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Date: __/__/__

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ID : 7769

Subject : Differential equation

Submitted to : Engr: Shomaila mazhar

24/4/020.

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Q No 1 Find the fourier series representation of

$$f(t) = 1 + t, \quad -\pi \leq t \leq \pi$$

Sol $f(t) = 1 + t, \quad -\pi \leq t \leq \pi$

here we use the formula

$$f(x) = a_0 + \sum_{n=1}^{\infty} a_n \cos t + \sum_{n=1}^{\infty} b_n \sin t \quad \text{eq (1)}$$

$$a_0 = \frac{1}{2\pi} \int_{-\pi}^{\pi} f(t)\, dt$$

$$a_0 = \frac{1}{2\pi} \int_{-\pi}^{\pi} (1+t)\, dt$$

$$a_0 = \frac{1}{2\pi} \left| t + \frac{t^2}{2} \right|_{-\pi}^{\pi}$$

$$a_0 = \frac{1}{2\pi} \left(\pi - (-\pi) + \frac{\pi^2}{2} - \left(\frac{-\pi^2}{2} \right) \right)$$

$$\boxed{a_0 = \frac{1}{2\pi} \left(2\pi + \frac{2\pi^2}{2} \right)}$$

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$$\boxed{a_0 \quad \frac{1}{2\pi}\left(2\pi + \pi^2\right)}$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} (1+t)\,(\cos nt)\,dt$$

$$a_n = \frac{1}{\pi}\left((1+t)\,\frac{\sin nt}{n}\Big|_{-\pi}^{\pi} - \int\left(\frac{\sin nt}{n}\,\frac{d}{dt}(1+t)\right)\right)$$

$$a_n = \frac{1}{\pi}\left((1+t)\,\frac{\sin nt}{n}\Big|_{-\pi}^{\pi} - \int\left(\frac{\sin n'}{n}\,\frac{d}{dt}(1+t)\right)\right)$$

$$a_n = \frac{1}{\pi}\left((1+t)\,\frac{\cancel{\sin nt}^{\,0}}{n} - \frac{\cos nt}{n^2}\Big|_{-\pi}^{\pi}\right)$$

$$a_n = \frac{-1}{n^2\pi}\left(\cos n\pi - \cos n(-\pi)\right)$$

$$a_n = \frac{-1}{n^2\pi}\left(-1 - (-1)\right)$$

$$\boxed{a_n = 0}$$

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$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} (1+t) \sin nt \, dt$$

$$b_n = \frac{1}{\pi}\left((1+t)\int_{-\pi}^{\pi} \sin nt - \int\left(\int \sin nt \cdot \frac{d}{dt}(1+t)\, dt\right)\right)$$

$$b_n = \frac{1}{\pi}\left(\frac{(1+t)(-\cos nt)}{n}\Big|_{-\pi}^{\pi} - \int\left(\frac{-\cos nt}{n}(1)\right)\right)$$

$$b_n = \frac{1}{\pi}\left(-\frac{(1+t)(\cos nt)}{n}\Big|_{-\pi}^{\pi} + \left(\frac{\cancel{\sin n x}^{\,0}}{n^2}\Big|_{-\pi}^{\pi}\right)\right)$$

$$b_n = \frac{-1}{n\pi}\left((1+\pi)(\cos n\pi) - (1+(\pi))(\cos n(\pi))\right)$$

$$b_n = \frac{-1}{n\pi}\left(\cancel{\cos} n\pi + \pi \cancel{\cos} n\pi - \cos n\pi + \pi \cos n\pi\right.$$

$$b_n = \frac{-1}{n\cancel{\pi}}\left(2\cancel{\pi} \cos n\pi\right.$$

here $\cos n\pi - \frac{(-1)^{n+1}}{n}$

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$$b_n = \frac{2}{n}(-1)^{n+1}$$

So eq become

$$\boxed{f(x) = \frac{1}{2\pi}\left(2\pi + \pi^2\right) + 0 + \sum_{n=1}^{\infty} \frac{2}{n}(-1)^{n+1} \sin t}$$

Q.No 2 Calculate the characteristic equation the Eigen values of the system where A is given by

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Sol A = $\begin{vmatrix} 1 & 0 & -1 \\ 3 & 1 & 4 \\ 0 & 2 & 2 \end{vmatrix}$

Eigen value = ?

