Date:

ALI HASNAIN

Sec -B

7966

CIVIL

Diff Eq

Maam Shumaila.

Assignment

Question # 01

$$x^3y''' + 2x^2y' + 2y - 10x + \frac{10}{x}$$

Solution:-

$$x^3\frac{d^3y}{dx^3} + 2x^2\frac{dy}{dx} + 2y = 10x + 10x^{-1}$$

$$x^3D^3y + 2x^2D^2 + 2y = 10x + 10x^{-1}$$

$$(x^3D^3 + 2x^2D + 2)y = 10x + 10x^{-1} \quad -①$$

Let $x = e^t \Rightarrow t = \ln x$

$xD = D$

$x^2D^2 = D(\Delta - 1) = \Delta^2 - D$

$x^3D^3 = \Delta(\Delta - 1)(\Delta - 2)$

Substituting into eq ①

$$(\Delta - 3\Delta^2 + 2D + 2(\Delta^2 - \Delta))y = 10^x + 10x^{-1}$$

$$(D^3 - D^2 + 2)y = 10x + 10x^{-1}$$

$$(m^3 - m^2 + 2)y = 10e^t + \frac{10}{e^t}$$

Using synthetic division

	1	-1	0	2
-1		-1	2	-2
	1	-2	2	0

$$D^2 - 2\Delta + 2 = 0$$

Now using quadratic formula.

$a = 1$, $b = -2$, $c = 2$

$$D = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Date:

$$D = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(2)}}{2a}$$

$$D = \frac{2 \pm \sqrt{4-8}}{2}$$

$$D = \frac{2 \pm \sqrt{-4}}{2} \Rightarrow \frac{2 \pm \sqrt{-1} + \sqrt{4}}{2}$$

$$D = \frac{(2+2i)}{2} \Rightarrow \frac{\cancel{2}(1+i)}{\cancel{2}}$$

$$D = 1 \pm i$$

Since roots are complex

$$y_c = e^{-x}(c_1 \cos t + c_2 \sin t)$$

Now Particular Integration:-

$$y_p = \frac{1}{D^3 - D^2 + 2} \cdot 10e^t + \frac{1}{D^3 - D^2 + 2} \cdot \frac{10}{et}$$

$$= \frac{10et}{(1)^3 - (1)^2 + 2} + \frac{10e^{-t}}{(1)^3 - (1)^2 + 2}$$

$$= \frac{\cancel{10}et}{\cancel{2}} + \frac{\cancel{10}e^t}{\cancel{2}}$$

$$= 5et + 5e^{-t}$$

$$y_p = 5et + 5e^{-t}$$

General solution:

$$y = y_c + y_p$$

$$y = e^{-x}(c_1 \cos t + c_2 \sin t) + 5et + 5e^{-t}$$

Put $e^t = x$, $t = \ln x$

$$y = e^{-x}(c_1 \ln x + c_2 \sin \ln x) + 5e^x + 5e^{-x}$$

Ans.

Date: Question #02 ③

$$x^3\frac{d^3y}{dx} + 4x^2\frac{d^2y}{dx} - 5x\frac{dy}{dx} - 15y = x^4$$

Solution:-

let $\frac{d}{dx} = D$

$x^3D^3y + 4x^2D^2y - 5xDy - 15y = x^4$

$(x^3D^3 + 4x^2D^2 - 5xD - 15)y = x^4$

let $x = e^t \Rightarrow t = \ln x$

$xD = D$

$x^2D^2 = D(D-1) = D^2 - D$

$x^3D^3 = D(D-1)(D-2) = D^3 - 3D^2 + 2D$

Now Substituting:-

$(x^3D^3 + 4x^2D^2 - 5xD - 15)y = x^4$

$(D^3 - 3D^2 + 2D + 4(D^2 - D) - 5(D) - 15y = e^{4t}$

$(D^3 + D^2 - 7D - 15)y = e^{4t}$

Synthetic division:-

	1	+1	-7	-15
5		3	12	15
	1	4	5	$\boxed{0}$

$D^2 + 4D + 5 = 0$

Quadratic division:-

$$D = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-4 \pm \sqrt{4^2 - 4(1)(5}}{2}$$

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Date:

$D = \frac{-4 \pm 2i}{2}$

$D = \frac{\cancel{2}(-2 \pm i)}{\cancel{2}}$

$y_c = e^{3x}(c_1 \cos t + c_2 \sin t)$

for $y_p = ?$

$y_p = \frac{1}{D^3 + D^2 - 7D - 15} \cdot e^{4t}$

$= \frac{1}{(4)^3 + (4^2) - 7(4) - 15} \cdot e^{4t}$

$= \frac{1}{64 + 16 - 28 - 15} e^{4t}$

$y_p = \frac{1}{37} e^{4t}$

Hence $y = y_c + y_p$

$y = (c_1 \cos t + c_2 \sin t) + \frac{1}{37} e^{4t}$

put $t = \ln x$, $x = \ln x$

$y = e^{3x}(c_1 \cos \ln x + c_2 \sin \log x) + \frac{1}{37} e^{4x}$

Ans.

