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SUBJECT # DIGITAL SIGNAL PROCESSING

DEPARTMENT # BEE

SEMESTER # 8TH

DATE # 24-08-2020

TEACHER NAME

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QUESTION NO 1

PART A

a) Consider the following analog signal

$$x_a(t) = 3\cos 100\pi t + 4\sin 200\pi t$$

1 Determine the minimum sampling rate required to avoid aliasing.

Ans// According to sampling Theorm

$F_1 = 100\,Hz$

$F_2 = 200\,Hz$

$f_s \geq F_{max}$ $\quad f = \frac{\omega}{2\pi}$

so

F_2 is max (greater than F_1)

$f_s > 2 \times 100$

$f_s = 200\,Hz$

ii) Suppose that the signal is sampled at the rate $f_s = 100$ Hz what is the discrete-time signal is obtained after sampling? Also explain the effect of this sampling rate on the newly generated discrete time signal?

Solution:-

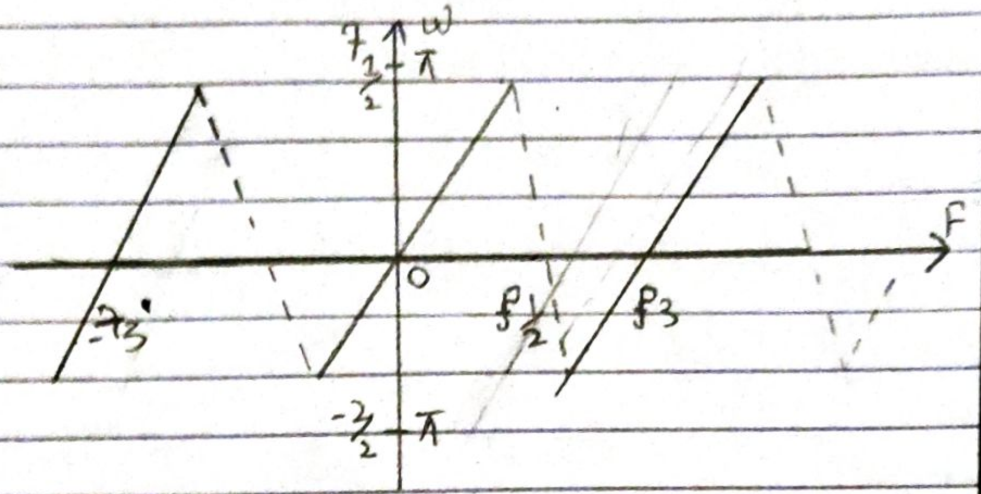
$$f_s = 100 \text{ Hz}$$

$$f = \frac{100}{2} = 50 \text{ Hz}$$

This is the max frequency that can be repesented uniquely by the sampled signal As

$$X_a[n] = 3\cos 2\pi\left(\frac{50}{100}\right)n + 4\sin 2\pi\left(\frac{100}{100}\right)n$$

$$= 3\cos\pi\left(\frac{5}{10}\right)n + 4\sin 2\pi n$$



The effect of Sampling rate on the newely generated discrete time signal is that There will be no display phenomenan mean. There will ~~be~~ not present unwanted component in Reconstruct of the signals The reconstruct original signals

iii) What is the analog signal $X_a(t)$ we can reconstruct from the samples If we use ideal interpolation?

Ans Folding frequency $= \frac{F_s}{2} = \frac{100}{2}$

$= 50$ Hz

$F_1 = 50\, Hz$

$F_2 = 100\, Hz$

Both frequency are either equal or the folding frequency.
Hence for ideal interpolation we can construct the orignal signal

$$X_a(t) = 3\cos 100\pi + 4\sin 200\pi t$$

Since only the frequency component at 100 Hz are present on the sampled signals the analog signal we can remove or reconstruct is

$$y_a(t) = 3\cos 100\pi t$$

Ans

QUESTION NO 1

PART B

b) Consider a discrete time signal which is given by

$$x(n) = \begin{cases} 0.5^n & , n \geq 0 \\ 0 & , n < 0 \end{cases}$$

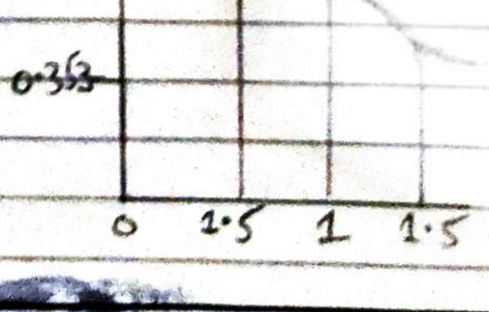
This signal is sampled at the rate $F_s = 2\,Hz$.

