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Subject: Hydrology

Semester: 6<sup>th</sup>

Q No 1

(1)

SOLUTION:

The pressure drop  $\Delta P$  is expected to depend upon the gate opening  $h$ , the overall depth  $d$ , the velocity  $V$ , density  $\rho$  and viscosity  $\mu$ .

Variable are

$$\Delta P, h, d, V, \rho, \mu$$

Dimensions:

$$\Delta P = MLT^{-2}$$

$$h = L$$

$$d = L$$

$$V = LT^{-1}$$

$$\rho = ML^{-3}$$

$$\mu = ML^{-1}T^{-1}$$

Number of variables,  $n = 6$

Number of independent dimensions,  $m = 3 (M, L, T)$

(2)  
Number of non-dimensional groups,  $n-m = 3$

Choose  $m (= 3)$  Scaling Variables:

geometric ( $l$ ), kinematic/line-dependent ( $v$ ),

dynamic  
mass-dependent ( $\rho$ )

For dimensionless group by non-dimensionalising the remaining variable,  $\Delta p$ ,  $h$  and  $l$ .

$$\Pi_1 = \Delta p l^a v^b \rho^c$$

$$M^0 L^0 T^0 = (M L^{-1} T^{-1}) (L)^a (L T^{-1})^b (M L^{-3})^c$$

$$= M^{1+c} L^{-1+a+b-3c} T^{-2-b}$$

$$M: 0 = 1+c \quad \Rightarrow c = -1$$

$$T: 0 = -2-b \quad \Rightarrow b = -2$$

$$L: 0 = -1+a+b+c \quad \Rightarrow a = 1+3c-b$$

$$\Pi_1 = \Delta p v^{-2} \rho^{-1} = \frac{\Delta p}{\rho v^2}$$

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$$\Pi = \frac{h}{d} \quad \text{for } h \text{ is length}$$

$$\Pi_3 = \mu d^a V^b \rho^c$$

$$M^0 L^0 T^0 = (ML^{-1}T^{-1}) (L)^a (LT^{-1})^b (ML^{-3})^c$$

$$= M^{1+c} L^{-1+a+b-3c} T^{-1-b}$$

$$M: \quad 0 = 1+c \quad \Rightarrow \quad c = -1$$

$$T: \quad 0 = -1-b+0 \quad \Rightarrow \quad b = -1$$

$$L: \quad 0 = -1+a+b-3c \quad \Rightarrow \quad a = 1+3c-b = -1$$

$$\Pi_3 = \mu d^{-1} V^{-1} \rho^{-1} = \frac{\mu}{\rho V d}$$

Recognition of the Reynold number suggest that we replace  $\Pi_3$  by

$$\Pi_3 = (\Pi_3)^{-1} = \frac{\rho V d}{\mu}$$

Hence, dimensional analysis yields

$$\pi_1 = f(\pi_2, \pi_3)$$

i.e

$$\frac{\Delta P}{\rho V^2} = f\left(\frac{h}{d}, \frac{\rho V d}{\mu}\right)$$

a) Dynamic similarities requires that all non-dimensional groups be the same in model and prototype i.e

$$\pi_1 = \left(\frac{\Delta P}{\rho V^2}\right) = \left(\frac{\Delta P}{\rho V^2}\right)_m$$

$$\pi_2 = \frac{h}{d} = \left(\frac{h}{d}\right)_m$$

$$\pi_3 = \left(\frac{\rho V d}{\mu}\right)_p = \left(\frac{\rho V d}{\mu}\right)_m$$

From the last we have velocity ratio

$$\frac{V_p}{V_m} = \frac{(\mu/\rho)_p}{(\mu/\rho)_m} \cdot \frac{d_m}{d_p} = \frac{0.002/800}{1.0 \times 10^{-6}} \times \frac{1}{5} = 0.5$$

Hence

$$V_m = \frac{V_p}{0.5} = \frac{3.0}{0.5} = 6.0 \text{ m/s} \quad (5)$$

b) The ratio of Quantities flow is

$$\frac{Q_p}{Q_m} = \frac{(\text{Velocity} \times \text{area})_p}{(\text{Velocity} \times \text{area})_m} = \frac{V_p}{V_m} \left( \frac{d_p}{d_m} \right)^2 =$$

$$0.5 \times 5^2 = 12.5$$

c) Finally for pressure drop

$$\pi_1 = \left( \frac{\Delta P}{\rho V^2} \right) = \left( \frac{\Delta P}{\rho V^2} \right) = \frac{(\Delta P)_p}{(\Delta P)_m} = \frac{\rho_p}{\rho_m} \left( \frac{V_p}{V_m} \right)^2$$

