Subject Advanced Mechanics ID 15345

Q No 1:

Solution:-

Moment at Section-A is

$M = 15345 \times 0.2$

$M = 3069$ N.m

Torque on the shaft is

$\tau = 15345 \times 0.15$

$\tau = 2301.75$

The Normal stress due to M at A is

$$\sigma = \frac{64Md}{2\pi d^4} = \frac{32M}{2\pi d^3}$$

& Maximum Shear stress due to T at A is

$$\tau = \frac{32Td}{2\pi d^4} = \frac{16T}{\pi d^3}$$

The Shear stress due to the shear force F is zero at A

$$\sigma_{1,3} = \frac{1}{2}\sigma \pm \frac{1}{2}(\sigma^2 + 4\tau^2)^{1/2}, \quad \sigma_2 = 0$$

(i) Max shear stress Theory

$$\tau_{max} = \frac{1}{2}(\sigma_1 - \sigma_3)$$

$$= \frac{1}{2}(\sigma_2 + 4\sigma\tau^2)^{1/2}$$

Scanned with CamScanner

(2)

$$= \frac{1}{2}\left(\frac{32}{\pi d^3}\right)\left(M^2+T^2\right)^{1/2}$$

$$= \frac{16}{\pi d^3}\left((3069)^2+(2301.75)^2\right)^{1/2}$$

$$= \frac{19547.7}{d^3}$$

With FOS N = 1.5, the value of τ_{max} becomes

$$N\,\tau_{max} = \frac{293216.56}{d^3}$$

This should not exceed the max shear stress value at yielding in the uniaxial tension test,

Thus

$$\frac{1}{d^3}(293216.56) = \frac{\sigma_y}{2} = \frac{207}{2}\times 10^6 \times d^3$$

$$293216.56 = 103.5\times 10^6 \times d^3$$

$$\frac{293216.56}{103.5\times 10^6} = d^3$$

$$d^3 = 2.83\times 10^{-4}\,m^3$$

or $d = 65.65\times 10^{-3}\,m$

$d = 6.565\,cm$

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(ii) Octahedral Shear stress theory:

$$\tau_{oct} = \frac{1}{3}\left[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2\right]^{1/2}$$

with $\sigma_2 = 0$

$$\tau_{oct} = \frac{1}{3}(2\sigma_1^2 + 2\sigma_3^2 - 2\sigma_1\sigma_3)^{1/2}$$

Substituting for σ_1 and σ_2 & Simplifying

$$\sigma_{oct} = \frac{\sqrt{2}}{3}(\sigma^2 + 3\tau^2)^{1/2}$$

$$= \frac{\sqrt{2}}{3\pi d^3}\left[(32M)^2 + 3(16T)^2\right]^{1/2}$$

$$= \frac{16\sqrt{2}}{3\pi d^3}(4M^2 + 3T^2)^{1/2}$$

$$= \frac{16\sqrt{2}}{3\pi d^3} \times 7316.7$$

$$\simeq \frac{\sqrt{2}}{3\pi d^3} \times 117067$$

Equating this ..

$$\frac{\sqrt{2}}{3\pi d^3} \times 1.5 \times 117067 = \sqrt{2}/3\ \sigma_y$$

or $2 \times 117067 = \pi d^3$
$\sigma_y = \pi d^3 \times 207 \times 10^6$

$d^3 = 10.56 \times 10^{-4} \implies \boxed{d = 10.18\,cm}$

(4)

Q#02

Solution:

Free body diagram :→ Bar BDE

$\Sigma B = 0$

$0 = -(15KN * 0.6m) + F_{CD} * 0.2m$

$= \dfrac{15KN * 0.6}{0.2m} = \dfrac{F_{CD} * 0.2m}{0.2m}$

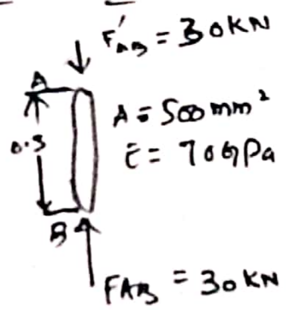
$F_{CD} = +45 KN$ (Tension)

