INTRODUCTION TO STRUCTURAL DYNAMICS & EARTHQUAKE ENGINEERING

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Section : A

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Qeustion #01

Given data:-

- A beam is Pulled in a downward direction = $\frac{1}{2}$ inch
- Ignore the self weight of beam as well as damping effect.
- $E = 29000$ Ksi
- $I = 150$ in^4
- δ_{st} = Deflection due to 7702 lb static load.

Required data:-

- Natural time Period of system = ?
- Develop and solve equation of motion for vibration.

Solution:-

As we know that

The general EOM for SDOF system is;

$$Ku + c\dot{u} + m\ddot{u} = P(t)$$

In our case system is undamped ($c=0$) undergoing free vibration ($P(t) = 0$)

Hence;

The general EOM becomes;

$$Ku + m\ddot{u} = 0 \longrightarrow (1)$$

Here

$$K = \frac{3EI}{L^3}$$

$$K = \frac{3 \times (29000) \times (150)}{(10 \times 12)^3}$$

$$K = 7.55 \text{ K/in}$$

Also;

$$K = 7.55 \text{ K/in} \times 1000 \times 12$$

$$K = 90625 \text{ lb/ft}.$$

Similarly ;

$$m = \frac{7702}{32.2} = 239.19 \text{ slug}$$

$$W_n = \sqrt{\frac{K}{m}} = \sqrt{\frac{90625}{239.19}}$$

$W_n = 19.464$ rad/sec

for natural time period of system

$$T_n = \frac{2\pi}{W_n} = \frac{2 * \pi}{19.464}$$

$$\boxed{T_n = 0.323 \text{ sec}}$$

Substituting the corresponding value in eq -1 we get

$$90625 u + 239.19 \ddot{u} = 0$$

where K is in lb/ft and m is in lb sec^2/ft

General solution to the EOM for undamped free vibration is;

$$u(t) = u(0)\cos(\omega_n t) + \frac{\dot{u}(0)}{\omega_n}\sin(\omega_n t)$$

$u(0) = \frac{1}{2}'' = \frac{1}{24}$ ft and $\dot{u}(0) = 0$

$$u(t) = \left(\frac{1}{24}\right) \times \cos(19.464t) + 0 =$$

$$\boxed{u_t = \frac{1}{24}\cos(19.464t)}$$

Equivalent static forces at any time "t" is

$$f_s(t) = K \cdot u(t) = \frac{90625 \times \cos(19.464t)}{24}$$

$$f_s(t) = 3776.041\cos(19.464t)$$

Amplitude of dynamic displacement, u_o for undamped free vibration is

$$u_o = \sqrt{\left[(u(0))^2 + \left(\frac{\dot{u}(0)}{\omega_n}\right)^2\right]}$$