$$= \frac{800}{1000} \times 0.5^2$$

$$= 0.2$$

$$\Delta P_m = 0.2 \times 60 = 12.0 \text{ KPa}$$

Settle

Step: 01

Here  $\eta$  is a function of  $(\beta, u, \omega, D, Q)$

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Q No 2

GIVEN:

$$\text{Max depth} = 78\text{m}$$

$$\text{Specific Gravity} = 2.4$$

$$G_{av} = 785\text{T/m}^2$$

$$\text{Height of wave} = 1.2\text{m}$$

Solution

①

$$H_{\text{limiting}} = \frac{G_{av}}{\gamma_w (G - Cu + 1)}$$

$$= \frac{785 \times 1000}{1000 (2.4 - 0 + 1)}$$

$$H_{\text{limiting}} = 230.8$$

② Top width: 'a'

$$\begin{aligned} \text{Free board} &= 1.5 \times h_{\text{wave}} \\ &= 1.5 \times 1.2 \\ &= 1.8 \end{aligned}$$

(7/

(2)

Height of Dam:  $H_w + F \cdot B$

$$78 + 1.8$$

$$H_D = 79.8$$

$$a = 14\% \text{ of } H_D$$

$$= 0.14 \times 79.8$$

$$= 11.172 \text{ m}$$

3) Base width:

$$b' = \frac{H_w}{\mu G} = \frac{78}{0.7 + 2.4}$$

$$= 46.42 \text{ m}$$

$$= 47 \text{ m}$$

4) for no tension criteria:

$$b' = \frac{H_w}{\sqrt{G}} = \frac{78}{\sqrt{2.4}}$$

$$= 50.34$$



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4) Depth of vertical portion on  $\frac{1}{3}$  side:

$$\begin{aligned}
 h' &= 2a\sqrt{4-c} \\
 &= 2 \times 11.17 \sqrt{4-0} \\
 &= 34.60 \\
 &= 35\text{m}
 \end{aligned}$$

upstream of b side:  $\frac{a}{16} = \frac{11.17}{16}$

$$= 0.6$$

Depth below the water level to the end of inclined portion  $\frac{1}{3} = 3.14a\sqrt{4}$

$$\begin{aligned}
 &= 3.14 \times 11.17 \sqrt{4} \\
 &= 54.33
 \end{aligned}$$

Total width of the base of the dam

$$\begin{aligned}
 b &= \frac{b'}{16} + \frac{a}{16} = 50.34 + \frac{11.17}{16} \\
 &= 51.03
 \end{aligned}$$

$$\tan \theta = \frac{b'}{H} = \frac{50.34}{78} \quad (9)$$

$$\theta = \tan^{-1}(0.64) \\ = 32.44.80^\circ$$

Depth of vertical portion on D/S (from WL on U/S side)

