IQRA NATIONAL UNIVERSITY PESHAWAR

DEPARTMENT OF CIVIL ENGINEERING

SUBJECT: ADVANCE FLUID MECHANICS

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Q.NO (01) A ANSWER

Question NO # 01 (A part)

1

⇒ Drag :- A body which is wholly Immersed in a homogenous fluid may be Subjected to two Kind of forces arising from relative motion b/w body and fluid these forces are termed as drag and lift. if the forces parallal to the motion then it is turmed as drag force.

There are two Components.

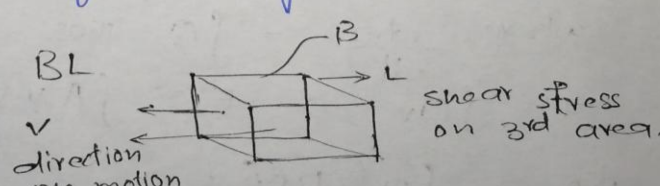
* Pressure Drag (Fp) :- it is equal to Integration of components in direction of motion of all pressure forces exerted on Surface of body.

$$F_p = C_p \int \frac{V^2}{2} A$$

∵ where Cp depends on shape.

* Friction Drag (Fƒ) :- it is equal to integ-ration of components of Shear Stress along Surface of body in direction of motion.

$$F_f = C_f \int \frac{V^2}{2} BL$$



⇒ Friction Drag of Boundry Layer.

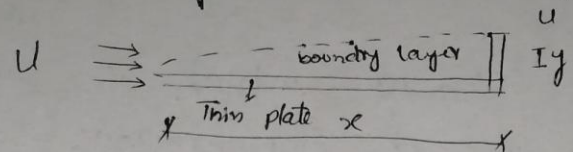
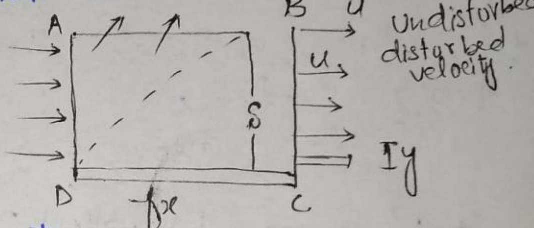


Fig shows growth of boondry layer along One site of Smooth plate inside the fluid.

Now consider a control vol.



Fluid undistarbed

where δ is Thicknes of boundry layer and U is Undisturbed velocity.

Thus

$-F_x$ = drag = (rate in momentum in x-dir).

leaving Through BC rate of momentum Through AB). - rate of momentum entring Through DA)

$\Delta P = P_{out} - P_{in}$.

Thus accourding to momentum.

$$\Sigma F = \frac{d}{dt}(P) = \frac{d\,m v}{dt}$$

where $\frac{dm}{dt} = \int Q$ Thus

$F = \int Q v \Rightarrow F = \int A \cdot v \cdot v \Rightarrow F = \int A v^2$

$DA \rightarrow \int U (UB\delta)$

$$BC \longrightarrow \rho B \int_0^{\delta} u^2 \cdot dy$$

$$AB \longrightarrow \rho v \left(UB\delta - B \int_0^{\delta} u \cdot dy \right)$$

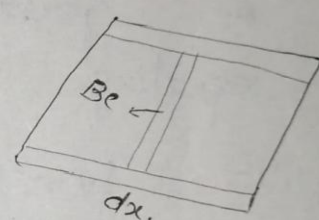
Putting value.

$$F_x = \rho B \int_0^{\delta} u (U - u)\, dy$$

$$F_x = \rho B U^2 \delta \alpha \qquad \because \alpha \text{ is Function of boundry layer.}$$

Now to find local wall shear stress.

$$\tau_0 = dF_x \Big/ B \cdot dx - \text{area}$$

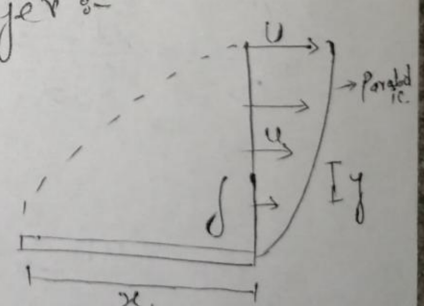


$$F_x = \rho B U^2 \delta \alpha$$

$$\tau_0 = \rho U^2 \alpha \frac{d\delta}{dx} \text{ is equation of Shear Stress.}$$