$$U_0 = \sqrt{\left(\frac{1}{24}\right)^2 + 0}$$

$$U_0 = \frac{1}{24} \text{ ft}$$

Amplitude of Equivalent static force,

f_{s0}

$$K u_0 = 90625 \times \frac{1}{24} = 3776.04 \text{ lb.}$$

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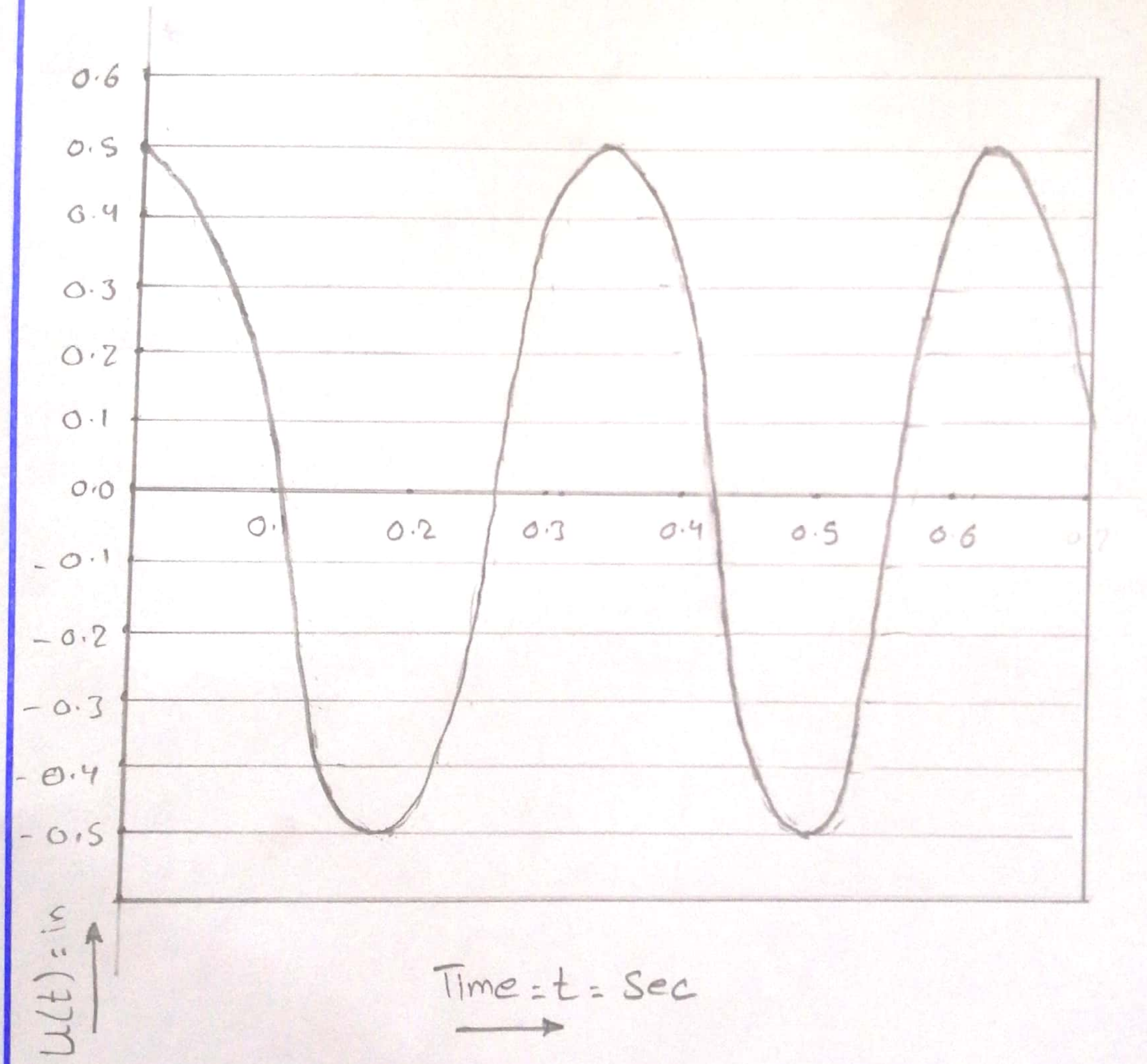
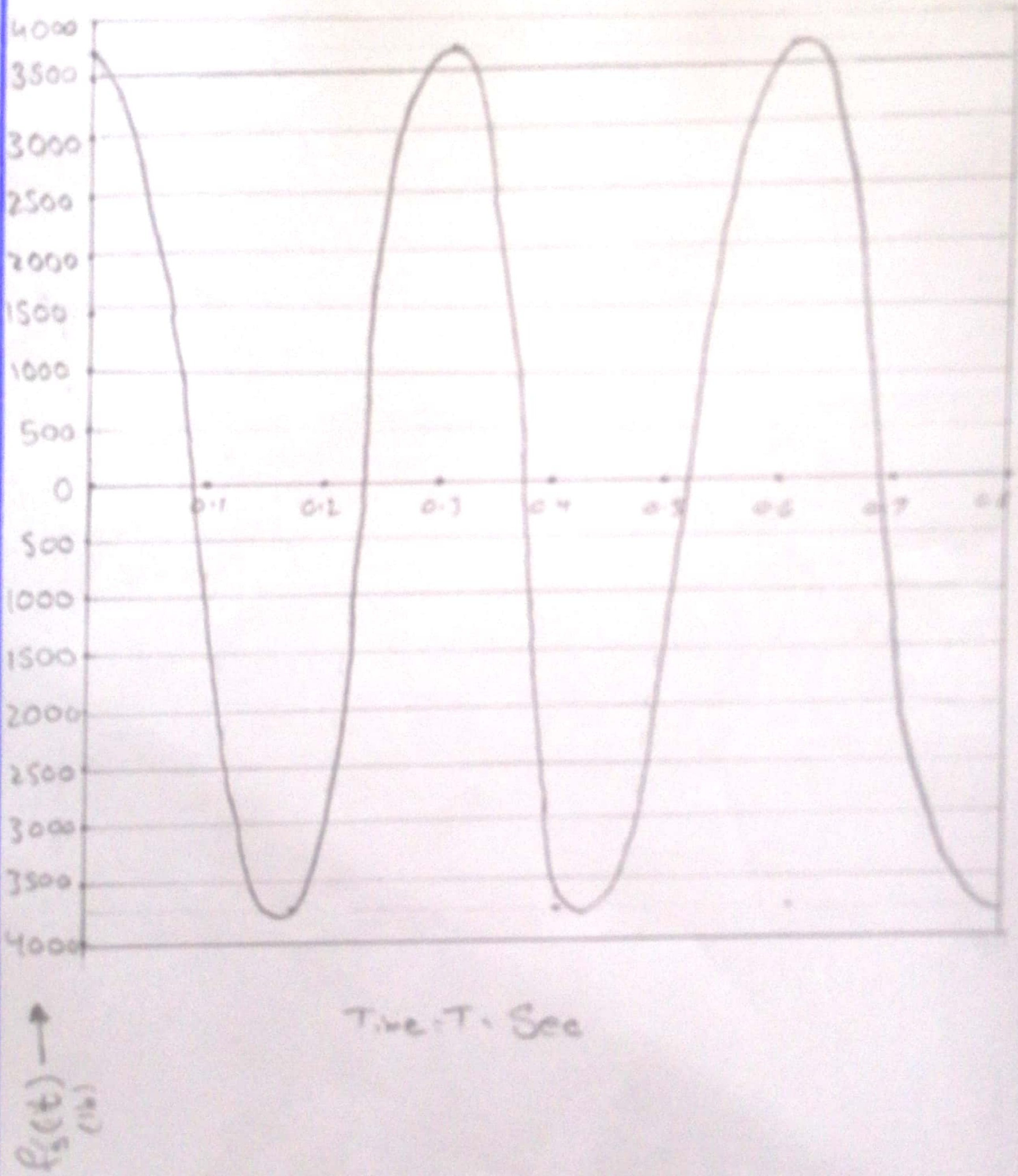
Graphs:

a- Showing Variation of displacement with time;



b. Showing Variation of Equivalent Static force with time :



Question NO 2:-

Given data:-

Damping ratio of reinforced concrete with considerable cracking = 3 - 5 %

→ So we take $\zeta = 3\%$

→ Other data are taken from Question No 1;

Required data:-

Develop and solve the equation of motion for vibration at free end = ?

Develop an equation showing variation in equivalent static forces with time = ?

Solution:-

As we know that;

EOM (equation of motion) for damped free vibration is;

$$Ku + c\dot{u} + m\ddot{u} = 0 \rightarrow (1)$$

As we know from question No:1 data

i.e

$K = 90625 \ lb/ft$

$m : 239.19 \ lb.\frac{sec^2}{ft}$

$W_n = 19.464 \ rad/sec$

As we know that:

$c = \xi \times 2 m W_n$

$c = 0.03 \times 2 \times 239.19 \times 19.464$

$c = 279.335 \ lb.sec/ft$

By Putting value's in eq (1)

we get;

$90625 u + 279.61 \dot{u} + 239.6 \ddot{u} = 0$

solution to the EOM for damped free vibration is:

$$u(t) = e^{-\xi w_n t}\left[u(0)\cos(w_D t) + \frac{1}{w_D}\left[\dot{u}(0) + u(0)\,\xi w_n\right]\sin(w_D t)\right]$$

Here;