$\sum M_D = 0$

$0 = 15KN * 0.4 - F_{AB} * 0.2$

$F_{AB} = -30 KN$ (Compression)

Displacement of B:



$\delta_B = \dfrac{PL}{AE}$

$= \dfrac{(-30 \times 10^3 N)(0.3m)}{(500 * 10^{-6} m^2)(70 * 10^9 Pa)}$

$= -257 * 10^{-6}$

$\boxed{\delta_B = 0.257 mm \uparrow}$

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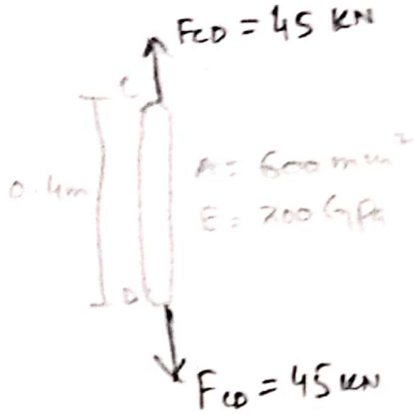
Displacement of D:



$$\delta_D = \frac{PL}{AE}$$

$$= \frac{(45 \times 10^3 N)(0.4m)}{(600 \times 10^{-6} m^2)(200 \times 10^9 Pa)}$$

$$= 150 \times 10^{-6}\ m$$

$$\boxed{\delta_D = 0.150\ mm}$$

$$\frac{BB'}{DD'} = \frac{BH}{HD} = \frac{0.257}{0.150} = \frac{(200\,mm) - x}{x}$$

$$\boxed{x = 36.85\ mm}$$

$$\frac{EE'}{DD'} = \frac{HE}{HD}$$

$$\frac{\delta_E}{0.150\ mm} = \frac{(400 + 36.85)}{36.85}$$

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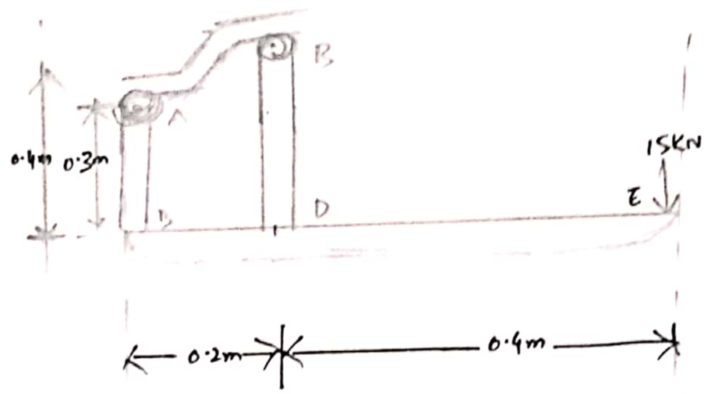
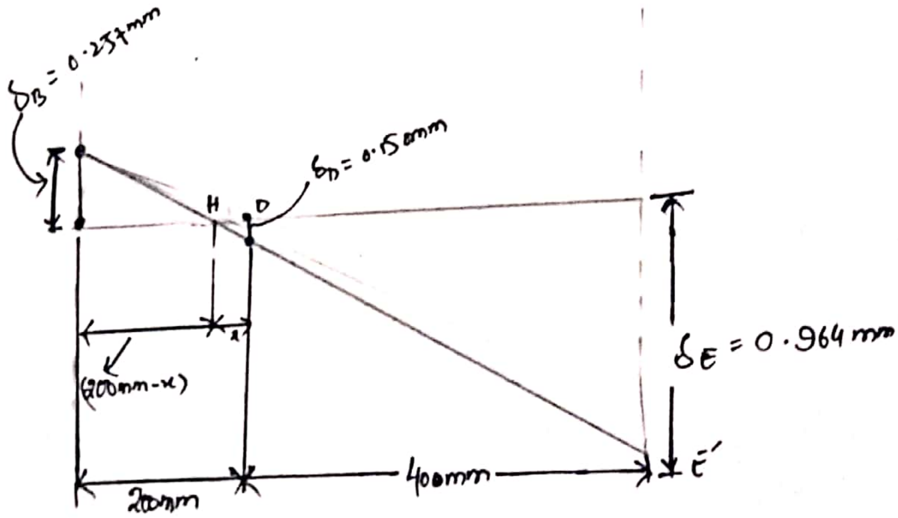
$$\delta_E = \frac{400 + 36.85}{36.85} \times 0.150$$

$$\boxed{\delta_E = 0.964\,mm \downarrow}$$





⑦

Q NO# 03 :-

Solution :-

⇒ Apply a statistics equilibrium analysis on two Shafts to find relationship b/w T_{CD} and T_0

