**Department of Electrical Engineering**
**Assignment**
**Date: 25/06/2020**

**Course Details**

**Course Title:** Signals & Systems **Module:** 04
**Instructor:** ______ **Total Marks:** 50

**Student Details**

**Name:** ______ **Student ID:** ______

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|---|---|---|---|
| Q1. | (a) | **Show** with a help of an equation that the differentiation of a function in time domain results in the multiplication by jw in frequency domain. | Marks 06+08 CLO 3 |
| | (b) | If $x[n] = 2\delta[n] - 4\delta[n-2] + 2\delta[n-3]$ <br> $h[n] = 3\delta[n] + \delta[n-1] + 2\delta[n-2]$ <br> **Produce** Y(z) and y[n] | |
| Q2. | | $f(x) = \begin{cases} -\pi/2 & -\pi \le x \le 0 \\ \pi/2 & 0 \le x \le \pi \end{cases}$ <br> **Retrieve** the Fourier series for the given function. | Marks 10 CLO 3 |
| Q3. | | If $X(z) = 2z^2 + 2z/(z^2 + 2z - 3)$ <br> **Retrieve** x[n] using inverse Z-transform method. | Marks 10 CLO 3 |
| Q4. | | **Express** the transfer function using the given data. <br> $A = \begin{bmatrix} -2 & -1 \\ 1 & 0 \end{bmatrix}$ $B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ $C = [1 \quad 2]$ $D = [0]$ | Marks 09 CLO 3 |
| Q5. | | **Apply** Fourier transform on the signal, **x (t) = e** $^{-a\|t\|}$ **u (t)** where u (t) is a unit step function. | Marks 07 CLO 3 |

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Q#1(a)

Solution Let $x(t)$ be a Continous time signal with a fourier transform of $X(j\omega)$

Then

$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(j\omega)\, e^{j\omega t}\, d\omega.$$

Differentiating both sides we get.

$$\frac{d}{dt} x(t) = \frac{d}{dt}\left[\frac{1}{2\pi} \int_{-\infty}^{\infty} X(j\omega) \cdot e^{j\omega t}\, d\omega\right.$$

$$= \frac{1}{2\pi} \int_{-\infty}^{\infty} X(j\omega)\, \frac{d}{dt} e^{j\omega t}\, d\omega$$

$$\frac{d}{dt} x(t) = j\omega \left[\frac{1}{2\pi} \int_{-\infty}^{\infty} X(j\omega)\, e^{j\omega t}\, d\omega\right]$$

So

$$\frac{d}{dt} x(t) = j\omega\, X(j\omega).$$

This equation shows the required relation.

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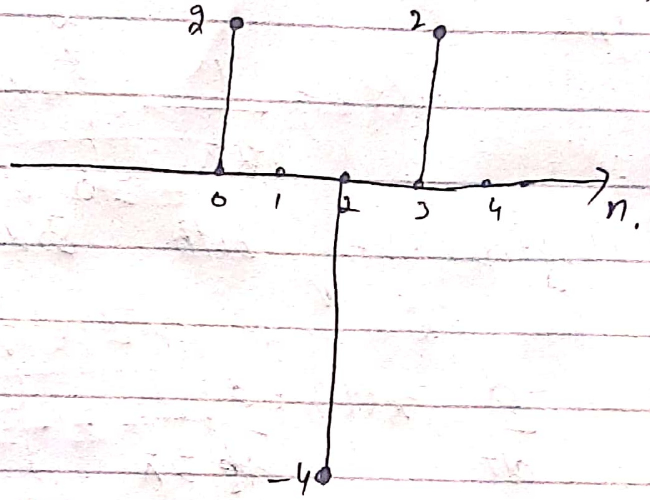
Q#1 (b)

$$x[n] = 2\delta[n] - 4\delta[n-2] + 2\delta[n-3]$$
$$h[n] = 3\delta[n] + \delta[n-1] + 2\delta[n-2].$$