$W_D = 19.464 \ rad/sec.$

$$u(t) = e^{-(0.03 \times 19.464 \times t)}\left[\frac{1}{24} \times \cos(19.464t) + \frac{1}{19.464} \times \left[0 + \frac{1}{24} \times 0.03 \times 19.464 \times \sin(19.464t)\right]\right]$$

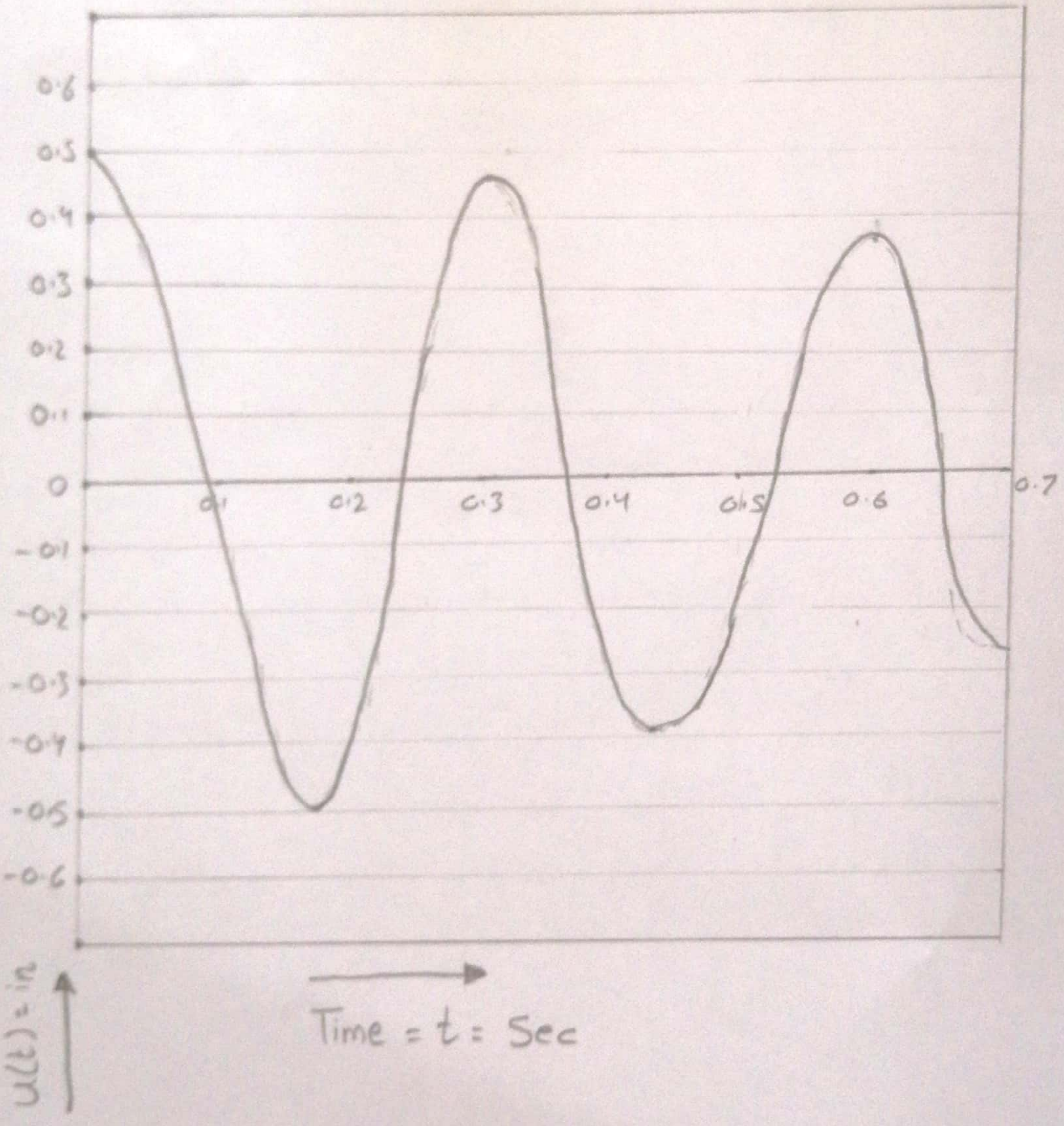
$$u(t) = e^{-(0.583t)}\left[0.0416 \times \cos(19.464t) + 0.024 \sin(19.464t)\right]$$