i) Draw the sampled signal.

$$f_s = \frac{1}{T} = T = 1/f_s$$

$$= 1/2 = 0.5 \text{ sec}$$

x_n	0.5^n
0	1
0.5	0.7071
1	0.5
1.5	0.353



ii) The samples of the signals are intended to carry 3 bits per sample.
Detemine the quantization level and quentization resolution to quantized the sampled signal achieved in part i.

Ans

$$L = 2n$$

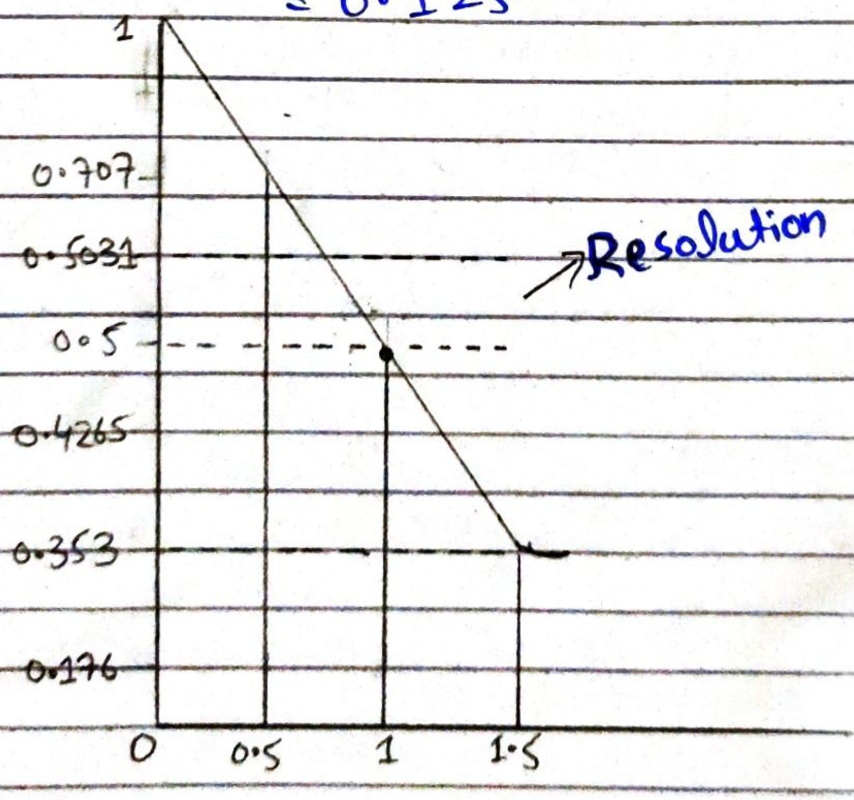
$$n = \text{bits } 3$$

$$L = 2^3 = 8 \text{ Levels}$$

$$\text{Resolution} = \frac{x_{max} - x_{min}}{L}$$

$$= \frac{1-0}{8}$$

$$= 0.125$$



iii)

	Discoel signal	Intoection	Reading	Error
0	1	1·0	1·0	0·0
1	0·8535	0·8	0·9	-0·2
2	0·707	0·7	0·7	0·0
3	0·6035	0·6	0·6	0·0
4	0·5	0·5	0·5	0·0
5	0·4265	0·4	0·4	0·0
6	0·353	0·3	0·4	-0·2
7	0·1765	0·1	0·1	-0·1

QUESTION NO 2

Compute the convolution $y(n)$ of the following signal

$$x(n) = \begin{cases} a^{n+1}, & -3 \le n \le 5 \\ 0, & \text{elsewhere} \end{cases}$$

$$h(n) = \begin{cases} 2^n, & 0 \le n \le 4 \\ 0, & \text{elsewhere} \end{cases}$$