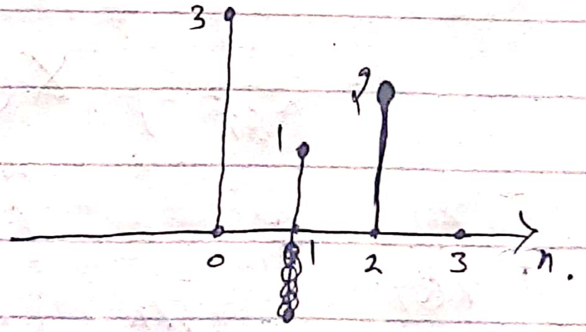
$$y[n] = x[n] * h[n].$$

$$y[n] = \left(2\delta[n] - 4\delta[n-2] + 2\delta[n-3]\right) * \\ 3\delta[n] + \delta[n-1] + 2\delta[n-2].$$

x[n]




h[n]




$$y[n] = 2\delta[n] - 4\delta[n-2] + 2\delta[n-3] + 2\delta[n-1] \\ -4\delta[n-3] + 2\delta[n-4] + 6\delta[n-3] + \\ 2\delta[n-4] + 4\delta[n-5].$$

$$y[n] = 2\delta[n] + 2\delta[n-1] - 4\delta[n-2] + 4\delta[n-3] \\ + 4\delta[n-4] + 4\delta[n-5].$$



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Now $Y(z) = 2 + 2z^{-1} - 4z^{-2} + 4z^{-4} + 4z^{-5}$

Ans

*——————————*

Q#3: if $X(z) = \dfrac{2z^2 + 2z}{z^2 + 2z - 3}$

find $x[n]$

Solution:

$$X(z) = \frac{2z^2 + 2z}{z^2 + 2z - 3}$$

$$X(z) = \frac{2z(z+1)}{z^2 + z - 3z - 3}$$

$$X(z) = \frac{2z(z+1)}{z(z+1) - 3(z+1)}$$

$$X(z) = \frac{2z\cancel{(z+1)}}{(z-3)\cancel{(z+1)}}$$

$$X(z) = \frac{2z}{z-3}$$

$$X(z) = 2\left(\frac{z}{z-3}\right).$$

Now $\quad x[n] = \dfrac{1}{2\pi j} \oint X(z) z^{n-1} dz.$

and $\frac{z}{z-a} \xleftrightarrow[\text{for } |z|>a.]{z^{-1}} a^n u[n]$.

$$\frac{z}{z-3} \xleftrightarrow{z^{-1}} 3^n u[n].$$

Multiplying both sides by 2 we get.

$$2\left(\frac{z}{z-3}\right) \xleftrightarrow{z^{-1}} 2.3^n u[n].$$

So $x[n] = 2 \cdot 3^n u[n]$ Ans.

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Q5 Apply Fourier transform on the Signal

$$x(t) = e^{-a|t|} u(t)$$

Sol:-

$$X(j\omega) = ?$$

The Fourier transform of the given function $x(t)$ is given by :

$$X(j\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt$$

by putting $x(t)$ we get .

$$X(j\omega) = \int_{-\infty}^{\infty} e^{-a|t|} u(t) e^{-j\omega t} dt .$$

as $$u(t) = \begin{cases} 1 & t \geq 0 \\ 0 & t < 0 . \end{cases}$$

So

$$X(j\omega) = \int_{0}^{\infty} e^{-a|t|} e^{-j\omega t} dt .$$

Now

$$e^{-a|t|} = \begin{cases} e^{-at} & \text{for } t \geq 0 \\ a^{-a(-t)} = e^{at} & \text{for } t < 0 . \end{cases}$$

So

$$X(j\omega) = 0 + \int_{0}^{\infty} e^{-at} e^{-j\omega t} dt .$$

$$\Rightarrow X(j\omega) = \int_{0}^{\infty} e^{-(a+j\omega)t} dt .$$

5 P.T.O

$$X(j\omega) = \left.\frac{e^{-(a+j\omega)t}}{-(a+j\omega)}\right|_0^{\infty}$$

$$X(j\omega) = -\frac{1}{a+j\omega}\left[e^{-\infty} - e^{0}\right].$$

$$X(j\omega) = \frac{1}{a+j\omega}$$ Ans.

