Department of Electrical Engineering
Assignment
Date: 14-04-2020

Course Details

Course Title: Electro Magnetic Field Theory **Module:** 4^{th} samester

Instructor: Dr.Rafiq Mansoor **Total Marks:** 30 .

Student Details

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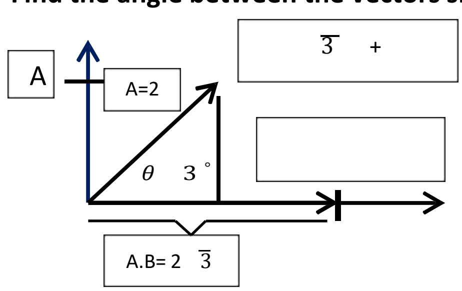
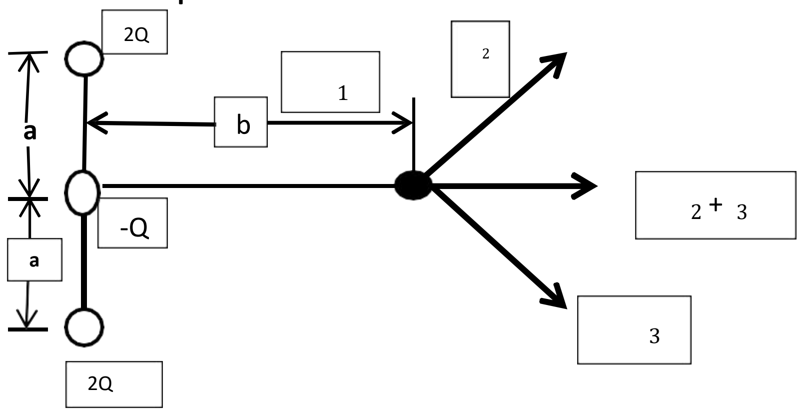
Q1: Solve the following short Question	(a)	Transform the vector ( ) located at point (-2,6,3) into cylindrical coordinates	Marks 2	CLO 1
	(b)	Convert the point (3,4,5) from Cartesian to spherical coordinates	Marks 2	CLO 1
	(c)	Find the spherical coordinates of A(2,3,-1)	Marks 2	CLO 1
	(d)	Find the Cartesian coordinates of B(4.25,120)	Marks 2	CLO 1
	(e)	Find the force between two charges when they are brought in contact and separated by 4cm apart, charges are 2nC and -1nC, in µN.	Marks 2	CLO 2
	(f)	Find the electric field intensity of two Charges -2C and -1C separated by a distance 1m in air	Marks 2	CLO 2
	(g)	Determine the charge that produce an electric field strength of 40 v/cm at a distance of 30cm in vacuum (in c)	Marks 2	CLO 2
	(h)	A charge of C is acted upon by a force of 0.1N. determine the distance to the other charge of C, both the charges are in vacuum	Marks 2	CLO 2
Q2:	**(a)**	**Find the angle between the vectors shown in figure.** 	**Marks 4**	**CLO 1**

	(b)	**Find the gradient of each of the following functions where a and b are constant** **(i)** **(ii)**	**Marks 4** **CLO 1**
Q3:		**Three pointer charges are placed on the y-axis as shown. Find the electric field at point P on the x-axis.**  	**Marks 6** **CLO 2**

Q1(a) :- Transform the vector $B = yi(x+z)j$ located at point $(-2, 6, 3)$ into cylindrical coordinates.

Sol

$$B = yi(x+z)j$$

Given points are $(-2, 6, 3)$

then

$$B = yi(xj + zj)$$

$$B = yxij + yzij$$

$$\Rightarrow \rho = \sqrt{x^2 + y^2}$$

$$\rho = \sqrt{(-2)^2 + (6)^2}$$

$$\rho = \sqrt{40}$$

$$\rho = 6.32$$

We know that

$$z = z$$

so $$z = 3$$

Now

$$\phi = \tan^{-1}(y/x)$$

$$\phi = \tan^{-1}(6/-2)$$

$\theta = \tan^{-1}(-3)$

$\phi = -72.56$

So

$B = 6.32 \quad , \quad -72.56, 3$

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(b) :- convert the point (3, 4, 5) from cartesian to spherical coordinates.