Solutions:-

We have

$$x(n) = x(k) = \{\alpha^{-2}, \alpha^{-1}, 1, \alpha, \alpha^2, \alpha^3, \alpha^4, \alpha^5, \alpha^6, 0, 0 \dots\}$$

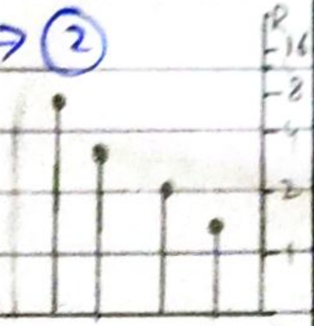
$$h(n) = h(k) = \{\dots 0, 1, 2, 4, 8, 16, 0 \dots\}$$

To find $y(n)$.

$$y(n) = \sum_{k=\infty}^{\infty} x(k)\, h(n-k)$$

for $n=0$ first to find $h(n-k)$
$h(0-k)$ so by inverting
$h(k)$ we get $h(-k)$

$= h(-k) = \{16, 8, 4, 2, 1\} \rightarrow ②$



so

$$y(0) = \sum_{k=-\infty}^{\infty} x(k) \times h(-k)$$

$$y(0) = (\alpha^{-2} \times 8) + (\alpha^{-1} \times 4) + (1 \times 2) + (\alpha \times 1)$$

$$y(0) = 8\alpha^{-2} + 4\alpha^{-1} + \alpha + 2$$
$$= \alpha - 2 + 4\alpha^{-2} + 4\alpha^{-2}$$

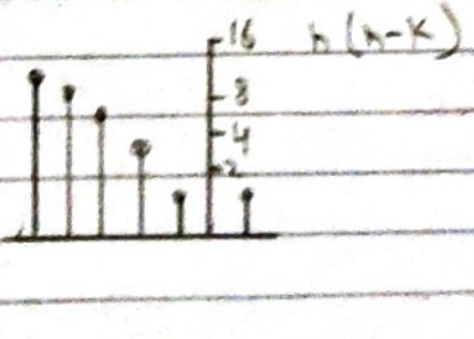
For $n = 1$

$$-h(1-k) = \{16, 8, 4, 2, 1\}$$

so

$$y(1) = (\alpha^{-2} \times 16) + (\alpha^{-2} \times 8) + (1 \times 4) + (\alpha \times 2) + \alpha^2 \times 1)$$

$$y(1) = 16\alpha^{-2} + 8\alpha^{-1} + 4 + 2\alpha + \alpha^2$$
$$= \alpha^2 + 2a + 4 + 8\alpha^{-1} + 16\alpha$$

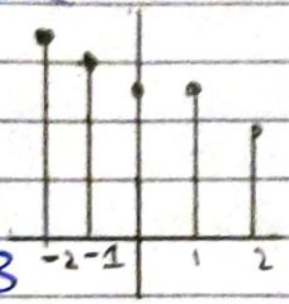


Now for $n = 2$

$$h(2-k) = \{16, 8, 4, 2, 1\}$$

$$y(2) = \{(\alpha^{-1} \times 16) + (1 \times 8) + (\alpha \times 4) +$$
$$(\alpha^2 \times 2) + (\alpha^3 \times 1)$$

$$= 16\alpha^{-2} + 8 + 4\alpha + 2\alpha^2 + \alpha^3$$

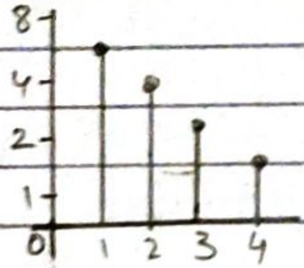


Similarly for $n=3$

$$h = (3-k) = \{16, 8, 4, 2, 1\}$$

$$y(3) = (1\times16) + (\alpha\times8) + (\alpha^2+4) + (\alpha^3\times2) + (\alpha^4\times1)$$

$$= 16 + 8\alpha + 4\alpha^2 + 2\alpha^3 + \alpha^4$$

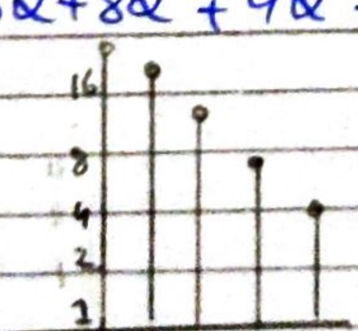


Now

$$h(4-k) = \{16, 8, 4, 2, 1\}$$

$$y(4) = (\alpha^1\times16) + (\alpha^2\times8) + (\alpha^3+4) + (\alpha^4+2) + (\alpha^5+1)$$