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Q#4: Express the tf using the given data.

$$A = \begin{bmatrix} -2 & -1 \\ 1 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad C = \begin{bmatrix} 1 & 2 \end{bmatrix}$$

$$D = [0].$$

Solution: As we know

$$G(S) = C(SI-A)^{-1}B + D \longrightarrow (1)$$

$$SI - A = S\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} -2 & -1 \\ 1 & 0 \end{bmatrix}$$

$$SI - A = \begin{bmatrix} S & 0 \\ 0 & S \end{bmatrix} - \begin{bmatrix} -2 & -1 \\ 1 & 0 \end{bmatrix}$$

$$SI - A = \begin{bmatrix} S+2 & 1 \\ -1 & S \end{bmatrix}.$$

$$(SI-A)^{-1} = \text{adj of} \begin{bmatrix} S+2 & 1 \\ -1 & S \end{bmatrix} \div \begin{vmatrix} S+2 & 1 \\ -1 & S \end{vmatrix}$$

$$\text{adj of } \begin{bmatrix} s+2 & 1 \\ -1 & s \end{bmatrix} = \begin{bmatrix} s & -1 \\ 1 & s+2 \end{bmatrix}$$

$\cancel{(sI-A)^{-1}} =$

$$\begin{vmatrix} s+2 & 1 \\ -1 & s \end{vmatrix} = s(s+2) - (1)(-1).$$

$$= s^2 + 2s + 1$$

$$(sI-A)^{-1} = \frac{1}{s^2+2s+1} \begin{bmatrix} s & -1 \\ 1 & s+2 \end{bmatrix}.$$

Now by putting the value in eqn ①

$$G(s) = [1 \quad 2] \cdot \frac{1}{s^2+2s+1} \begin{bmatrix} s & -1 \\ 1 & s+2 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} + 0.$$

$$G(s) = \frac{1}{s^2+2s+1} \begin{bmatrix} s \\ 1 \end{bmatrix} [1 \quad 2]$$

$$G(s) = \frac{1}{s^2+2s+1} \begin{bmatrix} (s+1) - (s+2) \\ 1 - 2 \end{bmatrix}.$$

$$G(s) = \frac{1}{s^2+2s+1} \begin{bmatrix} -1 \\ -1 \end{bmatrix}.$$

$$G(s) = -\frac{1}{s^2+2s+1} [1]$$

Ans

Q#2:

$$f(x) = \begin{cases} -\pi/2 & -\pi \le x < 0 \\ \pi/2 & 0 < x \le x \end{cases}$$

$$F[k] = \frac{1}{T}\int_T f(x)\, e^{-jk\omega_0 x}$$

$$F[k] = \frac{1}{2\pi}\left[\int_{-\pi}^{0} -\frac{\pi}{2}\, e^{-jk\omega_0 x}\, dx + \int_0^{\pi} \frac{\pi}{2}\, e^{-jk\omega_0 x}\, dx\right.$$

$$F[k] = \frac{1}{2\pi}\left[\left(-\frac{\pi}{2}\left(\frac{e^{-jk\omega_0 x}}{-jk\omega_0}\right)_{-\pi}^{0}\right) + \left(\frac{\pi}{2}\left[\frac{e^{-jk\omega_0 x}}{-jk\omega_0}\right]_{0}^{\pi}\right)\right]$$

$$F[k] = \frac{1}{2\pi}\left[\left[-\frac{\pi}{2}\left[\frac{1}{-jk\omega_0} + \frac{e^{jk\omega_0\pi}}{jk\omega_0}\right]\right] + \left[\frac{\pi}{2}\left(\frac{e^{-jk\omega_0\pi}}{-jk\omega_0} + \frac{1}{-jk\omega_0}\right)\right]\right]$$

$$F[k] = \frac{-1}{4jk\omega_0}\left[1 + e^{jk\omega_0\pi} + 1 + e^{-jk\omega_0\pi}\right]$$

$$F[k] = \frac{2 + e^{jk\omega_0\pi} + e^{-jk\omega_0\pi}}{-4jk\omega_0}$$

$$F[k] = \frac{-1}{2jk\omega_0}\sin k\omega_0\pi$$