Date:

Question No 3

$$x^2 y'' + 2xy' - by = 10x^2$$

Solution

$y(1) = 1$ and $y'(1) = -6$

$$x^2 \frac{d^2y}{dx^2} + 2x \frac{dy}{dx} - by = 10x^2$$

$$\Rightarrow \left(\frac{x^2 d^2}{dx^2} + 2x \frac{d}{dx} - 6 \right) y = 10x^2$$

Put $xD = \Delta \Rightarrow x^2 D^2 = D(D-1) = D^2 - D$

$x = e^t$ and $\log x = t$

$(\Delta^2 - \Delta + 2\Delta - 6) y = 10e^{2t}$

$(\Delta^2 + \Delta - 6) y = 10e^{2t}$

The characteristic equation

$\Delta^2 + \Delta - 6 = 0$

$\Delta^2 + 3D - 2\Delta - 6 = 0$

$\Rightarrow \Delta(\Delta+3) - 2(\Delta+3) = 0$

$\Rightarrow (\Delta+3)(D-2) = 0$

$D+3 = 0$, $\Delta - 2 = 0$

$D = 2$, $\Delta = -3$

Since roots are real and distinct

For $y_c = ?$

$y_c = c_1 e^{-3t} + 2(2 e^{2t})$

For $y_p = ?$

$$y_p = \frac{1}{\Delta^2 - \Delta - 6} 10^{2t}$$

$$= 10 \frac{1}{0} e^{2t} \quad \text{Fails}$$

Now

Date:

$$10 \frac{1}{d/d\Delta\,(\Delta^2+\Delta-6)}$$

$$\Rightarrow 10 \frac{t}{2\Delta+1} e^{2t}$$

$$= 10 \frac{1-t}{4+1} e^{2t}$$

$yp = 2te^{2t}$

General solution

$y = yc + yp$

$= C_1 e^{-3t} + C_2 e^{2t} + 2te^{2t}$

$y = C_1x^{-3} + C_2 x^2 + 2(\log x)\, x^2$ — (B)

Put $y(1) = 1$ i.e $x = 1$, $y = 1$ in (B)

$1 = C_1 (1)^{-3} + C_2 (1)^2 + 2\log(1)$

$1 = C_1 + C_2$

Now differentiate eq B w.r.t x

$y' = -3c_1 x^{-4} + 2C_2 x + \frac{2}{x}(x^2) + 4x\log x$

Now put $y'(1) = -6$ i.e $y'(1) = -6$ and $x = -6$

$-6 = -3C_1 + 2C_2 + 2 + 0$

$\Rightarrow -6 = -3C_1 + C_2 + 2 + 0$

$\Rightarrow -6-2 = -3C_1 + 2C_2 + 2$

$-8 = -3C_1 + C_2$ — (D)

$\times$ing eq (C) with (2) and $\div$ing from (D)

$2C_1 + 2C_2 = 2$

$\mp 3C_1 \pm 2C_2 = \mp 8$

$5C_1 = 10$

Date:

$c_1 = \frac{\cancel{16}\ 2}{\cancel{8}\ 1}$ $\quad c_1 = 2$

$-8 = -3(2) + 2c_2$

$-8 = -6 + 2c_2$

$2c_2 = -8+6$

$2c_2 = -2$

$c_2 = \frac{-\cancel{2}\ 1}{\cancel{2}}$

Now put the value of c_1 and in eq (B)

$y = 2x^{-3} - x^2 + 2 \ln(x(x^2)$

$y = \frac{2}{x^3} - x^2 + 2x^2 \log x$ Ans.

Question No 4

$x^2y'' + 7xy + 5y = x^5$

$y(0) = 2$ and $y'(1) = 2$

Solution

$x^2 \frac{dy^2}{dx^2} + 7x\frac{dy}{dx} + 5y = x^5$

$\left(x^2 \frac{d^2}{dx^2} + 7x\frac{dy}{dx} + 5\right)y = x^5$ — (A)

Put $xD = \Delta \Rightarrow x^2D^2 = \Delta(\Delta-1) = \Delta^2 - \Delta$

Date:

$x = e^t \Rightarrow \log x = t$ in eq (A)

$\Rightarrow (D^2 - D + 7\cancel{D} + 5)y = e^{5t}$

$\Rightarrow (D^2 + 6D + 5)y = e^{5t}$.