Sol

$P(3, 4, 5)$

$x = 3 \quad , \quad y = 4 \quad , \quad z = 5$

In spherical coordinates system

r, θ, ϕ

first we find r

$r = \sqrt{x^2 + y^2 + z^2}$

$r = \sqrt{(3)^2 + (4)^2 + (5^2)}$

$r = \sqrt{9 + 16 + 25}$

$r = \sqrt{50}$

$r = 7.07$

As

$\theta = \tan^{-1}(y/x)$

$\theta = \tan^{-1}(4/3)$

$\theta = \tan^{-1}(1.33)$

$\theta = 53.1^\circ$

As

$\phi = \tan^{-1}\left(\frac{\sqrt{x^2+y^2}}{z}\right)$

$\phi = \tan^{-1}\left(\frac{\sqrt{(3^2)+(4^2)}}{5}\right)$

$\phi = \tan^{-1}\left(\frac{\sqrt{9+16}}{5}\right)$

$\phi = \tan^{-1}\left(\frac{\sqrt{25}}{5}\right)$

$= \tan^{-1}\left(\frac{5}{5}\right)$

$= \tan^{-1}(1)$

$\phi = 45$

(c): Find the spherical coordinates of A (2, 3, -1)

Sol

r, θ, ϕ

As

$r = \sqrt{x^2 + y^2 + z^2}$

$r = \sqrt{(2^2) + (3^2) + (-1^2)}$

$= \sqrt{4 + 9 + 1}$

$r = \sqrt{14}$

$\boxed{r = 3.74}$

Now θ

$\theta = \tan^{-1}(y/x)$

$= \tan^{-1}(3/2)$

$= \tan^{-1}(1/5)$

$\boxed{\theta = 56.3^\circ}$

As $\phi = \tan^{-1}\left(\frac{\sqrt{x^2+y^2}}{z}\right)$

$= \tan^{-1}\left(\frac{\sqrt{(2)^2+(3)^2}}{-1}\right)$

$= \tan^{-1}\left(\frac{\sqrt{4+9}}{-1}\right)$

$= \tan^{-1}\left(\frac{\sqrt{13}}{-1}\right)$

$\phi = \tan^{-1}(-3.60)$

$\boxed{\phi = 74.4}$

$r = 3.74$, $\theta = 56.3^\circ$, $\phi = 74.4$

* ————— * * ————— *

(d) :- Find the cartesion coordinates of B (4, 25, 120)

Sol) The point B (4, 25, 120) is given in spherical (r, θ, ϕ)

Now we have to find (x, y, z).

As

$x = r \sin\theta \cdot \cos\phi$

$x = 4 \sin(25) \cdot \cos(120)$

$x = 4(0.42) \cdot (-0.5)$

$x = -0.84$

Also

$y = r \sin\theta \cdot \sin\phi$

$y = 4 \sin(25) \cdot \sin(120)$

$= 4(0.42) \cdot (0.86)$

$y = 2.45$

As

$z = r \cos\theta$

$z = 4 \cos(25)$

$z = 4(0.90)$

$z = 3.62$

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(e):- Find the force between two charges when they are brought in contact and separated by 4cm apart, charges are 2nc and -2nc, in μN.

Given data

$q_1 = 2\,nc$

$q_2 = -2\,nc$

$d = 4\,cm$

Required

$F = ?$

sol where

$$F = K \frac{q_1 q_2}{r^2}$$

As $K = \frac{1}{4\pi\epsilon_0}$

$$F = \frac{2\times10^{-9} \times -2\times10^{-9}}{4(3.14)\times 8.85\times10^{-12} \times (4\times10^{-2})^2}$$

$F = -2.224 \times 10^{-5}$

$F = -11.24\ \mu N$

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(f) :- Find the electric field intensity of two charges -2c and -1c separated by a distance 1m in air.

Given data

$q_1 = -2\,c$

$q_2 = -1\,c$

$d = 1m$

Required

$E = ?$

$\because K = 9 \times 10^9$

$$E_1 = \frac{K q_1}{d^2}$$

$$= \frac{9 \times 10^9 \times -2\,c}{(1)^2}$$

$E_1 = -18 \times 10^9$ V/m

Now

$$E_2 = \frac{K q_2}{(d^2)}$$

$$= \frac{9 \times 10^9 \times (-1C)}{(1)^2}$$

$$E_2 = -9 \times 10^9 \ V/m$$

As

$$E_T = E_1 + E_2$$

$$E_T = -18 \times 10^9 + -9 \times 10^9$$

$$E_T = -18 \times 10^9 - 9 \times 10^9$$

$$E_T = -27 \times 10^9 \ V/m$$

* ———— * x ———— x

(3) :- Determine the charge that produce an electric field strength of 40 v/cm at a distance of 30 cm in vaccum (in 10^{-8} c)