⇒ Apply a Kinematics analysis To relate the angular relation of The gears

⇒ Find the maximum allowable torque on each shaft - choose the Smallest

⇒ find the corresponding angle of twist for each shaft and net Angular rotation of end A.

⇒ Apply a Static equilibrium analysis on the two Shafts to find a relationship B/w T_{CD} and T_0.

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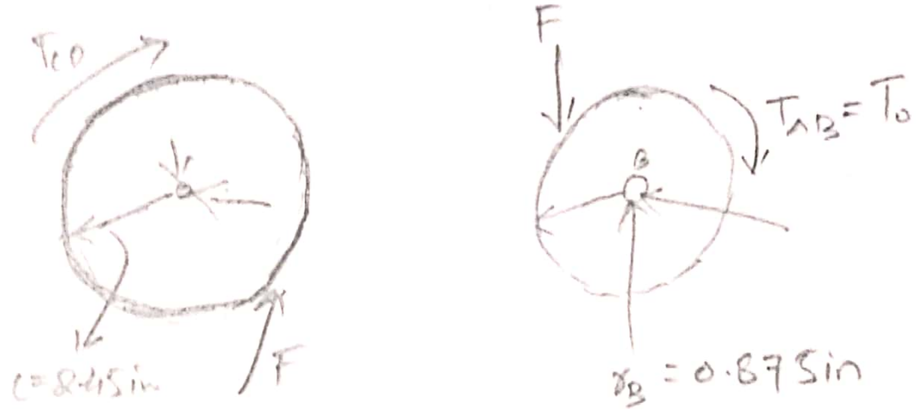
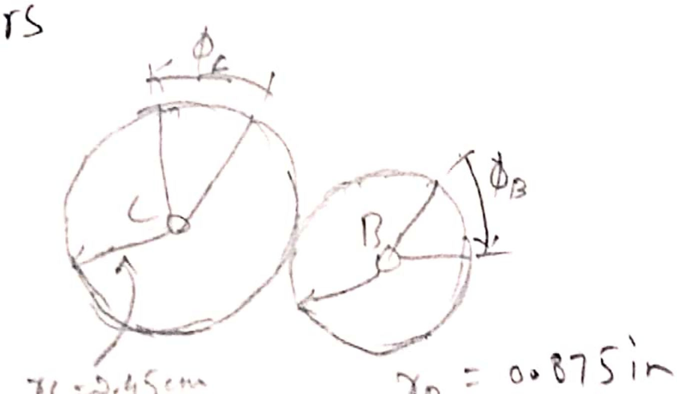


