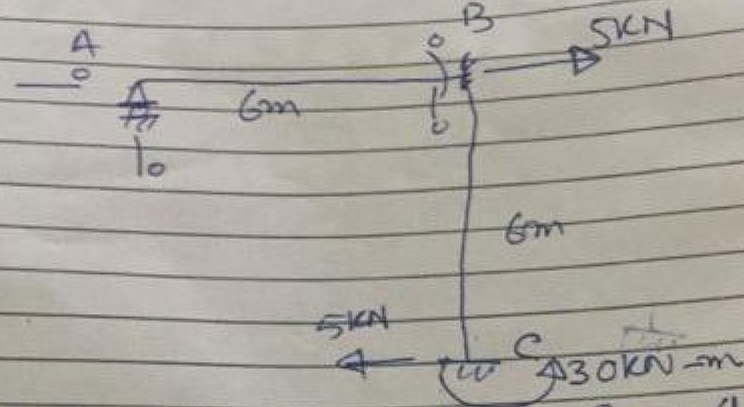
$E = 200\ GPa$
$I = 60 \times 10^6\ mm^4$
$A = 600\ mm^2.$

Deflection and Rotation @ 'B' = ?
Reactions @ Supports = ?
S.F.D and B.M.D = ?

$$\Rightarrow EI = 200 \times 10^9 * \frac{60 * 10^6}{(1000)^4} = 12 \times 10^6\ N\text{-}m^2$$

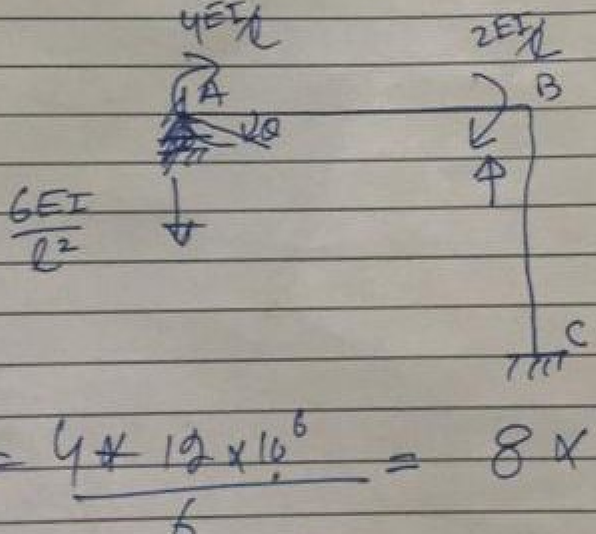
$$EA = 200 \times 10^9 * \frac{600}{(1000)^2} = 12 \times 10^6\ N\text{-}m^2.$$

Now Restrained Structure under existing Loading is



K.I = 3° i.e θ_A, θ_B and one Sway (horizontal)

Now applying unit rotation @ 'A' on restrained structure i.e



$4EI/l = \frac{4 * 12 \times 10^6}{6} = 8 \times 10^6$ N-m.

$2EI/l = 4 \times 10^6$ N-m.

$6EI/l^2 = 6 \times 12 \times 10^6 / 6 \times 6 = 2000K$

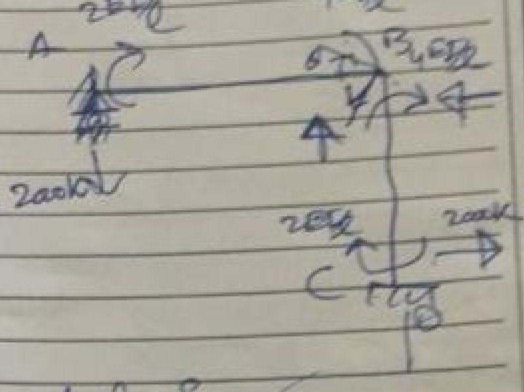
Date:

(3)

Applying unit rotation @ 'B'

