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Q1) Determine if the following system is consistent or not.

$$x_1 - (3^{rd}\ ID)x_2 + x_3 = 0$$
$$2x_2 - 8x_3 = 8 \qquad (3^{rd} ID = 8)$$
$$5x_1 \qquad - 5x_3 = 10$$

Sol :-

$$\left[\begin{array}{cccc} 1 & -8 & 1 & 0 \\ 0 & 2 & -8 & 8 \\ 5 & 0 & -5 & 10 \end{array}\right] \quad R_3 \leftrightarrow R_3 - 5R_1$$

$$\left[\begin{array}{cccc} 1 & -8 & 1 & 0 \\ 0 & 2 & -8 & 8 \\ 0 & 40 & -10 & 10 \end{array}\right] \quad R_2 \leftrightarrow \tfrac{1}{2}R_2$$

$$\left[\begin{array}{cccc} 1 & -8 & 1 & 0 \\ 0 & 1 & -4 & 4 \\ 0 & 40 & -10 & 10 \end{array}\right] \quad R_3 \leftrightarrow \tfrac{1}{10}R_3$$

$$\left[\begin{array}{cccc} 1 & -8 & 1 & 0 \\ 0 & 1 & -4 & 4 \\ 0 & 4 & -1 & 1 \end{array}\right] \quad R_3 \leftrightarrow R_3 - 4R_2$$

$$\left[\begin{array}{cccc} 1 & -8 & 1 & 0 \\ 0 & 1 & -4 & 4 \\ 0 & 0 & 15 & -15 \end{array}\right] \quad R_3 \leftrightarrow \frac{1}{15}R_3$$

$$\begin{bmatrix} 1 & -8 & 1 & 0 \\ 0 & 1 & -4 & 4 \\ 0 & 0 & 1 & -1 \end{bmatrix}$$

So from 3rd row we see that system can be consistant only if

$$0x_1 + 0x_2 + x_3 = -1$$

$$\boxed{x_3 = -1}$$

Q2) Find the inverse of $A = \begin{bmatrix} 3 & 4 & 5 \\ 2 & -1 & 4^{th}\ ID \\ 5 & -2 & 7 \end{bmatrix}$ by adjoint method.

Sol.-

$$\begin{bmatrix} 3 & 4 & 5 \\ 2 & -1 & 8 \\ 5 & -2 & 7 \end{bmatrix}$$

$(4^{th}\ ID = 8)$

$$A^{-1} = \frac{adj\ A}{|A|}$$

$$|A| = \begin{vmatrix} 3 & 4 & 5 \\ 2 & -1 & 8 \\ 5 & -2 & 7 \end{vmatrix}$$

$$= 3[(-1)(7) - (-2)(8)] - 4[(2)(7) - (5)(8)]$$
$$+ 5[(2)(-2) - (5)(-1)]$$

$$= 3[-7+16] - 4[14-40] + 5[-4+5]$$

$$= 3(9) - 4(-26) + 5(1)$$

$$= 27 + 104 + 5$$

$$\boxed{|A| = 136}$$

Now

$$\text{adj } A = \begin{bmatrix} 3 & 2 & 5 \\ 4 & -1 & -2 \\ 5 & 8 & 7 \end{bmatrix}$$

So Inverse of A^{-1}

$$A^{-1} = \frac{\text{adj } A}{|A|}$$

$$〃 = \frac{\begin{bmatrix} 3 & 2 & 5 \\ 4 & -1 & -2 \\ 5 & 8 & 7 \end{bmatrix}}{136}$$

$$〃 = \frac{1}{136}\begin{bmatrix} 3 & 2 & 5 \\ 4 & -1 & -2 \\ 5 & 8 & 7 \end{bmatrix}$$

$$= \begin{bmatrix} 3/136 & 2/136 & 5/136 \\ 4/136 & -1/136 & -2/136 \\ 5/136 & 8/136 & 7/136 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} 3/136 & 1/68 & 5/136 \\ 1/34 & -1/136 & -1/68 \\ 5/136 & 1/17 & 7/136 \end{bmatrix} \quad \underline{\text{Ans}}$$

Q3) Solve the following system of linear equation by Guass Jordan Method

$$2x + 2y + 4z = 18$$
$$x + 3y + 2z = 13$$
$$3x + 2y - 3z = 14$$

Sol :-

$$\begin{bmatrix} 2 & 2 & 4 & 18 \\ 1 & 3 & 2 & 13 \\ 3 & 1 & 3 & 14 \end{bmatrix} \quad R_1 = \tfrac{1}{2}R_1$$

$$\begin{bmatrix} 1 & 1 & 2 & 9 \\ 1 & 3 & 2 & 13 \\ 3 & 1 & 3 & 1 \end{bmatrix} \quad \begin{matrix} R_2 = R_2 - R_1 \\ R_3 = R_3 - 3R_1 \end{matrix}$$

$$\begin{bmatrix} 1 & 1 & 2 & 9 \\ 0 & 2 & 0 & 4 \\ 0 & -2 & -3 & -13 \end{bmatrix} \quad R_2 = \tfrac{1}{2}R_2$$

$$\begin{bmatrix} 1 & 1 & 2 & 9 \\ 0 & 1 & 0 & 2 \\ 0 & -2 & -3 & -13 \end{bmatrix} \quad R_3 = R_3 + 2R_2$$

$$\begin{bmatrix} 1 & 1 & 2 & 9 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & -3 & -9 \end{bmatrix} \quad \begin{matrix} R_1 = R_1 - R_2 \\ R_3 = -\tfrac{1}{3}R_3 \end{matrix}$$