$$\tan \theta = \frac{a}{d'} = \frac{485}{11.17}$$

$$d' = 17.30 \text{ m}$$

$$\tan \theta = \frac{11.17}{d'}$$

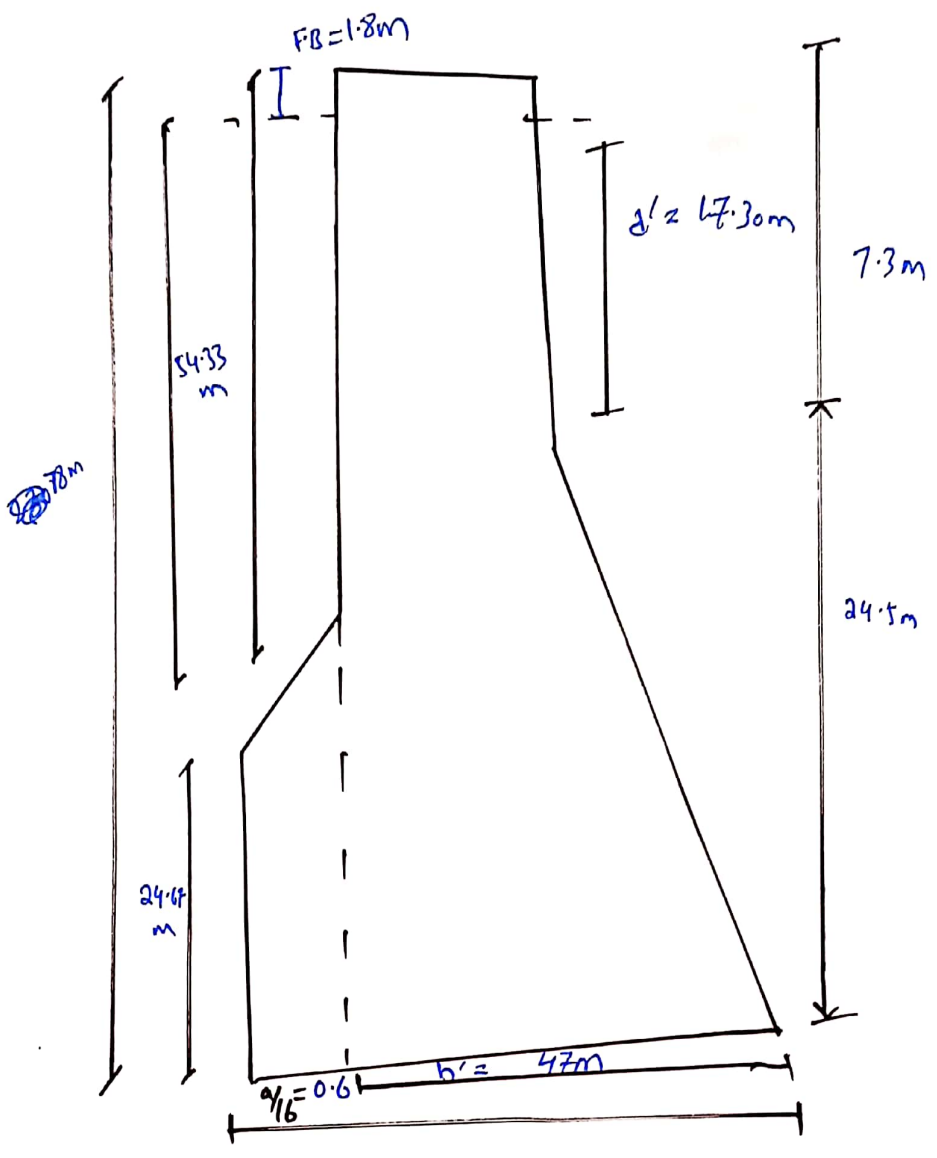
$$\left(\frac{839}{1300}\right) \times d' = 11.17$$

depth of vertical portion

$$d = d' + F.B \\ = 17.30 + 1.8 \\ = 19.1$$

(10)

2/16  
1/20  
1/20  
1/20



Q3:

ANSWER:

### DIMENSIONAL ANALYSIS AND SIMILITUDE:

Analysis using the fact that physical Quantities added to or equated with each other must be expressed in term length or time for inferences to be made about the relation between them

Buckingham  $\pi$  theorem:

The efficiency  $\eta$  of a fluid depends upon density  $\rho$ , dynamic viscosity  $\mu$  of the fluid, angular velocity  $\omega$ , diameter  $D$  of the rotor and the discharge  $Q$ . Express the efficiency  $\eta$  in terms of dimensionless parameters.

Step: 01

Here  $\eta$  is a function of  $\rho, \mu, \omega, D, Q$ .

$$\eta = f(\rho, \mu, \omega, D, Q)$$

depends

(12)

Step: 02

Variable  $\left\{ \begin{array}{l} \text{dependent } (\eta) \\ \text{Independent } (\rho, \mu, \omega, D, Q) \end{array} \right.$

The functional relationship between dependent and independent variable can be written as

$$f_1(\eta, \rho, \mu, \omega, D, Q) = 0$$

Step: 03

Total number of variable  $n = 6$

Step: 04

Dimension of Variable

$$\text{Efficiency } (\eta) = M^0 L^0 T^0$$

$$\text{Density } (\rho) = \frac{m}{V} = \frac{\text{kg}}{\text{m}^3} \quad M^1 L^{-3}$$

$$\mu = \frac{N \cdot s}{m^2} = \frac{\text{kg} \cdot m}{s^2} = M^1 L^{-1} T^{-1}$$

$$\text{Angular velocity } (\omega) = T^{-1}$$

$$\text{Dia } = m = L^1$$

$$\text{Discharge } , Q = L^3 T^{-1}$$

Step: 05

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Number of fundamental dimension for  
Problem

$$m = 3$$

Contain  $m+1$  variable

Step: 06

$$\begin{aligned} \text{Number of } \pi \text{ terms} &= n - m \\ &= 6 - 3 \\ &= 3 \end{aligned}$$

$$\Rightarrow f_1(C_1, n_1, n_2, n_3) = 0 \quad \text{--- (i)}$$

Geometric property ( $L, D, H$ )

flow property ( $V, \mu$ ),  $\omega, \alpha$

fluid property ( $\mu, \delta$ )

