Name Attaullah khan

ID 7933

Section B

Subject Diff equation

Exam Mid term

Department civil

Semister 4th

Scanned with CamScanner

Question No (1)

Solve the following objective ....

Part (i)

The order of matrix A is $m \times P$ and the order of B is $P \times n$ Then the order of matrix AB is ?

Sol: The order of matrix is equall to the no of its row multiply by no of column

$A = m \times P$ has "m" no of row and P no of column

Scanned with CamScanner

So

$B = P \times n$

Then its "P" no of row

and "n" is no of column

Also the number of column

in A is equal to the number

of rows in B so there

matrix are conformable for

multiplication - and there order

will be

$\boxed{AB = m \times n}$

Scanned with CamScanner

3

Part (II)

The number of non-zero rows in an echolon form?

Sol:

: one

Part (III)

If $B = \begin{bmatrix} 1 & 4 \\ 2 & a \end{bmatrix}$ is a singular matrix then $a = ?$

Sol:

For singular matrix $|B| = 0$

$$|B| = (1 \times a) - (4 \times 2) = 0$$

$$|B| = a - 8 = 0$$

So

The value of $\boxed{a = 8}$

Scanned with CamScanner

4 Part (iv)

If $A = \begin{bmatrix} 2i & i \\ i & -i \end{bmatrix}$ then $|A| = ?$

Sol:

$$A = \begin{bmatrix} 2i & i \\ i & -i \end{bmatrix}$$

$$|A| = \begin{vmatrix} 2i & i \\ i & -i \end{vmatrix}$$

$$|A| = (2i \times (-i)) - (i \times i)$$

$$|A| = -2i^2 - i^2$$

$$A = -2(-1) - (-1) \qquad \boxed{i^2 = -1}$$

$$|A| = 2 + 1$$

$$|A| = 3$$

Scanned with CamScanner

5 Part V

The matrix $A = \begin{bmatrix} 9 & 0 \\ 0 & 9 \end{bmatrix}$ is = ?

If each element of a principal diagonal of a matrix is same non zero scalar and all other element are zero this its scalar matrix
so it is

$A = \begin{bmatrix} 9 & 0 \\ 0 & 9 \end{bmatrix}$ is a scalar matrix

Scanned with CamScanner

(6) Part VI

Solution of $\frac{dy}{dx} + 2xy = y$?

Sol:-

$$\frac{dy}{dx} + 2xy = y$$

$$\frac{dy}{dx} = y - 2xy$$

multiplyin on b. sid by dx

$$dy = (y - 2xy)\, dx.$$

$$dy = y(1 - 2x)\, dx$$

$$\frac{1}{y} = (1 - 2x)\, dx$$

Taking integral on b. side.

$$\int \frac{1}{y}\, dy = \int (1-2x)\, dx$$

$$\ln y = x - \frac{\cancel{2}x^2}{\cancel{2}}$$

$$\ln y = x - x^2 + c$$

Scanned with CamScanner

(7) Part (VII)

The order and degree of diff equation ?

$$\left(\frac{dy}{dx}\right)^3 = \sqrt{1+\left(\frac{dy}{dx}\right)^2} \text{ is ?}$$

Sol:

order = 1

degree = 3

Part VIII

The order and degree of diff equation

$$\frac{d^2y}{dx^2} - 4xy = \sin\left(\frac{d^2y}{dx^2}\right) \text{ is ?}$$

Sol

Order = 2

Degree = 1

Scanned with CamScanner

(8)

Part IX :

The diff equation $2\frac{dy}{dx}+x^2y=2x+3$

$y(0)=5$ is ?