AB

$M_A = 4 \times 10^6$ N-m



$M_B = 8 \times 10^6$ N-m

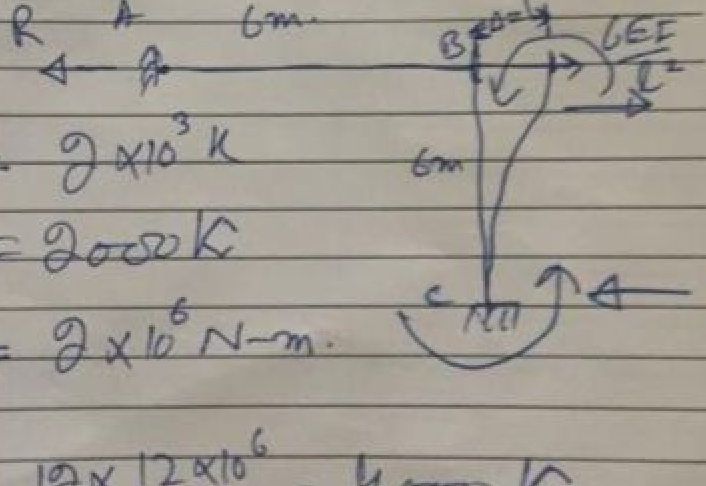
BC

$M_B = 8 \times 10^6$ N-m

$M_C = 4 \times 10^6$ N-m.

Applying unit horizontal Sway on 'AB' i.e

$R = \frac{EA}{L} \times \Delta$



$= \frac{12 \times 10^6}{6} = 2 \times 10^3$ K

$= 2000$ K

$M = \frac{6 \times 12 \times 10^6}{6 \times 6} = 2 \times 10^6$ N-m.

$R_C = \frac{12 EI}{L^3} = \frac{12 \times 12 \times 10^6}{6 \times 6 \times 6} = 4000$ K

$= 666.67$ K

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Date:

(4)

Now Coefficients of Stiffness Matrix are

Unit rotation @ A;

$$S_{11} = 8000 \text{ kN-m}$$

$$S_{21} = 4000 \text{ kN-m}.$$

$$S_{31} = 0$$

Unit rotation @ B;

$$S_{22} = 16000 \text{ kN-m}.$$

$$S_{12} = 4000 \text{ kN-m}.$$

$$S_{32} = -2000 \text{ k}$$

Now member end actions under unit sway i.e

$$S_{13} = 0$$

$$S_{23} = -2000 \text{ kN-m}.$$

$$S_{33} = +666.67 \text{ k}.$$

Date:

Now stiffness matrix is

$$[S] = \begin{bmatrix} 8000 & 4000 & 0 \\ 4000 & 16000 & -2000 \\ 0 & -2000 & \begin{matrix} -1333.33 \\ \cancel{666.67} \end{matrix} \end{bmatrix}$$

Now inverse of above matrix is

$$[S]^{-1} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \begin{matrix} -0.001 \\ \cancel{0.003} \end{matrix} \end{bmatrix}$$

Now

$$[D] = [S]^{-1} \{ [AD] - [ADL] \}.$$

$$AD = \begin{bmatrix} 0 \\ 0 \\ +5 \end{bmatrix}, \quad ADL = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow [D] = \begin{bmatrix} \cancel{-0.001} \\ \cancel{+0.002} \\ \cancel{+0.013} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -0.003 \end{bmatrix}$$

So

$$Q_A = -0.001 \;(C.C.W).$$

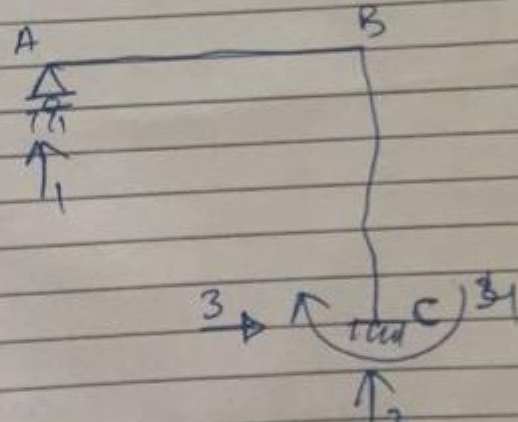
$$Q_B = +0.002 \;(C.W).$$

$$\Delta B = +0.013 \;(\text{towards right}).$$

Now Calculating Reactions @ Supports i.e.

