

Department of Electrical Engineering
Assignment
Date: 14-04-2020

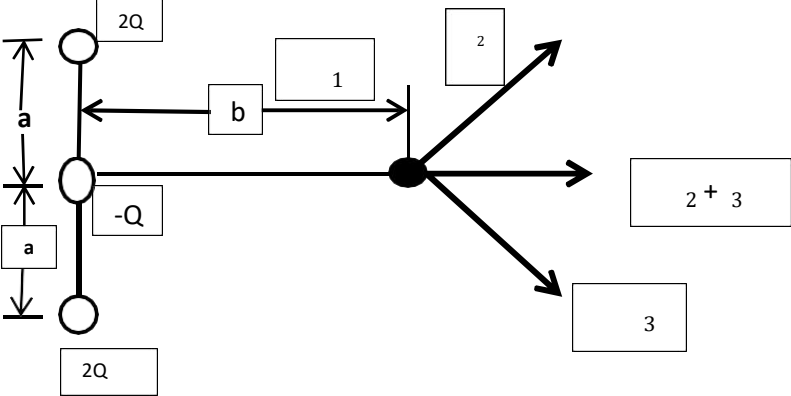
Course Details

Course Title: Electro Magnetic Field Theory Module: 4th semester
 Instructor: Dr.Rafiq Mansoor Total Marks: 30

Student Details

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Q1: Solve the following short Question	(a)	Transform the vector () located at point (-2,6,3) into cylindrical coordinates	Marks 2 CLO 1
	(b)	Convert the point (3,4,5) from Cartesian to spherical coordinates	Marks 2 CLO 1
	(c)	Find the spherical coordinates of A(2,3,-1)	Marks 2 CLO 1
	(d)	Find the Cartesian coordinates of B(4.25,120)	Marks 2 CLO 1
	(e)	Find the force between two charges when they are brought in contact and separated by 4cm apart, charges are 2nC and -1nC, in μN .	Marks 2 CLO 2
	(f)	Find the electric field intensity of two Charges -2C and -1C separated by a distance 1m in air	Marks 2 CLO 2
	(g)	Determine the charge that produce an electric field strength of 40 v/cm at a distance of 30cm in vacuum (in c)	Marks 2 CLO 2
	(h)	A charge of C is acted upon by a force of 0.1N. determine the distance to the other charge of C, both the charges are in vacuum	Marks 2 CLO 2
Q2:	(a)	<p>Find the angle between the vectors shown in figure.</p> <p>The diagram shows a coordinate system where vector \vec{A} is a vertical arrow pointing upwards with a magnitude of 2. Vector \vec{B} is an arrow pointing into the first quadrant. The angle between \vec{A} and \vec{B} is labeled as 3°. The angle between \vec{B} and the positive horizontal axis is labeled as θ. Below the diagram, a box contains the dot product $A \cdot B = 2\sqrt{3}$. To the right of the diagram, there are two empty boxes: the top one contains $\vec{3} +$ and the bottom one is empty.</p>	Marks 4 CLO 1

	(b)	<p>Find the gradient of each of the following functions where a and b are constant</p> <p>(i)</p> <p>(ii)</p>	<p>Marks 4</p> <p>CLO 1</p>
<p>Q3:</p>		<p>Three pointer charges are placed on the y-axis as shown. Find the electric field at point P on the x-axis.</p>  <p>The diagram shows a vertical y-axis with three point charges: a positive charge $2Q$ at the top, a negative charge $-Q$ in the middle, and a positive charge $2Q$ at the bottom. The distance between the top and middle charges is a, and the distance between the middle and bottom charges is a. A point P is located on the x-axis to the right of the y-axis, at a distance b from the y-axis. The electric field at P is represented by a vector pointing away from P. This vector is the sum of three component vectors: 1 (horizontal, pointing left), 2 (diagonal, pointing up and right), and 3 (diagonal, pointing down and right). The horizontal component of the total field is labeled $2 + 3$.</p>	<p>Marks 6</p> <p>CLO 2</p>