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$$\begin{bmatrix} 1 & 0 & 2 & 7 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{bmatrix} \quad R_1 = R_1 - 2R_3$$

$$\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{bmatrix}$$

$$\boxed{\begin{aligned} x &= 1 \\ y &= 2 \\ z &= 3 \end{aligned}}$$

Q4) Show that this matrix is Diagonisable.

$$\begin{bmatrix} 4 & 2 & -2 \\ -5 & 3 & 2 \\ -2 & 4 & 1 \end{bmatrix}$$

Sol :-

Matrix A is diagonisable if

$A = CDC^{-1}$

$\det(A - \lambda I_3) = 0$

$$A - \lambda I_3 = \begin{bmatrix} 4 & 2 & -2 \\ -5 & 3 & 2 \\ -2 & 4 & 1 \end{bmatrix} - \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix}$$

$$= \begin{bmatrix} 4-\lambda & 2 & -2 \\ -5 & 3-\lambda & 2 \\ -2 & 4 & 1-\lambda \end{bmatrix}$$

$$4-\lambda \begin{vmatrix} 3-\lambda & 2 \\ 4 & 1-\lambda \end{vmatrix} -2 \begin{vmatrix} -5 & 2 \\ -2 & 1-\lambda \end{vmatrix} -2 \begin{vmatrix} -5 & 3-\lambda \\ -2 & 4 \end{vmatrix} = 0$$

$$4-\lambda\left[(3-\lambda)(1-\lambda)-8\right] - 2\left[(-5)(1-\lambda)+4\right] - 2\left[(-20)+2(3-\lambda)\right] = 0$$

$$4-\lambda\left[3-3\lambda-\lambda+\lambda^2-8\right] - 2\left[-5+5\lambda+4\right] - 2\left[-20+6-2\lambda\right] = 0$$

$4-\lambda\left[\lambda^2-4\lambda-5\right]-2\left[5\lambda-1\right]-2\left[-14-2\lambda\right]=0$

$4\lambda^2+16\lambda-20-\lambda^3+4\lambda^2+5\lambda-10\lambda+28+4\lambda=0$

$-\lambda^3+8\lambda^2+15\lambda+10=0$

$\lambda = 9.65$

$\lambda = -0.82$

$\lambda = -0.829$

for $\lambda = -9.65$

$$A-\lambda I_3 = \begin{bmatrix} -5.65 & 2 & -2 \\ -5 & -6.65 & 2 \\ -2 & 4 & -8.65 \end{bmatrix}$$

for $\lambda = -0.82$

$$A-\lambda I_3 \begin{bmatrix} 4.82 & 2 & -2 \\ -5 & 8.82 & 2 \\ -2 & 4 & 1.82 \end{bmatrix}$$

By solving only 2 ~~eigenspaces~~ eigenspaces or 2 basis vectors in total

So matrix A is not diagonastive.

Q5) Determine if the following homogeneous system has a non trivial solution then describe the solution set.

$$3x_1 + 5x_2 - 4x_3 = 0 \quad \text{— (i)}$$
$$-3x_1 - 25x_2 + 4x_3 = 0 \quad \text{— (ii)}$$
$$6x_1 + x_2 - 8x_3 = 0 \quad \text{— (iii)}$$

Sol:-

$$A = \begin{bmatrix} 3 & 5 & -4 \\ -3 & -15 & 4 \\ 6 & 1 & -8 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

Add eq (i) & eq (ii)

$$3x_1 + 5x_2 - 4x_3 = 0$$
$$\underline{-3x_1 - 25x_2 + 4x_3 = 0}$$
$$-20x_2 = 0$$

$$x_2 = 0$$

Add eq (i) & eq (iii)

$$3x_1 + 5x_2 - 4x_3 = 0$$
$$\underline{6x_1 + x_2 - 8x_3 = 0}$$
$$9x_1 + 6x_2 - 12x_3 = 0$$

put $x_2 = 0$

$$9x_1 - 12x_3 = 0$$
$$9x_1 = 12x_3$$

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$$x_1 = \frac{4}{3} x_3$$

$$x_1 = 4/3\, x_3$$

$$x_2 = 0$$

$$x_3 = 0$$

Q6) Reduce the matrix to Normal form & find its rank.

$$\begin{bmatrix} 1 & 3 & 4 & 3 \\ 3 & 9 & 12 & 3 \\ 1 & 3 & 4 & 0 \end{bmatrix}$$

Sol:-

$$\begin{bmatrix} 1 & 3 & 4 & 3 \\ 3 & 9 & 12 & 3 \\ 1 & 3 & 4 & 0 \end{bmatrix}$$

Redue matrix to reduced row echelon form.

Swap matrix row $R_1 \leftrightarrow R_2$

$$\begin{bmatrix} 3 & 9 & 12 & 3 \\ 1 & 3 & 4 & 3 \\ 1 & 3 & 4 & 0 \end{bmatrix}$$

Cancel leading co-efficient in row R_2 by performing

$R_2 \leftarrow R_2 - 1/3 R_1$

$$\begin{bmatrix} 3 & 9 & 12 & 3 \\ 0 & 0 & 0 & 2 \\ 1 & 3 & 4 & 0 \end{bmatrix}$$

Cancel leading co-efficient in Row R_3 by performing

$$R_3 \leftarrow R_3 - \frac{1}{3} R_1$$

$$\begin{bmatrix} 3 & 9 & 12 & 3 \\ 0 & 0 & 0 & 2 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$

Rank of a matrix is the number of all zero row

So

Rand of $\begin{bmatrix} 1 & 3 & 4 & 0 \\ 3 & 9 & 12 & 3 \\ 1 & 3 & 4 & 0 \end{bmatrix}$

$$\boxed{= 2}$$