$$= 16\alpha + 8\alpha^2 + 4\alpha^3 + 2\alpha^4 + \alpha^5$$

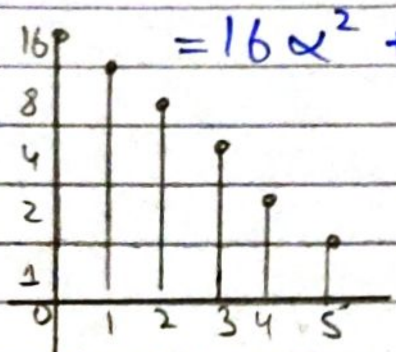


$\Rightarrow h(5-k) = \{0, 16, 8, 4, 2, 1\}$

$y(5) = (\alpha^1 \times 0) + (\alpha^2 \times 16) + (\alpha^3 \times 8) +$

$(\alpha^4 \times 4) + (\alpha^5 \times 2) + (\alpha^6 \times 1)$

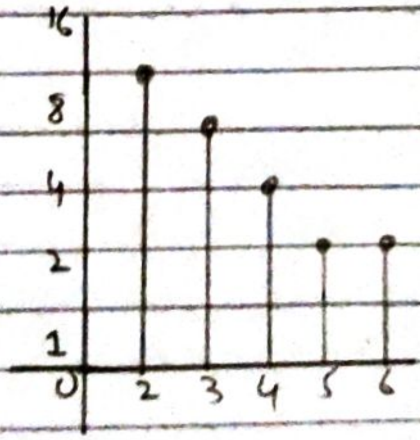
$= 16\alpha^2 + 8\alpha^3 + 4\alpha^4 + 2\alpha^5 + \alpha^6$



similarly if we calculate for rest of the values of n uptill there are any common values we get.

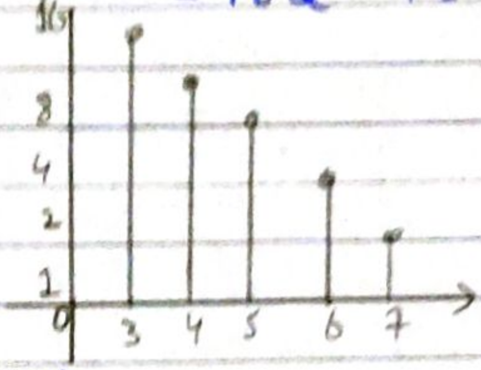
$y(6) = 0 + 0 + 16\alpha^3 + 8\alpha^4 + 4\alpha^5 + 2\alpha^6$

$= 16\alpha^3 + 8\alpha^4 + 4\alpha^5 + 2\alpha^6$



$y(7) = 0+0+0+16\alpha^4+8\alpha^5+4\alpha^6$

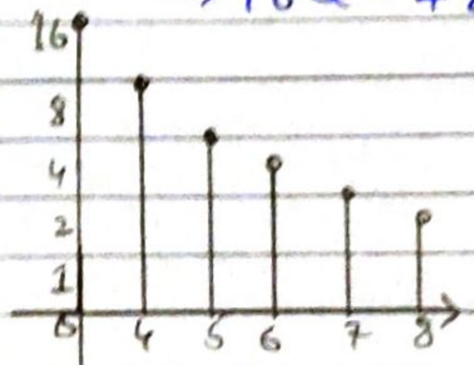
$= 16\alpha^4 + 8\alpha^5 + 4\alpha^6$



$h(7-k)$

$y(8) = 0+0+0+0+16\alpha^5+8+\alpha^6$

$\Rightarrow 16\alpha^5 + 8\alpha^6$

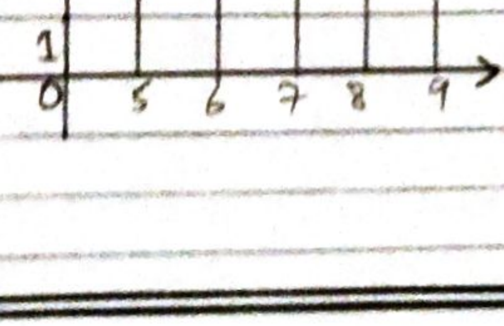


$h(8-k)$

$y(9) = 0+0+0+0+0+16\alpha^6$

$= 16\alpha^6$

$h(9-k)$



Ans

QUESTION No 3

Determine the z-transform of the following signals and also sketch its Regions of convergence (ROC).