Q1(a) :- Transform the vector $B = y i (x+z) j$ located at point $(-2, 6, 3)$ into cylindrical coordinates.

sol

$$B = y i (x+z) j$$

Given points are $(-2, 6, 3)$

then

$$B = y i (x j + z j)$$

$$B = y x i j + y z i j$$

$$\Rightarrow \rho = \sqrt{x^2 + y^2}$$

$$\rho = \sqrt{(-2)^2 + (6)^2}$$

$$\rho = \sqrt{40}$$

$$\rho = 6.32$$

We know that

$$z = z$$

$$\text{So } z = \rho$$

Now

$$\phi = \tan^{-1} (y/x)$$

$$\phi = \tan^{-1} (6/-2)$$

(2)

$$\theta = \tan^{-1}(-3)$$

$$\theta = -72.56$$

So

$$\rho = 6.32, \quad -72.56, \quad 3$$



(b) :- convert the point (3, 4, 5) from cartesian to spherical coordinates.

Sol

$$P(3, 4, 5)$$

$$x=3, \quad y=4, \quad z=5$$

In spherical coordinates system

$$\rho, \theta, \phi$$

first we find ρ

$$\rho = \sqrt{x^2 + y^2 + z^2}$$

$$\rho = \sqrt{(3)^2 + (4)^2 + (5)^2}$$

$$\rho = \sqrt{9 + 16 + 25}$$

3)

$$y = \sqrt{50}$$

$$y = 7.07$$

AS

$$\phi = \tan^{-1} (y/x)$$

$$\phi = \tan^{-1} (4/3)$$

$$\phi = \tan^{-1} (2.33)$$

$$\phi = 53.2^\circ$$

AS

$$\phi = \tan^{-1} \left(\frac{\sqrt{x^2 + y^2}}{z} \right)$$

$$\phi = \tan^{-1} \left(\frac{\sqrt{(3^2) + (4^2)}}{5} \right)$$

$$\phi = \tan^{-1} \left(\frac{\sqrt{9+16}}{5} \right)$$

$$\phi = \tan^{-1} \left(\frac{\sqrt{25}}{5} \right)$$

$$= \tan^{-1} \left(\frac{5}{5} \right)$$

$$= \tan^{-1} (1)$$

$$\phi = 45$$

(4)

(c): Find the spherical coordinates of A (2, 3, -1)

Sol r, θ, ϕ

As

$$r = \sqrt{x^2 + y^2 + z^2}$$

$$r = \sqrt{(2)^2 + (3)^2 + (-1)^2}$$

$$= \sqrt{4 + 9 + 1}$$

$$r = \sqrt{14}$$

$$r = 3.74$$

Now θ

$$\theta = \tan^{-1} (y/x)$$

$$= \tan^{-1} (3/2)$$

$$= \tan^{-1} (1.5)$$

$$\theta = \cancel{56.3} 56.3^\circ$$

5)

$$\begin{aligned}\phi &= \tan^{-1} \left(\frac{\sqrt{x^2 + y^2}}{z} \right) \\ &= \tan^{-1} \left(\frac{\sqrt{(2)^2 + (3)^2}}{-2} \right) \\ &= \tan^{-1} \left(\frac{\sqrt{4+9}}{-2} \right) \\ &= \tan^{-1} \left(\frac{\sqrt{13}}{-2} \right)\end{aligned}$$

$$\phi = \tan^{-1} (-3.60)$$

$$\phi = 74.4$$

$$r = 3.74, \quad \theta = 56.3^\circ, \quad \phi = 74.4$$



(d) :- Find the cartesian coordinates of B (4, 25, 120)

(50) } The point B (4, 25, 120) is given in spherical (r, θ , ϕ) now we have to find (x, y, z).