given data

$E = 40 \ v/cm$

$d = 30 \ cm$

Required

$Q = ?$

$$E = \frac{KQ}{d^2}$$

$$Ed^2 = KQ$$

$$\frac{Ed^2}{K} = Q \longrightarrow (1)$$

Now puttin values in eq (1)

$$Q = \frac{Ed^2}{K}$$

$$Q = \frac{40 \times (30)^2}{9 \times 10^9}$$

$$Q = \frac{40 \times (900)}{9 \times 10^9}$$

$$Q = 4 \times 10^{-6}\ C$$

OR

$$\boxed{Q = 4\ \mu C}$$

* ———— * ———— *

11) :- A charge 2×10^{-7} is acted upon by a force of 0.2 N. Determine the distance to the other charge of 4.5×10^{-7} C, both the charges are in vaccum.

Given data

$q_2 = 2 \times 10^{-7}$

$q_2 = 4.5 \times 10^{-7}$

$F = 0.2\ N$

$K = 9 \times 10^9$

Required d

$d = ?$

Sol{ Using the formula

$$F = K \frac{q_1 q_2}{d^2}$$

$$d^2 = K \frac{q_1 q_2}{F}$$

Now putting values

$$d^2 = \frac{9 \times 10^9 (2 \times 10^{-7})(4.5 \times 10^{-7})}{0.1}$$

$$d^2 = 8.1 \times 10^{-3}$$

As $d^2 = 0.0081$

Now Taking underroot on b·sides

$$\sqrt{d^2} = \sqrt{0.0081}$$

$$d = 0.09 m$$

$$d = 9 \times 10^{-2} m$$

$d = 0.09\,m$

OR

$d = 9\,cm$ Ans

Q2 (a) :- Find the angle between the vector show in the figure.

Sol :- As

$A \cdot B = |A||B| \cos\theta \rightarrow (i)$

$A \cdot B = 2\sqrt{3}$

$|A| = \sqrt{2^2}$ $|B| = \sqrt{2^2}$

$|A| = 2$ $|B| = 2$

put value in (i)

So (i) becomes as

$2\sqrt{3} = 2 \times 2 \cos\theta$

$2\sqrt{3} = 4 \cos\theta$

$\frac{\sqrt{3}}{2} = \cos\theta$

$\frac{\sqrt{3}}{2} = \cos\theta$

$\theta = \cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$

$= \cos^{-1}\left(\frac{1.73}{2}\right)$

$= \cos^{-1}(0.866)$

$\theta = 30°$

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Q2 :-

part (b) :- Find the gradient of each of the following where a and b are constant

(i) $f = ax^2 + by^3 z$

Sol:

$f = ax^2 + by^3 z$

$\frac{\partial f}{\partial x} = \frac{\partial}{\partial x}(ax^2 + by^3 z)$

$$\frac{\partial f}{\partial x} = 2ax$$

$$\frac{\partial f}{\partial y} = \frac{\partial}{\partial y}(ax^2 + by^3z)$$

$$\frac{\partial f}{\partial y} = 3by^2z$$

$$\frac{\partial f}{\partial z} = \frac{\partial}{\partial z}(ax^2 + by^3z)$$

$$\frac{\partial f}{\partial z} = by^3$$

$$\nabla f(x, y, z) = (2ax + 3by^2z,\ by^3)$$

* ———————— x   * ———————— *

(ii) $f = ar^2 \sin\phi + brz \cos 2\phi.$

Sol: gradient for spherical

$$\nabla f = \frac{\partial f}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial f}{\partial \theta}\hat{\theta} + \frac{1}{r\sin\theta}\frac{\partial f}{\partial \phi}\hat{\phi}$$

So

$$\nabla f = \frac{\partial}{\partial r}(ar^2 \sin\phi + brz\cos 2\phi)\hat{r} +$$
$$\frac{1}{r}\frac{\partial}{\partial \theta}(ar^2 \sin\phi + brz\cos 2\phi)\hat{\theta} +$$

$\frac{1}{r\sin\theta}\frac{\partial}{\partial\phi}(ar^2\sin\phi + brz\cos 2\phi)\hat{\phi}$.

Now Take partial derivative

$\Rightarrow \nabla f = (2ar\sin\phi + bz\cos 2\phi)\hat{r} + \frac{1}{r}(0)\ldots$
$+ \frac{1}{r\sin\theta}(ar^2\cos\phi - 2brz\sin 2\phi)\hat{\phi}$.