1) $x(n) = \begin{cases} (1/4), & n \geq 0 \\ (1/3), & n < 0 \end{cases}$

Solution:-

we know that

z-transform

$$X(z) = \sum_{n=0}^{\infty} \left(\frac{1}{4}\right)^n z + \sum_{n=-\infty}^{0} (1/3)^n z^n - 1$$

using geometric series

$$\Rightarrow \frac{1}{1-\frac{1}{4}z^{-1}} + \sum_{n=0}^{\infty} (1/3)^n z^n - 1$$

$$\Rightarrow \frac{1}{1-1/4\, z^{-1}} + \frac{1}{1-\frac{1}{3}} - 1$$

$$= \frac{1 - \frac{1}{\frac{1}{4} z^{-1}} + 8 - \frac{1}{\frac{1 - 1}{3}} - 1}{\left(1 - \frac{1}{4} z^{-1}\right)\left(1 - \frac{1}{3} z^{-3}\right)^{-1}}$$

$$= \frac{1 - \frac{1}{3} z + 1 - \frac{1}{4} z^{1} - \left(1 - \frac{1}{4} z^{-1}\right)\left(1 - \frac{1}{3} z\right)}{\left(1 - \frac{1}{4} z^{-1}\right)\left(1 - \frac{1}{3} z\right)}$$

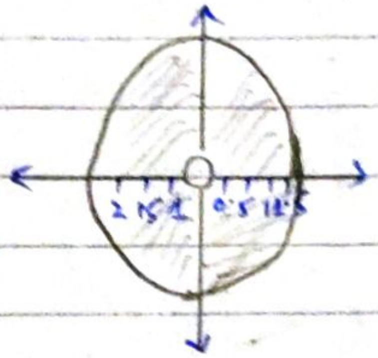
$$= \frac{1 - \frac{1}{3} z + 1 - \frac{1}{4} z^{-1} + \frac{1}{3} + \frac{1}{4} z + \frac{1}{1} z}{\left(1 - \frac{1}{4} z^{1}\right)\left(1 - \frac{1}{3} z\right)}$$

$$= \frac{1 - \frac{1}{12}}{\left(1 - \frac{1}{4} z^{1}\right)\left(1 - \frac{1}{3} z\right)}$$

$$= \frac{\frac{13}{12}}{\left(1 - \frac{1}{4} z^{-1}\right)\left(1 - \frac{1}{3} z\right)}$$

Hence the ROC is $\frac{1}{4} < |Z| < 3$

The sketch is under



ii) $x(n) = \begin{cases} \left(\frac{1}{2}\right)^n - 3^n, & n \geq 0 \\ 0, & \text{elsewhere} \end{cases}$

SOLUTION:-

Using The z-transform pair
i-e $x(n) = \alpha u(n) \rightarrow x(z) = \frac{1}{1-\alpha z^{-1}}$ eq (B)

By putting values

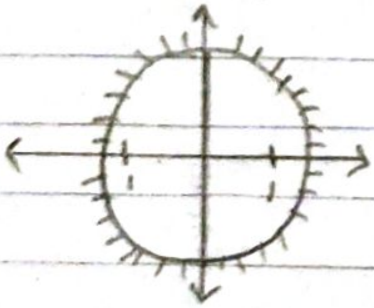
$$X_2(z) = \sum_{n=0}^{\infty} \left(\frac{1}{2}\right) z^{h-n} - \sum_{n=0}^{\infty} 2^n z^{-n}$$

$$= \frac{1}{1-\frac{1}{2}z^{-1}} - \frac{1}{1-2z^{-1}}$$

$$= \frac{-\frac{5}{2}z^{-1}}{\left(1-\frac{1}{2}z^{-1}\right)\left(1-2z^{-1}\right)}$$

As seen The ROC use
$|z| > 2$

The sketch are



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