AS

$$x = r \sin \theta \cdot \cos \phi$$

(6)

$$x = 4 \sin(25) \cdot \cos(120)$$

$$x = 4(0.42) \cdot (-0.5)$$

$$x = -0.84$$

Also

$$y = r \sin \theta \cdot \sin \phi$$

$$y = 4 \sin(25) \cdot \sin(120)$$

$$= 4(0.42) \cdot (0.86)$$

$$y = 2.45$$

As

$$z = r \cos \theta$$

$$z = 4 \cos(25)$$

$$z = 4(0.90)$$

$$z = 3.62$$

(7)

1e):- Find the force between two charges when they are brought in contact and separated by 4cm apart, charges are 2nC and -2nC, in μN .

Given data

$$q_1 = 2\text{ nC}$$

$$q_2 = -2\text{ nC}$$

$$d = 4\text{ cm}$$

Required

$$F = ?$$

sol } where

$$F = k \frac{q_1 q_2}{r^2}$$

$$\text{As } k = \frac{1}{4\pi\epsilon_0}$$

$$F = \frac{2 \times 10^{-9} \times -2 \times 10^{-9}}$$

$$4(3.14) \times 8.85 \times 10^{-12} \times (4 \times 10^{-2})^2$$

$$F = -1.224 \times 10^{-5}$$

(8)

$$F = -11.24 \text{ MN}$$

(f) :- Find the electric field intensity of two charges -2C and -1C separated by a distance 2m in air.

Given data

$$q_1 = -2\text{C}$$

$$q_2 = -1\text{C}$$

$$d = 2\text{m}$$

Required

$$E = ?$$

$$\therefore K = 9 \times 10^9$$

$$E_1 = \frac{K q_1}{d^2}$$

$$= \frac{9 \times 10^9 \times -2\text{C}}{(2)^2}$$

$$E_1 = -18 \times 10^9 \text{ V/m}$$

(9)

Now

$$E_1 = \frac{K q_1}{(d^2)}$$

$$= \frac{9 \times 10^9 \times (-2C)}{(1)^2}$$

$$E_1 = -9 \times 10^9 \text{ V/m}$$

As

$$E_T = E_1 + E_2$$

$$E_T = -18 \times 10^9 + -9 \times 10^9$$

$$E_T = -18 \times 10^9 - 9 \times 10^9$$

$$E_T = -27 \times 10^9 \text{ V/m}$$



(10)

(3) :- Determine the charge that produce an electric field strength of 40 V/cm at a distance of 30 cm in vacuum (in 10^{-8} C)

given data

$$E = 40 \text{ V/cm}$$

$$d = 30 \text{ cm}$$

Required

$$Q = ?$$

$$E = \frac{kQ}{d^2}$$

$$Ed^2 = kQ$$

$$\frac{Ed^2}{k} = Q \quad \longrightarrow (1)$$

Now putting values in eq (1)

$$Q = \frac{Ed^2}{k}$$

(11)

$$Q = \frac{40 \times (30)^2}{9 \times 10^9}$$

$$Q = \frac{40 \times (900)}{9 \times 10^9}$$

$$Q = 4 \times 10^{-6} \text{ C}$$

OR

$$Q = 4 \mu\text{C}$$

* ————— *

1h) :- A charge 2×10^{-7} is acted upon by a force of 0.2 N . Determine the distance to the other charge of $4.5 \times 10^{-7} \text{ C}$, both the charges are in vacuum.

Given data

$$q_1 = 2 \times 10^{-7}$$

$$q_2 = 4.5 \times 10^{-7}$$

$$F = 0.2 \text{ N}$$

$$K = 9 \times 10^9$$

(12)

Required d

$$d = ?$$

sol) Using the formula

$$F = K \frac{q_1 q_2}{d^2}$$

$$d^2 = K \frac{q_1 q_2}{F}$$

Now putting values

$$d^2 = \frac{9 \times 10^9 (2 \times 10^{-7}) (4.5 \times 10^{-7})}{0.2}$$

$$d^2 = 0.2 \times 10^{-3}$$

As $d^2 = 0.0002$

Now taking underroot on b. sides

$$\sqrt{d^2} = \sqrt{0.0002}$$

$$d = 0.01414 \text{ m}$$

$$d = 1.414 \times 10^{-2} \text{ m}$$

(13)

$$d = 0.09 \text{ m}$$

OR

$$d = 9 \text{ cm}$$

Ans



Q2 (a) :- Find the angle between the vector show in the figure.