⇒ Laminar boundry layer :-

$$\frac{u}{U} = F\left(\frac{y}{\delta}\right)$$



Assume.

$$\eta = y/\delta \quad \text{or} \quad y = \eta\delta.$$

Thus

$$\frac{u}{U} = F(\eta) \quad \text{or} \quad u = Uf(\eta)$$

⇒ In Case of laminar Flow.

$$\tau_0 = \mu\left(\frac{du}{dy}\right) = \frac{U}{\delta}\left(dy/dx\right)$$

$$= \frac{\mu u}{\delta}\left[\frac{df(x)}{dx}\right]$$

Solving the equation.

$$\tau_0 = \frac{\mu U B}{\delta} \text{ ——— } (1).$$

As genral equation is $\tau_0 = \rho u^2 \alpha \frac{ds}{dx}$.

equating both equation.

$$\frac{\mu U B}{\delta} = \rho v^2 \alpha \frac{ds}{dx} \quad \text{or} \quad \delta ds = \frac{\mu B}{\rho u \alpha} dx.$$

integrating the equation.

$$\frac{\delta^2}{2} = \frac{\mu B}{\rho u \alpha} x + C$$

Now at $x = 0$, $\delta = 0$ Thus

$C = 0$

$$\frac{\delta^2}{2} = \frac{\mu B}{\rho u \alpha} x \quad \text{or} \quad \delta = \sqrt{\frac{2B}{\alpha}} \cdot \sqrt{\frac{\mu x}{\rho u}}.$$

xing and ÷ing by "x"

$$\delta = \sqrt{\frac{2\beta}{\alpha}} \cdot \sqrt{\frac{\mu x}{\delta U}} \cdot \sqrt[3]{\beta}\sqrt{x} \cdot \sqrt{x}$$

$\because$ $\alpha = 0.135$
$\beta = 1.63$
$R_x = \frac{\delta u x}{\mu}$.

$$\delta = \frac{4.91}{\sqrt{R_x}} \cdot x \quad \text{or} \quad \frac{\delta}{x} = \frac{4.91}{\sqrt{R_x}}$$

Now

$$\tau_0 = \frac{\mu U B}{\delta}$$

Thus putting the values.

$$\boxed{\tau_0 = 0.332 \, \frac{\mu U}{x} \sqrt{R_x}}$$

where R_x is the local Reynold number

Now

$$F_f = B \int_0^L \tau_0 \, dx$$

stress.

Putting the values.

$$F_f = 0.664 \, B \sqrt{\delta \mu L U^3}.$$

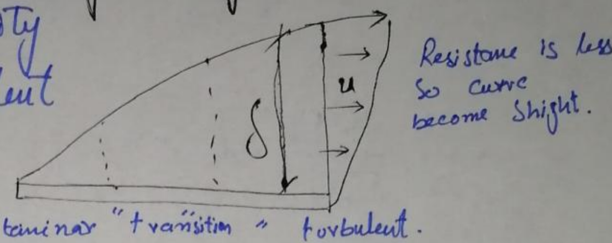
As general equation is

$$F_f = C_f \, \delta \, \frac{v^2}{2} \, BL \longrightarrow$$ equationg both equation.

$$C_f = 1.328 \sqrt{\frac{\nu}{\delta L V}} = \frac{1.328}{\sqrt{R}}$$

⇒ Turbulant boundry layer.

Fig show that velocity distribution in turbulent boundry layer.



Shows a much steepers gradient near wall and floor Flatter Through out remaning layer the Shear Stress is greater in turbulent Than in laminar layer.

As we have

$$\tau_0 = f\rho\frac{v^2}{8}$$ :: where v donates average velocity of pipe.