Sol:

$$2\frac{dy}{dx}+x^2y=2x+3$$

Xing "dx" on b. side.

and also apply integral

$$\int 2dy = \int (2x+3-x^2y)\,dx$$

$$2y = \frac{\cancel{2}x^2}{\cancel{2}}+3x-y\frac{x^3}{3}+c$$

$$y=\frac{x^2}{2}+\frac{3x}{2}-\frac{x^3y}{6}+c \quad \text{—} ①$$

Put $x=0$, $y=5$

$$5=0+0-0+c$$

$\boxed{c=5}$ ⟹ Put in eq ①

Then

$$y=\frac{x^2}{2}+\frac{3x}{2}-\frac{x^3y}{6}+5$$

Scanned with CamScanner

9) Part X:

$$\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} \text{ is ?}$$

Sol:

Expand by R_1

$$|A| = +1\begin{vmatrix} b & b^2 \\ c & c^2 \end{vmatrix} - a\begin{vmatrix} 1 & b^2 \\ 1 & c^2 \end{vmatrix} + a^2\begin{vmatrix} 1 & b \\ 1 & c \end{vmatrix}$$

$$|A| = 1(bc^2 - b^2c) - a(c^2 - b^2) + a^2(c - b)$$

$$|A| = bc^2 - b^2c - ac^2 + ab^2 + a^2c - a^2b$$

Scanned with CamScanner

(10)

Question No 2

Part (i)

Express the determinant

$$\begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \end{vmatrix}$$

as the productor of factor which are linear in a, b, c

Sol:

$$\begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \end{vmatrix}$$

Expand by R_1

Scanned with CamScanner

11

Expand by R_1

$$a\begin{vmatrix} b^2 & c^2 \\ b^3 & c^3 \end{vmatrix} - b\begin{vmatrix} a^2 & c^2 \\ a^3 & c^3 \end{vmatrix} + c\begin{vmatrix} a^2 & b^2 \\ a^3 & b^3 \end{vmatrix}$$

$$\Rightarrow a(b^2c^3 - b^3c^2) - b(a^2c^3 - c^2a^3) + c(a^2b^3 - b^2a^3)$$

$$= ab^2c^3 - ab^3c^2 - ba^2c^3 + bc^2a^3 + a^2b^3c - a^3b^2c$$

Taking common (abc)

$$\Rightarrow abc(bc^2 - b^2c - ac^2 + a^2c + ab^2 - a^2b)$$

$$\Rightarrow abc[bc(c-b) - ac(c+a) + ab(b-a)]$$

Scanned with CamScanner

12 Part (ii) Question 2nd

Find the Eigen value

$$\begin{bmatrix} 2 & -1 & -1 & 0 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ 0 & -1 & -1 & 2 \end{bmatrix}$$

Sol:

characteristic eq $|A - \lambda I| = 0 \rightarrow (1)$

$$\begin{bmatrix} 2 & -1 & -1 & 0 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ 0 & -1 & -1 & 2 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Now take determinant

$$|A - \lambda I| = 0$$

Scanned with CamScanner

13

$$\begin{bmatrix} 2-\lambda & -1 & -1 & 0 \\ -1 & 3-\lambda & -1 & -1 \\ -1 & -1 & 3-\lambda & -1 \\ 0 & -1 & -1 & 2-\lambda \end{bmatrix}$$

Expand by R_1

$$\Rightarrow 2-\lambda \begin{vmatrix} 3-\lambda & -1 & -1 \\ -1 & 3-\lambda & -1 \\ -1 & -1 & 2-\lambda \end{vmatrix} - (-1) \begin{vmatrix} -1 & -1 & -1 \\ -1 & 3-\lambda & -1 \\ 0 & -1 & 2-\lambda \end{vmatrix}$$

$$-1 \begin{vmatrix} -1 & 3-\lambda & -1 \\ -1 & -1 & -1 \\ 0 & -1 & 2-\lambda \end{vmatrix} = 0 \longrightarrow 11$$

<u>Again</u>

$$\begin{vmatrix} 3-\lambda & -1 & -1 \\ -1 & 3-\lambda & -1 \\ -1 & -1 & 2-\lambda \end{vmatrix}$$