Sol :- AS

$$A \cdot B = |A||B| \cos \alpha \rightarrow (1)$$

$$A \cdot B = 2\sqrt{3}$$

$$|A| = \sqrt{2^2}$$

$$|B| = \sqrt{2^2}$$

$$|A| = 2$$

$$|B| = 2$$

put value in (1)

So (1) becomes as

$$2\sqrt{3} = 2 \times 2 \cos \alpha$$

$$2\sqrt{3} = 4 \cos \alpha$$

$$\frac{2\sqrt{3}}{4} = \cos \alpha$$

(14)

$$\frac{\sqrt{3}}{2} = \cos \theta$$

$$\theta = \cos^{-1} \left(\frac{\sqrt{3}}{2} \right)$$

$$= \cos^{-1} \left(\frac{2.73}{2} \right)$$

$$= \cos^{-1} (0.866)$$

$$\theta = 30^\circ$$

* _____ *

* _____ *

Q2 :-

part (b) :- Find the gradient of each of the following where a and b are constant

(i) $f = ax^2 + by^3z$

Sol $f = ax^2 + by^3z$

$$\frac{\partial f}{\partial x} = \frac{\partial}{\partial x} (ax^2 + by^3z)$$

(15)

$$\frac{\partial f}{\partial x} = 2ax$$

$$\frac{\partial f}{\partial y} = \frac{\partial}{\partial y} (ax^2 + by^3z)$$

$$\frac{\partial f}{\partial y} = 3by^2z$$

$$\frac{\partial f}{\partial z} = \frac{\partial}{\partial z} (ax^2 + by^3z)$$

$$\frac{\partial f}{\partial z} = by^3$$

$$\nabla f(x, y, z) = (2ax + 3by^2z, by^3)$$

(ii) $f = ar^2 \sin \phi + brz \cos 2\phi$.

Sol

gradient for spherical

$$\nabla f = \frac{\partial f}{\partial r} \hat{r} + \frac{1}{r} \frac{\partial f}{\partial \theta} \hat{\theta} + \frac{1}{r \sin \theta} \frac{\partial f}{\partial \phi} \hat{\phi}$$

So

$$\nabla f = \frac{\partial}{\partial r} (ar^2 \sin \phi + brz \cos 2\phi) \hat{r} + \frac{1}{r} \frac{\partial}{\partial \theta} (ar^2 \sin \phi + brz \cos 2\phi) \hat{\theta} +$$

(16)

$$a \frac{1}{r \sin \theta} \frac{\partial}{\partial \phi} (ar^2 \sin \phi + brz \cos 2\phi) \hat{\phi}$$

Now take partial derivative

$$\rightarrow \nabla f = (2ar \sin \phi + bz \cos 2\phi) \hat{r} + \frac{1}{r} (0) \dots$$

$$+ \frac{1}{r \sin \theta} (ar^2 \cos \phi - 2brz \sin 2\phi) \hat{\phi}$$

So

$$\nabla f = (2ar \sin \phi + bz \cos 2\phi) \hat{r} + \frac{1}{r \sin \theta} \dots$$

$$(ar^2 \cos \phi - 2brz \sin 2\phi) \hat{\phi}$$

Now gradient for cylindrical

$$\nabla f = \frac{\partial f}{\partial \rho} \hat{\rho} + \frac{1}{\rho} \frac{\partial f}{\partial \phi} \hat{\phi} + \frac{\partial f}{\partial z} \hat{z}$$

