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Section B

Subject differential Equation.

Submitted To Shomaila Mazhar.

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Question 1.

$$\frac{dy}{dt} = e^{y-t} \sec(y)(1+t^2)$$

$y(0)=0$ So $x=0$ $y=0$

$$dy = e^{y} \cdot e^{-t} \sec(y)(1+t^2)\,dt.$$

$$\frac{1}{e^{y}\cdot \sec(y)}\,dy = (1+t^2)e^{-t}\,dt$$

as $\cos(y) = \frac{1}{\sec y}$

$$\int e^{-y}\cos y\,dy = \int (1+t^2)e^{-t}\,dt.$$

Using Integration by Parts.

$$e^{-y}\int \cos y\,dx - \int\left(\int \cos y \cdot \frac{d}{dy}e^{-y}\right) =$$

$$(1+t^2)\int e^{-t} - \int\left(\int e^{-t} \cdot \frac{d}{dt}(1+t^2)\right) \longrightarrow eq\,i$$

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L.H.S

$$e^{-y}\int \cos y\,dx - \int\left(\int \cos y \cdot \frac{d}{dy}e^{-y}\right)$$

$$e^{-y}\sin y - \int\left(\sin y \cdot e^{-y}(-1)\right)$$

$$e^{-y}\sin y + \int\left(\sin y \cdot e^{-y}\right)$$

$$e^{-y}\sin y + \int\left(e^{-y}\sin y\right)$$

Again Using Integration by Parts

$$= e^{-y}\sin y + e^{-y}(-\cos y) - \int\left(\int \sin y \frac{d}{dy}e^{-y}\right)$$

$$e^{-y}\sin y + e^{-y}(\cos y) - \int\left(-\cos y\,\frac{e^{-y}}{-1}\right)$$

$$= e^{-y}\sin y - e^{-y}\cos y - \int\left(\cos y\, e^{-y}\right)$$

Since $\int(\cos e^{-y}) = L.H.S.$

Since it is again Same to the First one L.H.S will become

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$L.H.S = e^{-y}(\sin y - \cos y) - L.H.S$

$2L.H.S = e^{-y}(\sin y - \cos y)$

$L.H.S = \frac{e^{-y}(\sin y - \cos y)}{2}$

Now taking R.H.S.

$\int (1+t^2)e^{-t}\,dt$

$\Rightarrow (1+t^2)\int e^{-t} - \int(\int e^{-t}\frac{d}{dt}(1+t^2)$

$\Rightarrow (1+t^2)e^{-t} - \int(-e^{-t}(2t))$

$\Rightarrow -(1+t^2)e^{-t} + \int(2t)e^{t})$

Again Using Integration by Parts.

$\Rightarrow -(1+t^2)e^{-t} + (2t(e^{-t} - \int(e^{t}\frac{d}{dt}2t))$

$\Rightarrow -(1+t^2)e^{-t} + (-2te^{-t} - \int(-e^{-t}2)$

$\Rightarrow -(1+t^2)e^{-t} + (-2te^{-t} + \int(2e^{-t}))$

$\Rightarrow -(1+t^2)e^{-t} + (-2te^{-t} + 2e^{-t}) + C$

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$\Rightarrow -(1+t^2)e^{-t} - 2te^{-t} - 2e^{-t} + c$

$\Rightarrow -e^{-t} - e^{-t}t^2 - 2te^{-t} - 2e^{-t} + c$

$\Rightarrow -(t^2+2t+3)e^{-t} + c = R.H.S.$

Now take L.H.S = R.H.S

$$\frac{e^{y}(\sin y - \cos y)}{2} = -(t^2+2t+3)e^{-t} + c$$

we know that

$x = 0 \quad y = 0$

Put it above

$\Rightarrow \frac{1}{2}(0-1) = -3 + c$

$c = 5/2$

Put the value of c

$$\frac{e^{y}}{2}(\sin y - \cos y) = -(x^2+2t+3)e^{-t} + 5/2$$

Answer

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Q2

$(\sqrt{x+y}+\sqrt{x-y})dx-(\sqrt{x+y}-\sqrt{x-y})dy=0$

$$\frac{dy}{dx}=\frac{\sqrt{x+y}+\sqrt{x-y}}{\sqrt{x+y}-\sqrt{x-y}} \longrightarrow (i)$$