$$f_s(t) = 90625 \times u(t)$$

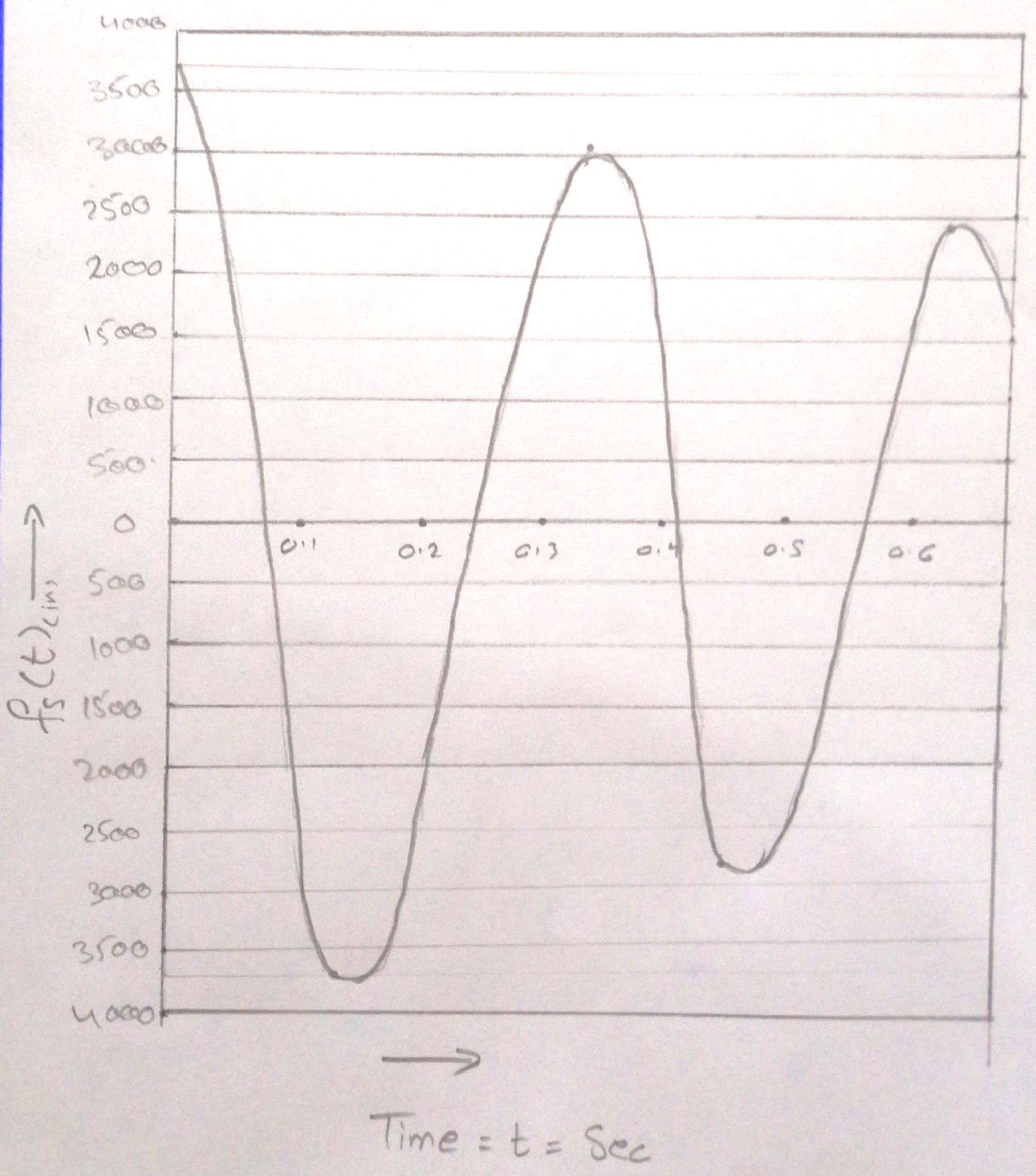
$$f_s(t) = e^{-0.584t}\left[3806.25 \cos(19.464t) + 2175 \sin(19.464t)\right]$$