Expand by R_1

Scanned with CamScanner

14

$$\Rightarrow 3-\lambda \begin{vmatrix} 3-\lambda & -1 \\ -1 & 2-\lambda \end{vmatrix} - (-1) \begin{vmatrix} -1 & -1 \\ -1 & 2-\lambda \end{vmatrix}$$

$$-1 \begin{vmatrix} -1 & 3-\lambda \\ -1 & -1 \end{vmatrix}$$

$$\Rightarrow (3-\lambda)\Big[\big((3-\lambda)(2-\lambda) - (-1)(-1)\big)$$

$$+1\big((-1)(2-\lambda) - (-1)(-1)\big)$$

$$-1\big((-1)(-1) - (-1)(3-\lambda)\big)\Big]$$

$$\Rightarrow (3-\lambda)(6-3\lambda-2\lambda+\lambda^2-1)+(-2+\lambda-1)$$

$$-(1+3-\lambda)$$

$$\Rightarrow (3-\lambda)(6-5\lambda-\lambda^2-1)+(-3+\lambda)$$

$$-(4-\lambda)$$

$$\Rightarrow (3-\lambda)(\lambda^2-5\lambda+5)+(-3+\lambda)-(4-\lambda)$$

15

$3\lambda^2 - 15\lambda + 15 - \lambda^3 + 5\lambda^2 - 5\lambda - 3$
$+ \lambda - 4 + \lambda$

$\Rightarrow \boxed{-\lambda^3 + 8\lambda^2 - 18\lambda + 8} \rightarrow$ (III)

$$\Rightarrow +1 \begin{vmatrix} -1 & -1 & -1 \\ -1 & 3-\lambda & -1 \\ 0 & -1 & 2-\lambda \end{vmatrix}$$

Expand by column 1

$$\Rightarrow -1 \begin{vmatrix} 3-\lambda & -1 \\ -1 & 2-\lambda \end{vmatrix} - (-1) \begin{vmatrix} -1 & -1 \\ -1 & 2-\lambda \end{vmatrix} + 0$$

$\Rightarrow -1(6 - 3\lambda - 2\lambda + \lambda^2 - 1) + 1(-2 + \lambda - 1)$

$\Rightarrow -\lambda^2 + 5\lambda - 5 - 3 + \lambda$

$\Rightarrow \boxed{-\lambda^2 + 6\lambda - 8} \Rightarrow$ (iv)

Scanned with CamScanner

16

$$\Rightarrow -1\begin{vmatrix} -1 & 3-\lambda & -1 \\ -1 & -1 & -1 \\ 0 & -1 & 2-\lambda \end{vmatrix}$$

Expand by C_1

$$\Rightarrow -\left[-1\begin{vmatrix} -1 & -1 \\ -1 & 2-\lambda \end{vmatrix} - (-1)\begin{vmatrix} 3-\lambda & -1 \\ -1 & 2-\lambda \end{vmatrix} + 0\right]$$

$$\Rightarrow -\left[-(-2+\lambda-1) + 1(6-3\lambda-2\lambda+\lambda^2-1)\right.$$

$$\Rightarrow -(3-\lambda+\lambda^2-5\lambda+5)$$

$$\Rightarrow -\lambda^2+5\lambda-5-3+\lambda$$

$$\Rightarrow \boxed{-\lambda^2+6\lambda-8} \longrightarrow V$$

Put (iii), (iv) and (v) in eq 2

$$(2-\lambda)\left[-\lambda^3+8\lambda^2-18\lambda+8\right]-\lambda^2$$
$$+6\lambda-8-\lambda^2+6\lambda-8$$

Scanned with CamScanner

17

$\Rightarrow -2\lambda^3 + 16\lambda^2 - 36\lambda + 16 + \lambda^4 - 8\lambda^3$
$+18\lambda^2 - 8\lambda - \lambda^2 + 6\lambda - 8 - \lambda^2 + 16\lambda - 8$

$\cancel{\Rightarrow \lambda^4 - 10\lambda^3}$

$\Rightarrow \lambda^4 - 2\lambda^3 - 8\lambda^3 + 16\lambda^2 + 18\lambda^2 - \lambda^2$
$-\lambda^2 - 36\lambda - 8\lambda + 6\lambda + 6\lambda + 16 - 16$