$$[A_R] = [A_{DL}] + [S][D].$$

$$[A_R] = \begin{bmatrix} R_1 \\ R_2 \\ R_3 \\ R_4 \end{bmatrix}$$



Now reactions due to external loads on restrained structure i.e

$$\begin{bmatrix} R_{DL} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -5 \\ -30 \end{bmatrix} + \begin{bmatrix} -2000 & -2000 & -2000 & 0 \\ 0 & 0 & 0 & 0 \\ +2000 & 0 & +2000 & +1333.33 \\ +4000 & 0 & 4000 & -2000 \end{bmatrix}$$

$$\times \begin{bmatrix} -0.001 \\ +0.002 \\ +0.013 \end{bmatrix}$$

Q NO 2:

Describe in detail the steps involved in Direct Stiffness Method (Computer based Stiffness method)

Answer :

Direct Stiffness Method (Computer based Stiffness method)

The following are the steps of Direct Stiffeness method :

(1) Identification of Structural Data. Information pertaining to the structure itself must be assembled and recorded. This information includes the number of members, the number of joints, the number of degrees of freedom, and the elastic properties of the material. The locations of the joints of the structure are specified by means of geometric coordinates. In addition, the section properties of each member in the structure must be given. Finally, the conditions of restraint at the supports of the structure must be identified. In computer programming, all such information is coded in some convenient way, as will be shown subsequently in this chapter and also in Chapter 5.

(2) Construction of Stiffness Matrix. The stiffness matrix is an inherent property of the structure and is based upon the structural data only. In computer programming it is convenient to obtain the joint stiffness matrix by summing contributions from individual member stiffness matrices (a

discussion of complete member stiffnesses is given in Sec. 4.3). The joint stiffness matrix to be considered is related to all possible joint displacements, including support displacements, as was discussed in Sec. 3.6. This array shall be called the *over-all joint stiffness matrix*. Its formation and rearrangement are described in Secs. 4.4 and 4.6, respectively.

(3) Identification of Load Data. All loads acting on the structure must be specified in a manner suitable for computer programming. Both joint loads and member loads must be given. The former may be handled directly, but the latter are handled indirectly by supplying as data the fixed-end actions caused by the loads on the members.

(4) Construction of Load Vector. The fixed-end actions due to loads on members may be converted to *equivalent joint loads*, as described previously in Sec. 1.12. These equivalent joint loads may then be added to the actual joint loads to produce a problem in which the structure is imagined to be loaded at the joints only. Formation and rearrangement of the load vector are described in Secs. 4.5 and 4.6, respectively.

(5) Calculation of Results. In the final phase of the analysis all of the joint displacements, reactions, and member end-actions are computed. The calculation of member end-actions proceeds member by member (see Eq. 4-5) instead of for the structure as a whole. Such calculations require the use of complete member stiffness matrices for member directions, a subject that

Q NO (4): Differentiate between flexibility and stiffness method.

ANSWER :

FLEXIBILTY METHOD :

1.Determine the degree of S.I (degree of redundancy), n

2. Choose the redundants.

3. Assign Coordinates 1,2,…,n to the redundants.

4. Remove all the deduced once to obtain the release structure.

5. Determine [ΔL], the displacements at the coordinates due to the applied loads acting on the released structure.[ΔL] ΔRL.

6. Determine [ΔR], the displacements at the coordinates.due to the redundants acting on the released strc.

7.Compute the net displacement at the coordinates.

$[\Delta] = [\Delta L] + [\Delta R]$

$[\Delta] = [\Delta L] + [F][R]$

$[\Delta RS] = [\Delta RL] + [F][AR]$

$[AR] = [F]^{-1} [\Delta RS-\Delta RL]$

8. Use the compatibility of displacement to compute the redundance.

9. Knowing the redundants, compute the internal member forces by using equations of statics.

STIFFNESS METHOD:

1. Determine the degree of K.I (degree of freedom), n.

2. Identify the independent displacement components.

3. Assign Coordinates 1 to n to the independent displacement components.

4. Prevent all the displacement component to restrain strc.

5. Determine [ΔL], the actions at the coordinates in the restrained strc due to the loads other than those acting at the coordinates.

6. Determine the forces required at the coordinates in the unrestrained strc to cause the independent displacement components, Δ . AB.

7. Compute the net forces at the coordinates.

$[A] = [AL] + [A\Delta] [S] [\Delta]$

8. Use the conditions of equations to compute the displacements.

$[\Delta] = [S]^{-1} [A-AL]$

9. Knowing the displacements, compute the internal member forces by using slope deflection equations.