Graph:

a. Shows variation of displacement with time;



b- Showing Variation of Equivalent Static force with time:



Question No 3

Given data:-

- Amplitude cable force = 60 Kips
- Horizental displacement of tank = $\frac{7702}{1000}$

  $= 7.702$ in

- Cycles = 7
- Cycle completion time = 3.57 sec
- Amplitude of displacement = 2.286 cm

  $= 0.9$ in

Required data:

complete the following:

- Damping ratios
- Natural period of undamped vibration
- Stiffness of structures
- weight of tank
- Damping co-efficient
- Number of cycles to reduce the displacement amplitude to 0.5"

Solution:

As given in question;

$u_1 = 7.702\ in$

After $J = 7$

$u_{J+1} = u_8 = 2.286\ cm = 0.9\ in$

a- **ζ = Damping ratio = ?**

$$J = \frac{1}{2\pi\zeta} \ln\left[\frac{u_1}{u_{J+1}}\right]$$

By putting values we get;

$$7 = \frac{1}{2\pi\zeta} \ln\left[\frac{7.702}{0.9}\right]$$

$$\zeta = \frac{1}{2\pi 7} \ln\left[\frac{7.702}{0.9}\right]$$

$$\zeta = 0.0488 = 4.88\%\ \text{Ans.}$$

$$\zeta = 4.88\%$$

6> Natural Period of undamped vibration

According to question,

7 cycle's of vibration's are completed in 3.57 sec

Time Period to complete one cycle = $\frac{3.57}{7}$

$T_D = 0.51$ sec

As we know that

$$\omega_D = \omega_n\sqrt{(1)-(\zeta)^2}$$

$$\frac{2\pi}{\omega_D} = \frac{2\pi}{\omega_n\sqrt{(1)-(\zeta)^2}}$$

$$T_D = T_n / \sqrt{(1)-(\zeta)^2}$$

$$T_n = T_D\sqrt{(1)-(\zeta)^2}$$

By putting values we get;

$$T_n = 0.5\sqrt{(1)-(0.0488)^2}$$

$$T_n = 0.5094$$

$$T_n = 0.509 \text{ sec}$$

c: stiffness of structure = K

As we know that

$$K = \frac{60 \cos(60)}{7.702}$$

$K = 3.89$ K/in

$K = 46741.10$

d: weight of tank = W =

As we know that

$$\omega_n = \sqrt{K/m} = \sqrt{\frac{K}{(W/g)}}$$

$$= \sqrt{\frac{K \cdot g}{W}}$$

$$\omega_n^2 = \frac{K \cdot g}{W}$$

$$W = \frac{K \cdot g}{\omega_n^2} \longrightarrow (a)$$

~~A~~lso: $\omega_n = \frac{2\pi}{T_n}$

eq (a) => $W = \frac{K \cdot g \times T_n^2}{4\pi^2}$

Put value's:

$$W = \frac{46741.10 \times 32.2 \times (0.51)^2}{4 \times \pi^2}$$

$W = 9530.925$ lb

$W = 9.530$ K

e: Damping coefficient = c =

As we know that

$$\xi = \frac{c}{2 m \omega_n}$$

$$c = \xi \times 2 m \omega_n = \xi \times 2m \times \left(\frac{2\pi}{T_n}\right)$$

$$c = \frac{\xi \times 4 \times \pi \times m}{T_n}$$

By putting value's we get;

$$c = \frac{0.0488 \times 4 \times \pi \times \left(\frac{9530.925}{32.2}\right)}{0.51}$$

$$\boxed{c = 355.908 \text{ lb.sec/ft}}$$

f- Number of cycles to reduce the displacement amplitude to 0.5": j =

As we know that;-

$$j = \frac{1}{2\pi\zeta} \ln\left[\frac{u_1}{u_{j+1}}\right]$$

$$j = \frac{1}{2\times\pi\times0.0488} \ln\left[\frac{7.702}{0.5}\right]$$

$J = 8.918$ say 9 cycles.

$J = 8.9 = 9$ cycles