$$\Sigma M_B = 0 = F(0.875\,in) - T_0$$

$$\Sigma M_c = 0 = F(2.45\,in) - T_{CD}$$

$$T_{CD} = 2.8\,T_0$$

⇒ Apply a Kinetic analysis to relate the angular rotation of the gears

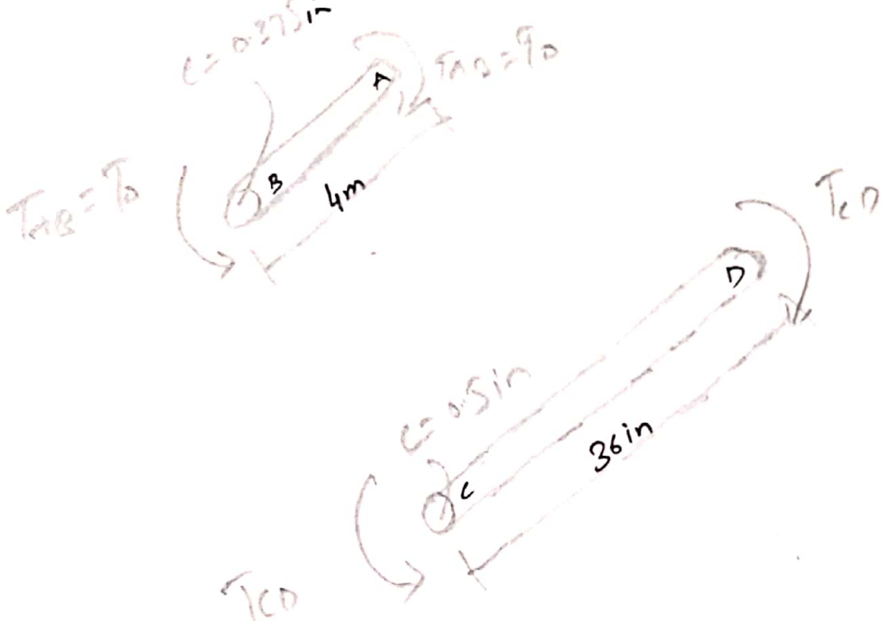


$$r_B \phi_B = r_c \phi_c$$

$$\phi_B = \frac{r_c}{r_B} \phi_c = \frac{2.45}{0.875} \phi_c$$

(a)

⇒ Find the T_0 for the maximum allowable torque on each Shaft, choose The Smallest



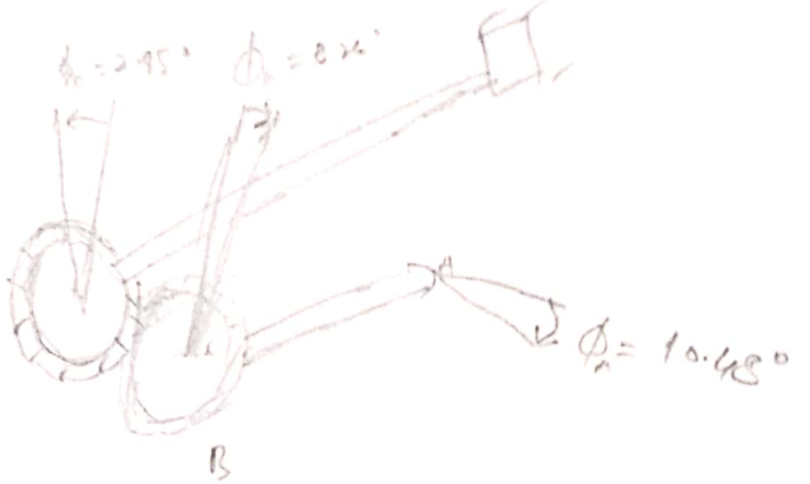
$$\tau_{max} = \frac{T_{AB}c}{J_{AB}} \Rightarrow 10{,}000\ \text{Psi} = \frac{T_0(0.375\ \text{in})}{\frac{\pi}{2}(0.375\ \text{in})^4}$$

$$= 10000\ \text{Psi} = \frac{T_0(0.375\ \text{in})}{0.0310\ \text{in}^4}$$

$$\therefore T_0 = \frac{310\ \text{in}^2\ \text{lb}}{0.375\ \text{lb}}$$

$$\Rightarrow \boxed{T_0 = 826\ \text{lb}}$$

→ Find the corresponding angle of twist for each shaft and the net angular rotation of end A.



$$\phi_{A/B} = \frac{T_{AB}L}{J_{AB}G} = \frac{(826\ lb.in)(24\ in)}{\frac{\pi}{2}(0.375)^4(15\times10^6\ psi)}$$

$$\phi_{AB} = 0.0425\ rad = 2.43°$$

$$\phi_{C/D} = \frac{T_{CD}L}{J_{CD}G} = \frac{2.8(826\ lb.in)(24\ in)}{\frac{\pi}{2}(0.5\ in)^4(15\times10^6\ psi)}$$