Step: 07

$$\pi_1 = D^{a_1} \omega^{b_1} \rho^{c_1}$$

$$\pi_2 = D^{a_2} \omega^{b_2} \rho^{c_2}$$

$$\pi_3 = D^{a_3} \omega^{b_3} \rho^{c_3}$$

Step: 08 (14)

$$\pi_1 = D^a, \omega^b, \rho^c, \eta$$

Substituting Dimension on both sides

$$M^0 L^0 T^0 = L^{a1} (T^{-1})^{b1} (M L^{-3})^{c1} M^0 L^0 T^0$$

$$M^0 L^0 T^0 = M^{c1} L^{a1-3c1} T^{-b1}$$

Power M =  $c_1 = 0$

Power L =  $c_1 - 3c_1 = 0$

$$c_1 = 0$$

Power of T

$$-b_1 = 0$$

$$b_1 = 0$$

Step: 09

Substituting values of  $a, b, c$  in  $\pi_1 = \omega$

$$\pi_1 = D^0 \omega^0 \rho^0 \eta$$

$$\pi_1 = \eta$$

$$\pi_2 = D^{a_2} \omega^{b_2} \rho^{c_2} \mu$$

Substituting Dimension

$$M^0 L^0 T^0 = L^{a_2} (T^{-1})^{b_2} (ML^{-3})^{c_2} M^1 L^{-1} T^{-1}$$

$$M^0 L^0 T^0 = M^{c_2+1} L^{a_2-3c_2-1} T^{-b_2-1}$$

Power of M  $c_2 + 1 = 0$   $c_2 = -1$

Power of L  $a_2 - 3c_2 - 1 = 0$   
 $a_2 = -2$

Power of T  $-b_2 - 1 = 0$   
 $b_2 = -1$

$$\pi_2 = D^{-2} \omega^{-1} \rho^{-1} \mu$$

$$\pi_2 = \frac{\mu}{D^2 \omega \rho}$$

$$\pi_3 = D^{a_3} \omega^{b_3} \rho^{c_3} Q$$



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$$M^0 L^0 T^0 = L^{a_3} (T^{-1})^{b_3} (M L^{-3})^{c_3} L^3 T^{-1}$$

$$= M^{c_3} L^{a_3 - 3c_3 + 3} T^{-b_3 - 1}$$

Power of M  $\boxed{c_3 = 0}$

Power of L  $c_3 - 3c_3 + 3 \Rightarrow c_3 = -3$

Power of T  $-b_3 - 1$   
 $b_3 = -1$

$$\pi_3 = D^{-3} \omega^{-1} \rho^0 Q$$

$$\pi_3 = \frac{Q}{\omega D^3}$$

Substituting the value of  $\pi_1, \pi_2, \pi_3$  in

$$f_1 \left( \eta, \frac{\mu}{D^2 \omega \rho}, \frac{Q}{\omega D^3} \right)$$

$$\eta = \phi \left[ \frac{\mu}{D^2 \omega \rho}, \frac{Q}{\omega D^3} \right]$$

Q 4

(17)

a

ANSWER:


Particle Density:

Density of the particle is directly proportional to the rate of fall of velocity since particle with high density tends to settle down early.

Particle density  $\propto$  fall velocity

Particle Concentration:

Concentration of particle size will considerably effect its fall velocity as the section having greater concentration will be settle down at the place thus causing more fall velocity comparing with section of low concentration.



## Particle Shape: (11)

Particle having regular shape tends to be affected more than irregular shapes. Since regular shape particles have even surface which offers very little or no friction while particles with irregular shape offer more friction, as the particles with smaller surface area are more likely to be affected due to their less resistance.

## VISCOSITY OF WATER:

From the experimental study we can see that parameters such as temperature and pressure changes the magnitude of viscosity of section of water having more temperature and pressure will fall objectively more due to increase in kinetic energy so fall velocity will be more.

Fall velocity  $\propto$  Viscosity of water

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## Turbulence of water:

Turbulence of water depends upon the different factor such as velocity zigzag motion thus the velocity varies at every point which is why it effect ball velocity, moreover increase in K.E tends to effect the ball velocity compared with steady flow.

## Particle Diameter:

Dia of Particle is directly proportional to ball velocity i.e

greater the size it will tend to settle faster so greater will be fall velocity

Particle Dia  $\propto$  fall velocity