Now we have obtained an approximate relation b/w v and U by using pipe factor equation of

$$\frac{V}{U_{max}} = \frac{1}{1 + 1.33\sqrt{F}}$$

Using Friction Factor of 0.028 From chart which is middle critical value

so $\boxed{U = 1.235\, V}$

NO we have

$$\tau_0 = f\rho\frac{v^2}{8}$$

As we have

$$F = \frac{0.316}{R_{0.25}}$$

Thus

$$\tau_0 = \frac{0.316}{\left(\frac{DU}{\nu}\right)^{1/4}} \frac{\rho v^2}{8}$$

where

$$V = \frac{U}{1.235} \quad \text{Thus}$$

$$\tau_0 = \frac{0.316}{\left(\frac{D}{\nu}\left(\frac{u}{1.235}\right)\right)^{1/4}} \cdot \rho/_8 \left(\frac{V}{1.235}\right)^2$$

$\varepsilon \quad D = 25$

Thus

$$\tau_0 = \frac{0.023 \, \rho v^2}{\left(\delta v/_{\nu}\right)^{1/4}}$$

$\Rightarrow$ As we have

$$\tau_0 = \rho v^2 \alpha \frac{d\delta}{dx}$$

Equating both and integrating For boundry condition of. $x = 0$, $\delta = 0$

Thus

$$\delta = \left(\frac{0.0287}{\alpha}\right)^{4/5} \left(\nu/_{Ux}\right)^{1/5} x.$$

For $\alpha = 0.0972$

$$\Rightarrow \boxed{\frac{\delta}{x} = \frac{\cancel{00}\ 0.377}{(R_x)^{1/5}}}$$

Putting values in equation.

$$\tau_0 = 0.0587 \frac{\rho v^2}{2} \left(\frac{\nu}{Ux}\right)^{1/5}$$

Now $F_f = B \int_0^2 \tau_0 \, dx$

$$F_f = 0.0735 \rho U^2/2 \left(\frac{\nu}{UL}\right) BL.$$

As

$$F_f = c_f \rho \frac{v^2}{2} BL$$

Equating Both

$\because$ R less than 10^7 for $800,000 < R < 10^7$

$$\boxed{c_f = \frac{0.0735}{R^{1/5}}}$$

For $R > 10^7$

$$\boxed{c_f = \frac{0.455}{(\log R)^{2.58}}}$$

Ans

Q.NO (01) B	ANSWER

QNO 01 (B part).

As specific energy.

$$E = y + \frac{V^2}{2g}$$

The Flow Q per unit width b can be expressed as

$$q = Q/b$$

Now average velocity will be

$$V = Q/A = \frac{qb}{by} = q/y$$

Thus

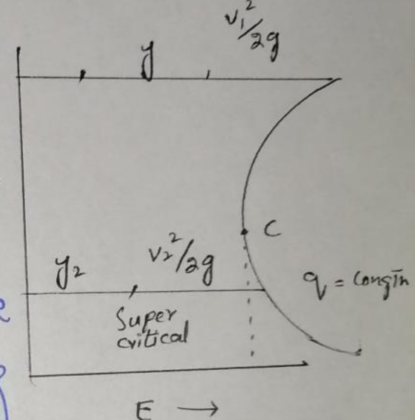
$$E = y + \frac{V^2}{2g} \Rightarrow y + \frac{1}{2g}\left(\frac{q^2}{y^2}\right)$$

$$(E - y) = \frac{1}{2g}\left(q^2/y^2\right) \text{ or}$$

$$(E - y)\,y^2 = q^2/2g$$

Thus E vs Y will be parabolic

For particular q, there will be two kind of possible values of y, for a given E.



The Equation is cubic with three roots, with Third root being negative Point c, represents dividing Point b/w two regine of flow thus for given q_1 and value of E is minimum and Flow at that point is critical Flow. Depth of Flow at that Point is critical depth y_c and velocity at that point is critical velocity V_c