Step ①

$\Rightarrow$ We have

$(A - \lambda I) X = 0$ A = Given matrix

Step ②

Now we have; the characteristic equation is given by

$|A - \lambda I| = 0$

$$\begin{vmatrix} 1 & 0 & -1 \\ 3 & 1 & 4 \\ 0 & 2 & 2 \end{vmatrix} - \lambda \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

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$$\begin{vmatrix} 1-\lambda & 0 & -1 \\ 3 & 1-\lambda & 4 \\ 0 & 2 & 2-\lambda \end{vmatrix} = 0$$

Step ③

$\lambda^3 - |\text{Sum of diagonal element}| \lambda^2 + \text{Sum of diagonal minors} \, | \lambda - |A| = 0$ ⑬

Sum of diagonal element $= 1+1+2 = 4$

Sum of diagonal minors =

$$\begin{vmatrix} 1 & 4 \\ 2 & 2 \end{vmatrix} + \begin{vmatrix} 1 & -1 \\ 0 & 2 \end{vmatrix} + \begin{vmatrix} 1 & 0 \\ 3 & 1 \end{vmatrix}$$

$= (-6) + (2) + (1)$

$= -6 + 2 + 1$

$= -3$

By putting values in eq (B)

$\lambda^3 - 4\lambda^2 - 3\lambda - |A| = 0$ ——— (C)

$$|A| = \begin{vmatrix} 1 & 0 & 1 \\ 3 & 1 & 4 \\ 0 & 2 & 2 \end{vmatrix} = 1\begin{vmatrix} 1 & 4 \\ 2 & 2 \end{vmatrix} - 0\begin{vmatrix} 3 & 4 \\ 0 & 2 \end{vmatrix} + 1\begin{vmatrix} 3 & 1 \\ 0 & 2 \end{vmatrix}$$

$= 1(2-8) - 0 + 1(6-0)$

$= -6 + 6$

$= 0$

By putting value in eq (C)

$\lambda^3 - 4\lambda^2 - 3\lambda - 0 = 0$

$\lambda^3 - 4\lambda^2 - 3\lambda = 0$

$\lambda(\lambda^2 - 4\lambda - 3) = 0$

$\lambda = 0$

$\lambda^2 - 4\lambda - 3 = 0$

Using Quadratic formula.

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$$\lambda = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(-3)}}{2(1)}$$

$$= \frac{4 \pm \sqrt{16+12}}{2} = \frac{4 \pm \sqrt{28}}{2}$$

$$\lambda = \frac{4+\sqrt{28}}{2}, \quad \lambda = \frac{4-\sqrt{28}}{2}$$

We have eigenvalues:

$$\lambda = \left(0, \frac{4+\sqrt{28}}{2}, \frac{4-\sqrt{28}}{2}\right.$$

required solution

Q No 3

$$5x + 0 + 4z + 2m = 3$$
$$x - y + 2z + m = 1$$
$$4x + y + 2x + 0 = 1$$
$$x + y + z + m = 0$$

Sol

$$\left[\begin{array}{cccc|c} 5 & 0 & 4 & 2 & 3 \\ 1 & -1 & 2 & 1 & 1 \\ 4 & 1 & 2 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 \end{array}\right] \quad R_4 \quad R_2$$

$$\left[\begin{array}{cccc|c} 5 & 0 & 4 & 2 & 3 \\ 1 & -1 & 2 & 1 & 1 \\ 4 & 1 & 2 & 0 & 1 \\ 0 & 2 & -1 & 0 & -1 \end{array}\right]$$

$$\left[\begin{array}{cccc|c} 5 & 0 & 4 & 2 & 3 \\ 1 & -1 & 2 & 1+4/5 & 1 \\ 0 & -1 & +6/5 & \cancel{}4/5 & 3/5 \\ 0 & 2 & -1 & \cancel{4/5}\ 0 & -1 \end{array}\right] \quad -1/5 \times R_3$$

$$\left[\begin{array}{cccc|c} 5 & 0 & 4 & 2 & 3 \\ 1 & -1 & 2 & 1 & 1 \\ 0 & -1 & 6/5 & 4/5 & 3/5 \\ 0 & 0 & 7/5 & 8/5 & 1/5 \end{array}\right] \quad 5\times R_3 \text{ and } 5\times R_4$$

$$\left[\begin{array}{cccc|c} 5 & 0 & 4 & 2 & 3 \\ 1 & -1 & 2 & 1 & 1 \\ 0 & -5 & 6 & 4 & 2 \\ 0 & 0 & 7 & 8 & 1 \end{array}\right] \quad 5R_3 \text{ and } 5R_4$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 4/5 & 2/5 & 3/5 \\ 1 & -1 & 2 & 1 & 1 \\ 0 & -5 & 6 & 4 & 3 \\ 0 & 0 & 7 & 8 & 1 \end{array}\right] \quad 1/5\times R_1$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 4/5 & 2/5 & 3/5 \\ 0 & -1 & 6/5 & 1/5 & 2/5 \\ 0 & -5 & 6 & 4 & 3 \\ 0 & 0 & 7 & 8 & 1 \end{array}\right] \quad R_2\times 5$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 4/5 & 2/5 & 3/5 \\ 0 & -5 & 6 & 1 & 2 \\ 0 & 0 & 0 & 3 & 1 \\ 0 & 0 & 7 & 8 & 1 \end{array}\right] \quad R_3 - R_2$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 4/5 & 2/5 & 3/5 \\ 0 & -5 & 6 & 1 & 2 \\ 0 & 0 & 1 & 8/7 & 1/7 \\ 0 & 0 & 0 & 1 & 1/3 \end{array}\right]$$