$$\phi_{C/D} = 0.0376\ rad = \boxed{2.15°}$$

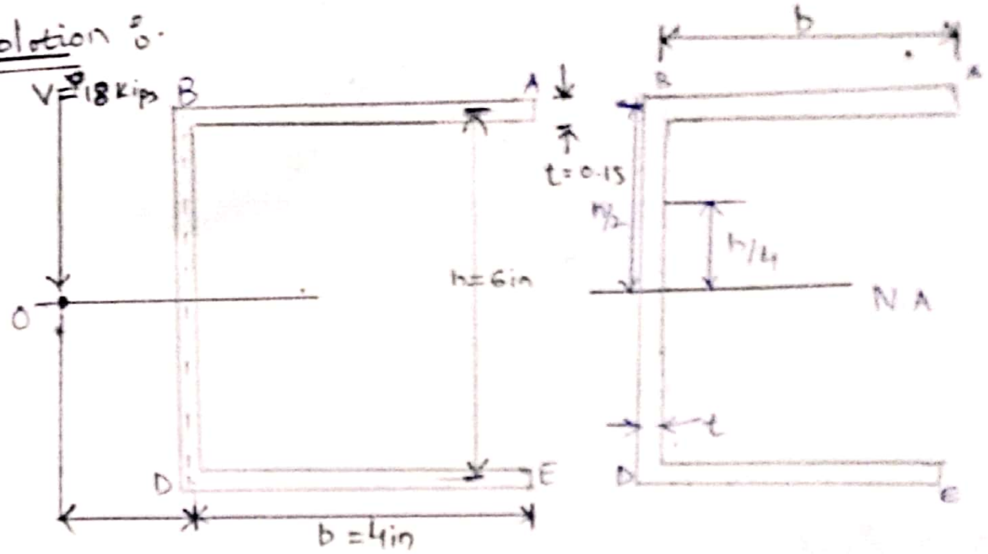
$$\phi_B = 2.8\,\phi_C = 2.8(2.15) = 6.02°$$

$$\phi_A = \phi_B + \phi_{A/B} = 6.02° + 2.43°$$

$$\Rightarrow \boxed{\phi_A = 8.45°}$$

Q NO. (04) Solution :-

Figures :→



I.D = 15345

So, $V = 15 + 3 = 18\text{Kips}$

⇒ Shear Stresses in the flange,

$$\tau = \frac{VQ}{It} = \frac{V}{It}(St)\,\frac{h}{2} = \frac{Vh}{2I}S$$

$$\tau_B = \frac{Vhb}{2(\frac{1}{12}th^2)(6b+h)} = \frac{6Vb}{th(6b+h)}$$

$$\tau_B = \frac{6(18\text{kips})(4\text{in})}{(0.15\text{in})(6\text{in})(6\times 4\text{in}+6)}$$

$$\boxed{\tau_B = 16\ \text{Kips}}$$

⇒ Shear stress in the Web,

$$\tau_{max} = \frac{VQ}{It} = \frac{V(\frac{1}{8}ht)(4b+h)}{\frac{1}{12}th^2(6b+h)t}$$

$$= \frac{3V(4b+h)}{2th(6b+h)}$$

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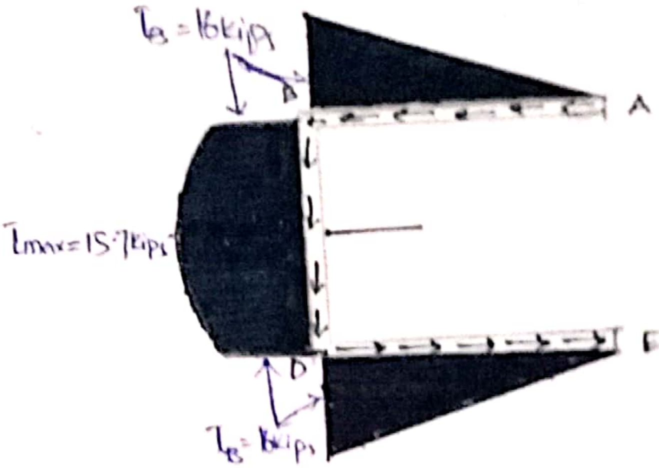
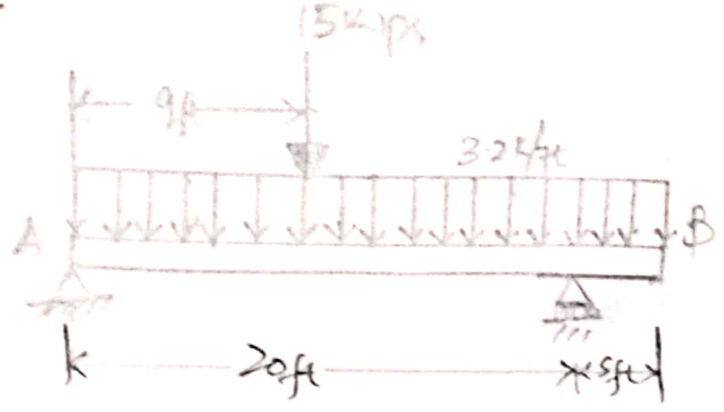
$$\tau_{max} = \frac{3(18\,kips)(4 \times 4\,in + 6\,in)}{2(0.15\,in)(6\,in)(6 \times 6\,in + 6\,in)}$$