Thus

$$E = y + \frac{1}{2g}\left(q^2/y^2\right)$$

For minimum specific energy $\frac{dE}{dy} = 0$

Thus

$$dE/dy = 1 - 2/2g\left(q^2/y^3\right) = 0$$

$$\Rightarrow \quad q^2/gy^3 = 1 \Rightarrow q^2 = gy^3$$

$$\Rightarrow \quad q^2/g = y^3 \Rightarrow \left(q^2/g\right)^{1/3} = y_{cr}$$

Now

$$\Rightarrow \quad q^2 = gy^3$$

and

$$\Rightarrow \quad q = vy \Rightarrow v^2y^2 = gy^3$$

or $v^2 = gy_{cr}$

or

$$\boxed{v_{cr} = \sqrt{gy_{cr}}}$$

Q.NO (02) ANSWER

Question NO # 02

⇒ Given Data

water flow at rate of = $Q = 3.5\, m^3/s$

Bed slope, $S_o = 0.0008$

$n = 0.0219$

width of bed is ID 7776

⇒ Required

Depth of Rectangular chanel = ?

critical depth $y_c = ?$

critical velocity $= v_c = ?$

Flow is - Critical or Sub- Critical = ?

Solution

$$Q = \left(\frac{1}{n} R_n^{2/3} S_0^{1/2} \right) A \quad \text{—— (1)}$$

Area $= 7.776 \times d$

Parameter $= d + 7.776 + d$

hydrulic Radius $= R_n = \dfrac{\text{Area}}{\text{Parameter}}$

As we know that

$$Q = \left(\frac{1}{n} R_n^{2/3} S_0^{1/2} \right) A$$

Putting the values.

$$3.5 = \left(\frac{1}{0.0219} \times \left(\frac{7.776 \times d}{2d + 7.776} \right)^{2/3} \times (0.0008)^{1/2} \right.$$
$$\times 7.776 \times d$$

As $q = Q/b$

$$q = \frac{3.5}{7.776} = 0.450$$

For critical depth

$$y_{cr} = \left(\frac{q^2}{g}\right)^{1/3}$$

$$y_{cr} = \left(\frac{(0.450)^2}{9.81}\right)^{1/3}$$

$$\boxed{y_{cr} = 0.274\,m}$$

⇒ Critical velocity, V_{cr}

using equation $V_{cr} = \sqrt{g y_{cr}}$

$$V_{cr} = \sqrt{(9.81)(0.274)}$$

$V_{cr} = 1.639 \ m/s$

$V = Q/A$

$$V = \frac{3.5}{7.776 \times 0.831}$$

$$\boxed{V = 0.541 \ m/s}$$

$y = 0.840 m$, $y_{cr} = 0.274 m$

$V = 0.541 \ m/s$, $V_{cr} = 1.639 \ m/s$

As $y > y_{cr}$ and $V < V_{cr}$

So Flow is Sub-Critical

①

Question No 03

Given data: width of Smooth plate

$= B = 200\,mm$

$B = 0.2\,m$

length of plate $L = 800\,mm$

$L = 0.8\,m$

specific gravity of oil $\rho = 0.89$

undisturbed velocity $V = 5\,m/s$

kinematic viscosity $\nu = 0.93 \times 10^{-4}\,m^2/s$.

⇒ Required data:

Friction drag $F_f = ?$

⇒ Solution

As we know That.

The Friction drag is given by the equation.

$$F_f = C_f\, \rho\, \frac{V^2}{2}\, BL \quad \text{——} ①$$

where c_f depends on viscosity.

Reynold's number is given by

$$R = \frac{\rho v L}{\mu} = \frac{vL}{\nu} \quad \because \nu = \frac{\mu}{\rho}$$

Putting valus we get.

$$R = \frac{5\ m/s \times 0.8\ m}{0.93 \times 10^{-4}\ m^2/s}$$

$$R = 43{,}010.75$$

which is less Than 500,000.

So,

$$c_f = \frac{1.328}{\sqrt{R}} = \frac{1.328}{\sqrt{43011}} = 0.00640.$$

Now As Specific gravity $= \dfrac{\rho_{oil}}{\rho_{water}}$

ρ_{oil} = Specific gravity $\times \rho_{water}$.

$$\rho_{oil} = 0.89 \times 1000$$

$$\boxed{\rho_{oil} = 890}$$

Putting values in equation ① we get

$$F_f = 0.00640 \times 0.89 \times 1000 \times \frac{(5)^2}{2} \times 0.2 \times 0.8$$

$$\boxed{F_f = 11.392\ N}$$

So,

The Friction drag on One side of Smooth plate is 11.392 N.

THE END