So

$\nabla f = (2ar\sin\phi + bz\cos 2\phi)\hat{r} + \frac{1}{r\sin\theta}\ldots$
$(ar^2\cos\phi - 2brz\sin 2\phi)\hat{\phi}$

Now gradient for cylindrical

$\nabla f = \frac{\partial f}{\partial\rho}\hat{\rho} + \frac{1}{\rho}\frac{\partial f}{\partial\phi}\hat{\phi} + \frac{\partial f}{\partial z}\hat{z}$.

$\Rightarrow \nabla f = \frac{\partial}{\partial\rho}(ar^2\sin\phi + brz\cos 2\phi)\hat{\rho} + \frac{1}{\rho}\frac{\partial}{\partial\phi}\ldots$
$(ar^2\sin\phi + brz\cos 2\phi)\hat{\phi} + \frac{\partial}{\partial z}(ar^2\sin\phi + \ldots$
$brz\cos 2\phi)\hat{z}$.

Now we Take partial derivative so the first term become zero

$\Rightarrow \nabla f = \frac{1}{\rho}(ar^2\cos\phi - 2brz\sin 2\phi)\hat{\phi} + \ldots$
$(br\cos 2\phi)\hat{z}$.

So

$$\nabla f = \frac{1}{\rho}\left(ar^2\cos\phi - 2brz\sin 2\phi\right)\hat{\phi} + \cdots$$

$$+ \left(br\cos 2\phi\right)\hat{z}$$

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Q3:- Three pointer charges are placed on the y-axis as shown. Find the electric field at point p on the x-axis.

Sol:- The distance between charges 2Q and point "p" is

$$r^2 = b^2 + a^2$$

so

$$r = \sqrt{b^2 + a^2}$$

let assume that charges 2Q make angle (α) and $(-\alpha)$ with x-axis.

magnitude of $|\vec{E}_1| = |\vec{E}_2| = k\frac{Q}{r^2}$

$$= k\frac{(2Q)}{r^2}$$

$$= \frac{K(2Q)}{b^2+a^2}$$

so resultant of $\vec{E_1}$ and $\vec{E_2}$ is

$$\vec{E}_{1+2} = \vec{E_1} + \vec{E_2} = \vec{E}_{1x} + \vec{E}_{2x}$$

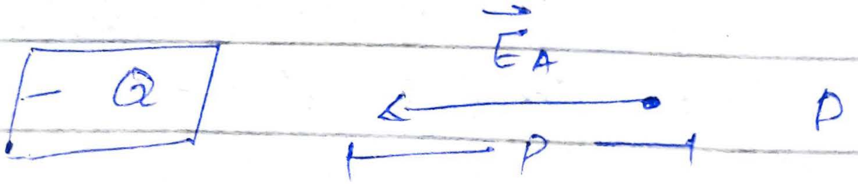
(y-component will be cancel)

$$= \frac{K(2Q)}{b^2+a^2} \cos(\alpha))$$

$$E_{1+2} = \frac{4KQ\cos(\alpha)}{b^2+a^2} \longrightarrow (i)$$

Now electric field at point "P" due to charge "-Q".

→ As charge is Negative Electric field at point will be direct towards charge "-Q"



$$\vec{E}_A = \frac{-K(Q)}{b^2}$$

Net electric field at point P will be

$$\vec{E}_{net} = \vec{E}_A + (\vec{E}_1 + \vec{E}_2)$$

$$= \frac{-K(Q)}{b^2} + \frac{4KQ\cos\alpha}{b^2 + a^2}$$

$$= \frac{-KQ(a^2+b^2) + 4KQb^2\cos\alpha}{b^2(a^2+b^2)}$$

$$= \frac{KQ}{b^2(a^2+b^2)}\left[4b^2\cos\alpha - (a^2+b^2)\right]$$

where $k = 9 \times 10^9 \, Nm^2/c^2$

$$\vec{E}_{Net} = \frac{9\times10^9 Q}{b^2(a^2+b^2)}\left[4b^2\cos\alpha - (a^2+b^2)\right]$$

Now $\alpha = \tan^{-1}\left(\frac{a}{b}\right)$

so $$\vec{E}_{Net} = \frac{9\times10^9 Q}{b^2(a^2+b^2)}\left[4b^2\cos\left[\tan^{-1}\left(\frac{a}{b}\right)\right] - (a^2+b^2)\right]$$

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