$\Rightarrow \lambda^4 - 10\lambda^3 + 32\lambda^2 - 32\lambda = 0$

By Synthetic division

we get $\lambda(\lambda - 2)(\lambda^2 - 8\lambda + 16) = 0$

$\lambda = 0$

$\lambda - 2 = 0 \Rightarrow \lambda = 2$

$\lambda^2 - 8\lambda + 16 = 0$

By factorization method

Scanned with CamScanner

18

By factorisation method

$$\lambda^2 - 4\lambda - 4\lambda + 16 = 0$$

$$\lambda(\lambda - 4) - 4(\lambda - 4) = 0$$

$$(\lambda - 4)(\lambda - 4) = 0$$

$$\lambda = 4 , \lambda = 4$$

$$\boxed{\lambda_1 = 0 , \lambda_2 = 2 , \lambda_3 = 4 , \lambda_4 = 4}$$

Scanned with CamScanner

19)

Question No = 3

$(x^2 + 3y^2)dx - 2xy\,dy = 0$

$x = 2$, $y = 6$

Sol:

$(x^2 + 3y^2)dx - 2xy\,dy = 0$

$(x^2 + 3y^2)dx = 2xy\,dy$

Divide b·side by $2xy\,dx$

$$\frac{dy}{dx} = \frac{x^2 + 3y^2}{2xy}$$

$$\frac{dy}{dx} = \frac{x^{\cancel{2}}}{2\cancel{x}y} + \frac{3y^{\cancel{2}}}{2x\cancel{y}}$$

$$\frac{dy}{dx} = \frac{1}{2}\left[\frac{x}{y} + \frac{3y}{x}\right] \rightarrow 1$$

let $y = vx$

Scanned with CamScanner

Diff

$$dy = v\,dx + x\,dv$$

Dividing by dx

$$\frac{dy}{dx} = v + x\frac{dv}{dx} \rightarrow (ii)$$

Put (ii) in eq (1)

$$v + x\frac{dv}{dx} = \frac{1}{2}\left[\frac{x}{xv} + 3\frac{vx}{x}\right]$$

$$\Rightarrow v + x\frac{dv}{dx} = \frac{1}{2}\left[\frac{1}{v} + 3v\right]$$

multiplying b.side by "2"

$$2v + 2x\frac{dv}{dx} = \frac{1}{v} + 3v$$

$$2x\frac{dv}{d} = \frac{1}{v} + 3v - 2v$$

Scanned with CamScanner

$$2x\frac{dv}{dx} = \frac{1}{v} + v$$

$$2x\frac{dv}{dx} = \frac{1+v^2}{v}$$

xing b. side by $\frac{dx}{dv}$
we get

$$2x\,dv = \frac{1+v^2}{v}\,dx$$

xing b.side by $\frac{v}{x(1+v^2)}$

$$2\frac{v}{1+v^2}\,dv = \frac{1}{x}\,dx$$

Take integral on b.side

$$\int\frac{2v}{1+v^2}\,dv = \int\frac{1}{x}\,dx + c$$

$$\ln(1+v^2) = \ln x + \ln c$$

Scanned with CamScanner

22

Take e on b. side

$$e^{\ln|1+v^2|} = e^{\ln|xc|}$$

$$1+v^2 = xc$$

Put $v = y/x$

$$1+\left(y/x\right)^2 = xc$$

$$\frac{x^2+y^2}{x^2} = xc$$

$$x^2+y^2 = x^3c \rightarrow \star$$

Put $x=2$, $y=6$ in eq $\star$

$$(2)^2+(6)^2 = (2)^3c$$

$$4+36 = 8c$$

$$c = \frac{40}{8} = \boxed{c=5} \rightarrow \text{Put in } \star$$

Scanned with CamScanner

23

So

$$x^2 + y^2 = 5x^3$$

$$y^2 = 5x^3 - x^2$$

$$y^2 = x^2(5x - 1)$$

Taking square root on b. side

$$y = x\sqrt{5x-1} \, , \; y = -x\sqrt{5x-1}$$

or

$$y = \pm x\sqrt{5x-1}$$

Ans

Scanned with CamScanner