This is homogenous Differential eq in x and y to solve this

Put $y=vx$

$$\frac{dy}{dx}=v+x\frac{dv}{dx}$$

Thus eq (i) becomes

$$v+x\frac{dv}{dx}=\frac{\sqrt{x+vx}+\sqrt{x-vx}}{\sqrt{x+vx}-\sqrt{x-vx}}$$

$$v+x\frac{dv}{dx}=\frac{\sqrt{1+v}+\sqrt{1-v}}{\sqrt{1+v}-\sqrt{1-v}}$$

$$v+x\frac{dv}{dx}=\frac{\sqrt{1+v}+\sqrt{1-v}}{\sqrt{1+v}-\sqrt{1-v}}\times\frac{\sqrt{1+v}+\sqrt{1-v}}{\sqrt{(1+v)}+\sqrt{1-v}}$$

$$v+x\frac{dv}{dx}=\frac{1+\cancel{v}+1-\cancel{v}+2\sqrt{1-v^2}}{2v}$$

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$$v + x\frac{dv}{dx} = \frac{2(1+\sqrt{1+v^2})}{2v}$$

$$v + x\frac{dv}{dx} = \frac{1+\sqrt{1-v^2}}{v}$$

$$x\frac{dv}{dx} = \frac{1+\sqrt{1-v^2}}{v} - v$$

$$x\frac{dv}{dx} = \frac{1+\sqrt{1-v^2}-v^2}{v}$$

$$x\frac{dv}{dx} = \frac{\sqrt{1-v^2}(1+\sqrt{1-v^2})}{v}$$

$$\frac{v\,dv}{\sqrt{1-v^2}(1+\sqrt{1-v^2})} = \frac{dx}{x}$$

Taking Integral on both Sides.

$$\int \frac{v\,dv}{\sqrt{1-v^2}(1+\sqrt{1-v^2})} = \int \frac{dx}{x}$$

Put $1+\sqrt{1-v^2} = t$

$$\Rightarrow \frac{1}{2}(1-v^2)^{-1/2}(-2v)\,dv = dt$$

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$$\frac{v\,dv}{\sqrt{1-v^2}} = -dt$$

$$\int \frac{-dt}{t} = \int \frac{dx}{x}$$

$$-\ln t = \ln x + \ln c$$

$$-\ln(1+\sqrt{1-v^2}) = \ln cx$$

$$\ln(1+\sqrt{1-v^2}) = -\ln cx$$

$$\ln(1+\sqrt{1-v^2}) = \ln(cx)^{-1}$$

$$1+\sqrt{1-v^2} = \frac{1}{cx}$$

$$1+\sqrt{1-\frac{y^2}{x^2}} = \frac{1}{cx}$$

$$1+\sqrt{\frac{x^2-y^2}{x^2}} = \frac{1}{cx}$$

$$x+\sqrt{x^2+y^2} = \frac{1}{c}$$

$$x+\sqrt{x^2-y^2} = c_1 \qquad \frac{1}{c} = c_1$$

~~ac~~ which is a Required Solution

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Question. #3

$(D^4-D^2)y = 3x^2+4\sin x-2\cos x$

Solution.

$(D^4+D^2)y = 3x^2+4\sin x-2\cos x$

$F(D)y = F(x)$

Complementry Solution y_c.

$y = y_c + y_p$

$\Rightarrow D^4-D^2=0 \Rightarrow D^2(D^2+1)=0.$

Either $D^2=0 \Rightarrow \boxed{D=0}$

$D^2+1=0 \quad D^2=-1$

$D=\sqrt{-1} \Rightarrow D=i \quad D=0+i$

Root are real & Complex

$y_c = c_1e^{0.x}+c_2e^{0x}(c_2\cos x+c_3\sin x)$

$y = c_1+c_2\cos x+c_3\sin x$

$y_p = \frac{1}{F(D)}F(x)$

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$$y_p = \frac{1}{D^4 D^2}(3x^2 + 4\sin x - 2\cos x)$$

$$= \frac{3x^2}{D^4+D^2} + \frac{4\sin x}{D^4+D^2} - \frac{2\cos x}{D^4+D^2}$$

$f(D) = D^4 + D^2$

at $D=0$ $\quad 4D^3 + 2D$

Now also For $D=0 \Rightarrow f(D)=0$

again differentiating.

$f''(D) = 12D+2$

So for $D=0$

$f''(0) = 12(0)+2 = 2$

So replacing $\frac{1}{f(0)}$ with $\frac{x^2}{f''(D)}$

$$\Rightarrow y_p = \frac{x^2 3x^2}{12D+2} + \frac{x^2}{12D+2} \cdot 4\sin x - \frac{x^2 2\cos x}{12D+2}$$

Putting $D=0$ in all.

$$y_p = \frac{x^2 3x^2}{12(0)+2} + \frac{x^2 . 4\sin x}{12(0)+2} - \frac{2x^2\cos x}{12(0)+2}$$

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$$y_p = \frac{3x^4}{2} + \frac{4x^2 \sin x}{2} - \frac{2x^2 \cos x}{2}$$

$$= \frac{3x^4}{2} + 2x^2 \sin x - x^2 \cos x$$

So

Putting in equation (i)

$$y = C_1 + C_2 \cos x + C_3 \cos x + \frac{3}{2} x^4 + 2x^2 \sin x$$

$$- x^2 \cos x$$

$$y = C_1 + (C_2 - x^2) \cos x + (C_3 + 2x^2) \sin x + \frac{3}{2} x^4$$