# (Differential Equation)



Muhammad Asim
ID :- 12362.
Dept :- Electrical Engineering
Submitted to:-
SIR HIMAYAT ULLAH.

Q1:- Estimate the general solution of

$$4y'' - 20y' + 25y = 0 \quad - (A)$$

Sol :- As $\lambda^2 + a\lambda + b = 0$ — (1).
For a general form of;

$$y'' + ay' + by = 0 \quad - (2)$$

So, first to modify the given eq (A) ÷ing it by 4.

$$\frac{4y''}{4} - \frac{20y'}{4} + \frac{25y}{4} = \frac{0}{4} \quad (2)$$

$$\Rightarrow y'' - 5y' + \frac{25y}{4} = 0 \quad - (3)$$

From (2) & (3).

$a = -5$ & $b = 25/4$

putting a & b in eq (1).

$$\lambda^2 - 5\lambda + \frac{25}{4} = 0$$

$\Rightarrow \quad 4\lambda^2 - 20\lambda + 25 = 0$

using quardartic equation

$$\lambda_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2(a)}.$$

$$= \frac{-(-20) \pm \sqrt{(-20)^2 - 4(4)(25)}}{2(4)}$$

$$= \frac{20 \pm \sqrt{400-400}}{8}$$

$$= \frac{20}{8} \Rightarrow 5/2.$$

the roots are real & repeated/equal

So, the general solution is;

$$y = (c_1 + c_2 x)\, e^{\lambda x}$$

$$\Rightarrow \boxed{y = (c_1 + c_2 x)\, e^{5x/2}}$$

is the required all estimated/general solution.

or $\boxed{y(x) = c_1 e^{5/2 x} + c_2 x e^{5/2 x}}$

answer.

Q2:- Estimate the general Solution.

$$y' = (x+2)\, y^2$$

Solution :-

We can write the given Sol. as;

$$\frac{dy}{dx} = (x+2)\, y^2$$

$$\Rightarrow \frac{dy}{y^2} = (x+2)\, dx$$

taking integral on BHS;

$$\int \frac{1}{y^2}\, dy = \int (x+2)\, dx$$

$$\int y^{-2}\, dy = \left(\frac{x^2}{2} + 2x\right) + c$$

$$\frac{y^{-2+1}}{-2+1} = \frac{x^2}{2} + 2x + c$$

$$\frac{y^{-1}}{-1} = \frac{x^2}{2} + 2x + c$$

$$y^{-1} = -\frac{x^2}{2} - 2x + c$$

$$\Rightarrow \boxed{y = -\frac{1}{x^2/2 + 2x + c}}$$

Q3:- Calculate the initial value

$$y'' + 2y' + y = 0$$

$$y(0) = 4 \quad , \quad y'(0) = -6.$$

Solution:- We can write the given eq

$$d^2y/dx^2 + 2\frac{dy}{dx} + y = 0$$

& we can write it as differential operate as;

$$D^2y + 2D + y = 0$$

& its auxillary equation will be;

$$D^2 + 2D + 1 = 0$$

$$D^2 + 1D + 1D + 1 = 0$$

$$D(D+1) + 1(D+1) = 0$$

$$(D+1)(D+1) = 0$$

$$\Rightarrow \quad D = 1, 1$$

So, the roots are real & same/equal

so the general solution $y_c$ will be;

$$y = y_c = (c_1 + c_2 x)e^{1x} \quad \text{—(1)}$$

Now using the initial condition $y(0) = 4$.

$$\Rightarrow \quad 4 = (c_1 + c_2(0))\, e^{1(0)}$$

$$\Rightarrow \quad 4 = c_1$$

$$c_1 = 4 \quad = (4).$$

taking derivative of eq ①.

$$\frac{dy}{dx} = \frac{dy_c}{dx} = \frac{d}{dx}\left\{(c_1 + c_2 x)e^x\right\}$$

$$\Rightarrow \quad y' = y_c' = \frac{d}{dx}\left(c_1 e^x + c_2 x e^x\right).$$

$$= c_1 \frac{d}{dx} e^x + c_2 \frac{d}{dx} x e^x.$$

$$= c_1 e^x + c_2\left(x e^x + e^x\right).$$

$$y' = y_c' = c_1 e^x + c_2 e^x + c_2 x e^x.$$

Now using the second initial equation:

$$y'(0) = -6.$$

$$\Rightarrow \quad -6 = c_1 e^{(0)} + c_2 e^{(0)} + c_2 \cancel{(0)}^{0} e^{(0)}$$

$$\Rightarrow \quad c_2 = -10$$

putting values of $c_1$ & $c_2$ in eq ①.

$$\boxed{y = y_c = (4 - 10x)\, e^x}$$ is the req solution.