By aluadratic formula.

$$D = \frac{b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$D = \frac{-6 \pm \sqrt{6^2 - 4(1)(5)}}{2(1)}$$

$$= \frac{-6 \pm \sqrt{36 - 20}}{2}$$

$$= \frac{-6 \pm \sqrt{16}}{2}.$$

$$= -6 \pm \sqrt{4^2}$$

$$= \frac{\cancel{2}(-3 \pm 2)}{\cancel{2}}$$

$D = -3 \pm 2$

Since roots arereal and distinct

Date:

$y_c = C_1 e^{-5t} + C_2 e^{-t}$

For $y_p = ?$

$y_p = \frac{1}{D^2 + 6D + 5} e^{5t}$

$= \frac{1}{(5)^2 + 6(5) + 5} e^{5t}$

$= \frac{1}{60} e^{5t}.$

Now General solution is.

$y = y_c + y_p.$

$y = C_1 e^{-5t} + C_2 e^{-t} + \frac{1}{60} e^{5t}.$

$y = C_1 x^{-5} + C_2 x^{-1} + \frac{1}{60} x^5 \longrightarrow (13)$

$x = 0$ put in this equation

No in eq (13) $e^0 = 1$

put $y(2) = 2$ i-e $y = 2$ and $x = 2$.

$2 = C_1(2)^{-5} + C_2(2)^{-1} + \frac{1}{60}(2)^5.$

$2 = -32C_1 - 2C_2 2C_2 + \frac{1}{\cancel{60}\,15}(\cancel{32}\,8)$

Date:

$\pm 2/3 = \pm 320\,c_1 \pm 4\,c_2$

$\frac{34}{15} = -256\,c_1$

$c_1 = \frac{34 \times 256}{15}$

$\boxed{c_1 = 580}$

put the value of c_1 in eq (c)

$\frac{22}{15} = -32(580) - 2\,c_2$

$\Rightarrow \frac{22}{15} = -18560 - 2\,c_2$

$\Rightarrow \frac{22}{15} + 18568 = -2c_2$

$\Rightarrow \frac{18561}{-2} = c_2$

$-\boxed{9280 = c_2}$

Now put the value of c_1 and c_2 in eq (B)

$y = 580x^{-5} - 9280x^{-1} - \frac{1}{60}x^5$

$y = \frac{580}{x^5} - \frac{9280}{x} + \frac{1}{60}x^5$

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Question = (05).

$(x+1)^2 y'' - 3(x+1)' y' + 4y = x^2$.

Solution:-

$(x+1)^2 \frac{d^2y}{dx^2} - 3(x+1) \frac{dy}{dx} + 4y = x^2$.

$\Rightarrow ((x+1)^2 \frac{d^2}{dx^2} - 3(x+1) \frac{d}{dx} + 4)y = x^2$.

$\Rightarrow [(x+1)^2 D^2 - 3(x+1) D + 4] y = x^2 \rightarrow$ (A)

put $(x+1)D = \Delta \Rightarrow (x+1)^2 D^2 = D(D-1) = \Delta^2 - \Delta$.
$x = e^t$ in eq (A).

$\Rightarrow [D^2 - D - 3D + 4] y = e^{2t}$.

$\Rightarrow [D^2 - 4D + 4] y = e^{2t}$.

$\Rightarrow (D^2 - 4D + 4) y = e^{2t}$

$\Rightarrow (D^2 - 4D + 4) 2 = e^{2t}$.

for y_c we find the roots.

$D^2 - 4D + 4 = 0$.

$D^2 - 2D - 2D + 4 = 0$.

$D(D-2) - 2(D-2) = 0$.

Date:

$D-2=0$, $D=2$.

$D-2=0$, $D=2$.

So the roots are real and reaped.

The genral solution are

$$y = (c1 + c2x)^{mx}.$$

$$y = (c1 + c3x)^{2x}.$$

For $yp = ?$

$$yp = \frac{1}{D^2-4D+4}.$$

$(2)^2 - 4(2) + 4.$

$\Rightarrow 0.$

$$yp = \frac{2}{2D-4} e^{2t}.$$

if we put $2^{(1)}$

$2D-4 \Rightarrow 2(2)-4=0.$

we take again derivatee.

$$yp = \frac{\cancel{2}}{\cancel{2}} \cdot e^{2t}.$$

$$y = (c1 + c2x)^{2t} + e^{5t} \rightarrow \text{general solution}.$$

Ans.