$$\boxed{\tau_{max} = 15.71\,kips}$$



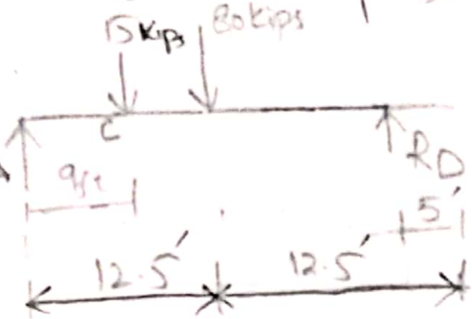
(17)

Q NO # 05 ⇒ Given DATA :- $S_{roll} = 15+4 = 19$ & $C_{roll} = 15+1 = 16$



Solution :- ① Determine Reaction at A & D

$\circlearrowleft + \sum M_A = 0$



$-15 \times 9 - 80 \times 12.5 + R_D \times 20.0$

$\Rightarrow -135 - 1000 + 20R_D = 0$

$\Rightarrow -1135 + 20R_D = 0$

$\Rightarrow 20R_D = 1135$

$\Rightarrow R_D = \frac{1135}{20}$

$\Rightarrow \boxed{R_D = 56.75\,\text{kips}} \longrightarrow$ Eq ⊛

Now $+\uparrow \sum F_y = 0$

$R_A + R_D - 15\,\text{Kips} - 80\,\text{Kips} = 0$

$R_A + R_D - 95\,\text{Kips} = 0$

$R_A + R_D = 95\,\text{Kips} \longrightarrow$ Eq #

Put equation ⊛ in equation #

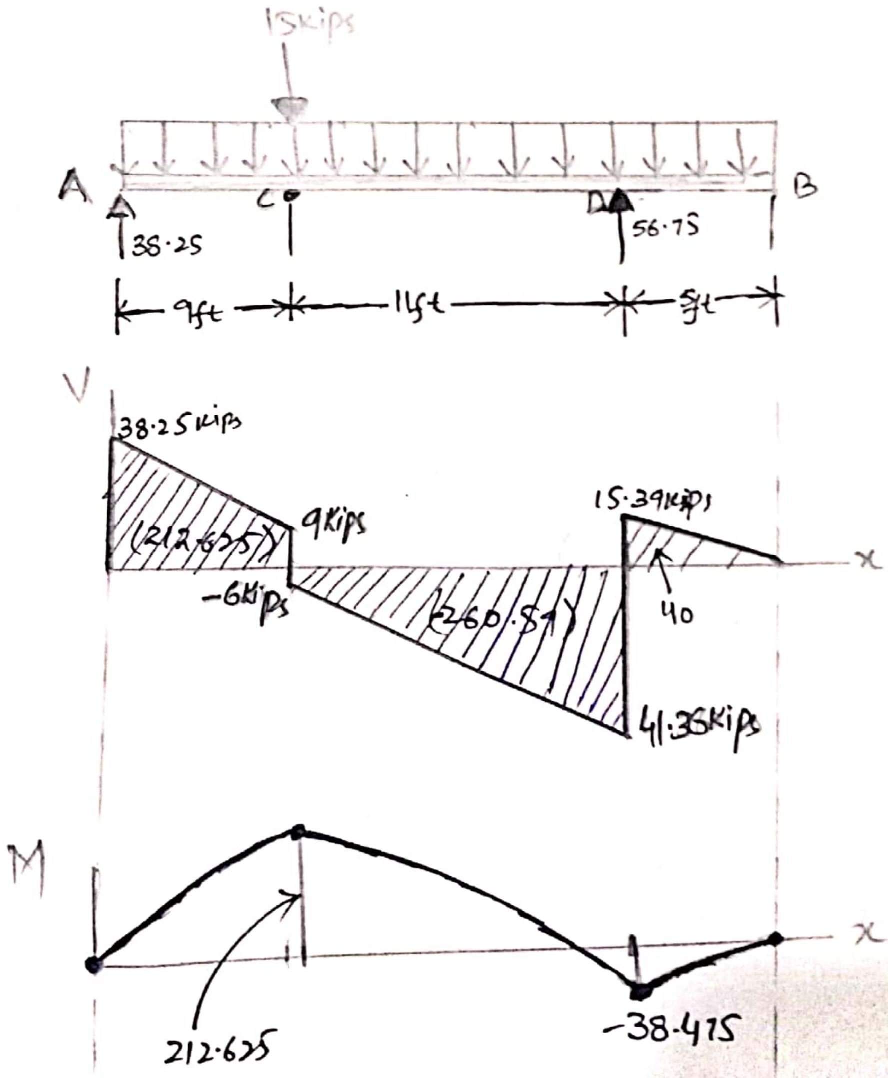
(12)