$$\Rightarrow \nabla f = \frac{\partial}{\partial \rho} (ar^2 \sin \phi + brz \cos 2\phi) \hat{\rho} + \frac{1}{\rho} \frac{\partial}{\partial \phi} \dots$$

$$(ar^2 \sin \phi + brz \cos 2\phi) \hat{\phi} + \frac{\partial}{\partial z} (ar^2 \sin \phi +$$

$$brz \cos 2\phi) \hat{z}$$

Now we take partial derivative

so the first term become zero

$$\Rightarrow \nabla f = \frac{1}{\rho} (ar^2 \cos \phi - 2brz \sin 2\phi) \hat{\phi} + \dots$$

$$(br \cos 2\phi) \hat{z}$$

so

(27)

$$\nabla f = \frac{1}{\rho} (ar^2 \cos \phi - 2brz \sin 2\phi) \hat{\phi} + \dots$$

$$* (br \cos 2\phi) \hat{z}.$$

Q3:- Three positive charges are placed on the y-axis as shown. Find the electric field at point p on the x-axis.

So :- The distance between charges $2Q$ and point "p" is

$$r^2 = b^2 + a^2$$

so

$$r = \sqrt{b^2 + a^2}$$

let assume that charges $2Q$ make angle (α) and $(-\alpha)$ with x-axis.

$$\text{magnitude } \vec{E}_1 = \vec{E}_2 = k \frac{2Q}{r^2}$$

$$= k \frac{(2Q)}{r^2}$$

$$= \frac{K (2Q)}{b^2 + a^2}$$

so resultant of \vec{E}_1 and \vec{E}_2 is

$$\vec{E}_{1+2} = \vec{E}_1 + \vec{E}_2 = E_{1x} + \vec{E}_{2x}$$

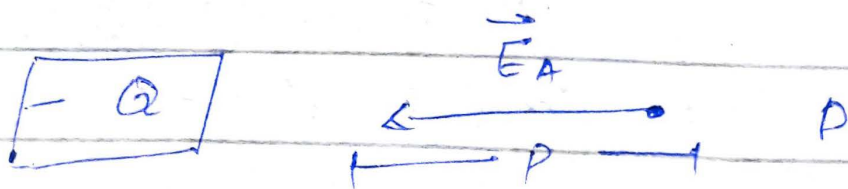
(y-component will be cancel)

$$= \frac{K (2Q) (\cos(\alpha))}{b^2 + a^2}$$

$$E_{1+2} = \frac{4K Q (\cos(\alpha))}{b^2 + a^2} \rightarrow \hat{i}$$

Now electric field at point "p" due to charge "-Q".

→ AS charge is Negative Electric field at point will be direct towards charge "-Q"



$$\vec{E}_A = \frac{-K (Q)}{b^2}$$

Net electric field at point P will be

(19)

$$\vec{E}_{\text{net}} = \vec{E}_A + (\vec{E}_1 + \vec{E}_2)$$

$$= \frac{-K(Q)}{b^2} + \frac{4KQ \cos \alpha}{b^2 + a^2}$$

$$= \frac{-KQ(a^2 + b^2) + 4KQb^2 \cos \alpha}{b^2(a^2 + b^2)}$$

$$= \frac{KQ}{b^2(a^2 + b^2)} [4b^2 \cos \alpha - (a^2 + b^2)]$$

where $k = 9 \times 10^9 \text{ Nm}^2/\text{C}^2$

~~$$\vec{E} = \frac{KQ}{b^2(a^2 + b^2)}$$~~

$$\vec{E}_{\text{net}} = \frac{9 \times 10^9 Q}{b^2(a^2 + b^2)} [4b^2 \cos \alpha - (a^2 + b^2)]$$

now $\alpha = \tan^{-1} \left(\frac{a}{b} \right)$

so

$$\vec{E}_{\text{net}} = \frac{9 \times 10^9 Q}{b^2(a^2 + b^2)} [4b^2 \cos \left[\tan^{-1} \left(\frac{a}{b} \right) \right] - (a^2 + b^2)]$$

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