Q4:- Analyze the general Solution

$$x^2y'' + 3xy' + y = 0.$$

Solution:- Second order Euler homogeneous eq

$a = 3$ , $b = 1$.

$$m^2 + (a-1)m + b = 0$$

$$m^2 + (3-1)m + 1 = 0.$$

$$m^2 + (2)m + 1 = 0$$

$$m^2 + 2m + 1 = 0$$

$$m^2 + m \cdot m + 1 = 0$$

$$m(m+1) + 1(m+1) = 0$$

$$(m+1)(m+1) = 0$$

$$m = -1 \ , \ m = -1$$

Roots are real & equal

$$y = (c_1 + c_2 \ln x)x^m$$

$$\boxed{y = (c_1 + c_2 \ln x)x^{-1}}$$

Q5:- Examine the method :-

$$y'' + y' - 6y = 6x^3 - 3x^2 + 12x \quad -① $$

For Homogenous equation

$a = 1 \quad , b = -6$

$\lambda^2 + \lambda - 6$

$\Rightarrow \quad \lambda^2 + 3\lambda - 2\lambda - 6.$

$\lambda(\lambda+3) - 2(\lambda+3)$

$\Rightarrow \quad \lambda - 2 = 0 \quad , \lambda + 3 = 0$

$\lambda = 2 \quad , \quad \lambda = 3$

So,

$$y = C_1 e^{2x} + C_2 e^{-3x}$$

$\Rightarrow y_P = K_3 x^3 + K_2 x^2 + K_1 x^1 + K_0$

$y_P = 3K_3 x^3 + 2K_2 x^2 + K_1$

$y_P = 6K_3 x + 2x_2$

Put in eq ①

$\Rightarrow \underbrace{6x_3 x + 2x}_{y''} + 3K_3 x^2 + 2K_2 x + x_1$
$-6K_3 x^3 - 6x_2 x^2 - 6x_1 x - 6x_0$

$\Rightarrow \quad 6x_3 x^3 - 3K_2 x^2 + 12x_1$

$\Rightarrow -6K_3x^2 + (3K_3 - 6K_2)x^2 + 6K_3 + 2K_2) - 6K_1x$
$+ 2K_2 + K_1 - 6K_0 = 6x^3 - 3x^2 + 12x - 6K_3 = 6.$

$\Rightarrow \boxed{K_3 = -1}$

$3K_3 - 6K_2 = -3.$

$3(-1) - 6K_2 = -3.$

$-K_2 = 0 \Rightarrow K_2 = 0.$

$6K_3 - 2K_2 - 6K_1 = 12$

$\Rightarrow 6(-1) + 1(0) - 6K_1 = 12.$

$-6K_1$
$K_1 = -18/6.$

$\boxed{K_1 = -3}$

$-2K_1 + K_1 + K_0 = 0$
$-2(-3) - 2 + K_2 = 0.$

$\boxed{K_0 = -1/2}$

So, ~~yP = $-x^3$~~

$yP = -x^3 - 3x - 1/2 = -x^3 - 3x - 1/2.$

Q 6:- Examine the method.

$$y'' - 4y' + 4y = x^2 e^{2x}.$$

Sol:- Since the auxilary equation is

$$m^2 - 4m + 4 = 0.$$

$$m^2 - 2m - 2m + 4 = 0.$$

$$m(m-2) - 2(m-2) = 0$$

$$(m-2)(m-2) = 0$$

$$m = 2, \quad m = 2.$$

$$y_c = c_1 e^{2x} + c_2 x e^{2x}.$$

$$y_1 = e^{2x} \text{ and } y_2 = x e^{2x}.$$

We next Compute;

$$w(e^{2x}, xe^{2x}) = \begin{vmatrix} e^{2x} & xe^{2x} \\ 2e^{2x} & 2xe^{2x} + e^{2x} \end{vmatrix}$$

$$= 2xe^{4x} + e^{4x} - 2xe^{4x}$$

$$= \cancel{2x}e^{4x} + e^{4x} - \cancel{2x}e^{4x}$$

$$= e^{4x}$$

Now as $f(x) = x^2 e^2 x.$

$$w_1 = \begin{vmatrix} 0 & xe^{2x} \\ x^2e^{2x} & 2xe^{2x}+e^{2x} \end{vmatrix}$$

$$= -x^3e^{4x}.$$

$$w_2 = \begin{vmatrix} e^{2x} & 0 \\ 2e^{2x} & x^2e^{2x} \end{vmatrix}$$

$$w_2 = x^2e^{4x} - 0$$

$$w_2 = x^2e^{4x}.$$

$$U_1' = \frac{w_1}{w}$$

$$U_1' = \frac{-x^3e^{4x}}{e^{4x}}$$

$$U_1' = -x^3$$

$$U_2' = \frac{w_2}{w} \Rightarrow \frac{x^2e^{4x}}{e^{4x}}$$

$$U_2' = x^2$$

It follows that.

$$U_1' = -x^3$$

$$\int U_1' = -\int x^3$$

$$u_1 = \frac{-x^4}{4}$$

and

$$u_2' = x^2 \Rightarrow \int u_2' = \int x^2$$

$$u_2 = \frac{x^3}{3}$$

Therefore,

$$y_p = \left(\frac{-x^4}{4}\right) e^{2x} + \left(\frac{x^3}{3}\right) x e^{2x}$$

$$y_p = \left(-\frac{x^4}{4} + \frac{x^4}{3}\right) e^{2x}$$

Hence $y = y_c + y_p$.

$$y = c_1 e^{2x} + c_2 x e^{2x} + \left[-\frac{x^4}{4} + \frac{x^4}{3}\right] e^{2x}$$

ans.

Q 7:- Identify an ODE

$y'' + ay' + by = 0 \longrightarrow 1, e^{-3x}$.

Solution:- So,

Basis are $1, e^{-3x}$.

$y = c_1 e^{x} + c_2 e^{-3x}$.

$\lambda = 0 \quad , \quad \lambda = -3$.

$\lambda_1 - 0 = 0 \quad , \quad \lambda + 3 = 0$.

$\lambda(\lambda + 3) = 0$.

$\lambda^2 + 3\lambda = 0$

$a = 3 \quad , \quad b = 0$.

$y'' - 3y' + 0y = 0$

$\boxed{y'' - 3y' = 0}$ answer