$$\Rightarrow R_A + 56.75 = 95$$

$$\Rightarrow R_A = 95 - 56.75$$

$$\Rightarrow \boxed{R_A = 38.25 \text{ kips}}$$

(ii) To determine shear force and bending moment



$|M|_{max} = 239.4 \text{ Kip-in}$ with $V = 9 \text{ Kips}$

$|V|_{max} = 41.36 \text{ Kips}$

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(iii) Calculate the required section modulus and select appropriate beam section:

Shape	$S\,in^3$
W24×68	154
W_{21}×62	127
W18×72	146
W16×77	134
W14×82	123
W12×96	131

$$S_{min} = \frac{|M|_{max}}{\sigma_{all}}$$

$$S_{min} = \frac{2274\ kip\cdot in}{19\ ksi}$$

$$S_{min} = 119.7\ in^3$$

Select W_{21}×62 beam section

(iv) maximum normal stress

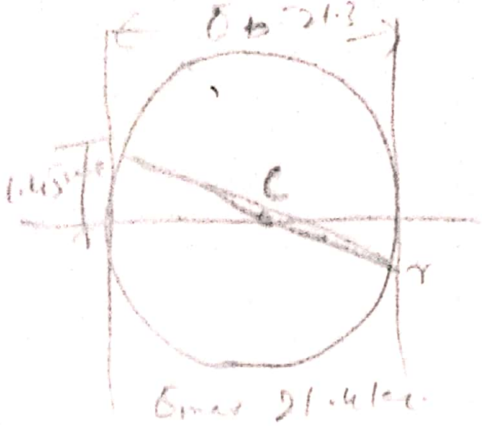
$$\sigma_a = \frac{M_{max}}{S} = \frac{2873 \times 60\ kip\cdot in}{127\ in^3} = 22.6\ ksi$$

$$\sigma_b = \sigma_a \frac{y_b}{c} = (22.6\ ksi)\frac{9.8}{10.5} = 21.3\ ksi$$

$$\tau_b = \frac{V}{A_{web}} = \frac{12.2\ kips}{8.4} = 1.45\ kips$$

$$\sigma_{max} = \frac{21.3\ ksi}{2} + \sqrt{\left(\frac{21.3\ ksi}{2}\right)^2 + (1.45\ ksi)^2}$$

$$\sigma_{max} = 21.4 < 24\ ksi$$



Find the max Shearing Stress

$$\tau_{max} = \frac{V_{max}}{A_{web}} = \frac{43\ \text{kips}}{8.40\ \text{in}^2} = 5.12\ \text{Kip}$$

5.12 Kip < 14.5 Kip

The END