$R_3 \leftrightarrow R_4$

$1/7 \times R_3$

$1/3 \times R_4$

$$\left[\begin{array}{cccc|c} 1 & 0 & 4/5 & 2/5 & 3/5 \\ 0 & -5 & 6 & 1 & 2 \\ 0 & 0 & 1 & 1 & -4/21 \\ 0 & 0 & 0 & 1 & 1/3 \end{array}\right]$$

$C_2 \times -5$

$$\left[\begin{array}{cccc|c} 1 & 0 & 4/5 & 2/5 & 3/5 \\ 0 & 1 & 6 & 1 & 2 \\ 0 & 0 & 1 & 1 & -4/21 \\ 0 & 0 & 0 & 1 & 1/3 \end{array}\right]$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 4/5 & 2/5 & 3/5 \\ 0 & 1 & 0 & -5 & 26/21 \\ 0 & 0 & 1 & 0 & -11/21 \\ 0 & 0 & 0 & 1 & 1/3 \end{array}\right]$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 4/5 & 2/5 & 3/5 \\ 0 & 1 & 0 & 0 & 31/21 \\ 0 & 0 & 1 & 0 & -11/21 \\ 0 & 0 & 0 & 1 & 1/3 \end{array}\right]$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 1 & 1/2 & 3/4 \\ 0 & 1 & 0 & 0 & 31/21 \\ 0 & 0 & 1 & 0 & -11/21 \\ 0 & 0 & 0 & 1 & 1/3 \end{array}\right]$$

$5/4 \times R_1$

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$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 1/2 & 126/84 \\ 0 & 1 & 0 & 0 & 31/21 \\ 0 & 0 & 1 & 0 & -11/21 \\ 0 & 0 & 0 & 1 & 1/3 \end{array}\right]$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 1/2 & 1/2 \\ 0 & 1 & 0 & 0 & 31/21 \\ 0 & 0 & 1 & 0 & -11/21 \\ 0 & 0 & 0 & 1 & 1/3 \end{array}\right]$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 3/4 \\ 0 & 1 & 0 & 0 & 31/21 \\ 0 & 0 & 1 & 0 & -11/21 \\ 0 & 0 & 0 & 1 & 1/3 \end{array}\right]$$

$$(x, y, z, m) = (3/4, 31/21, -11/21, 1/3)$$

$x = 3/4$

$y = 31/21$

$z = -11/21$

$m = 1/3$

Q.no 4 Verify that

$$u(x,t) = \sin(x + 2t)$$

is a solution of the one-dimensional wave equation.

Sol:: Given that

$$u(x,t) = \sin(x + 2t)$$

diff w.r.t x partialy

$$\frac{\partial u}{\partial x} = \frac{\partial}{\partial x} \sin(x+2t)$$

$$\frac{\partial u}{\partial x} = \cos(x + 2t) \frac{\partial}{\partial x}(x + 2t)$$

$$\frac{\partial u}{\partial x} = \cos(x + 2t)(1+0)$$

$$\frac{\partial u}{\partial x} = \cos(x + 2t)$$

$$\frac{\partial^2 u}{\partial x^2} = \frac{\partial}{\partial x} \cos(x + 2t)$$

$$\frac{\partial^2 u}{\partial x^2} = -\sin(x+2t) \cdot \frac{\partial}{\partial x}(x+2t)$$

$$\frac{\partial^2 u}{\partial x^2} = -\sin(x+2t)(1+0)$$

$$\boxed{\frac{\partial^2 u}{\partial x^2} = -\sin(x+2t}$$

and

$$u(x,t) = \sin(x+2t)$$

diff w.r.t "t"

$$\frac{\partial u}{\partial t} = \frac{\partial}{\partial t}\sin(x+2t)$$

$$\frac{\partial u}{\partial t} = \cos(x+2t)(0+2)$$

$$\frac{\partial u}{\partial t} = 2\cos(x+2t)$$

$$\frac{\partial^2 u}{\partial t^2} = (2) - \sin(x+2t)(0+2)$$

$$\boxed{\frac{\partial^2 u}{\partial t^2} = -4\sin(x+2t)}$$

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we know that one-dimensional wave equation is

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$$

$$-4 \sin(x+2t) = c^2 [-\sin(x+2t)]$$

$$-4 \sin(x+2t) = -c^2 \sin(x+2t)$$

$$-4 \sin(x+2t) + c^2 \sin(x+2t) = 0$$

For the orbitrary constant $c = \pm 2$

$$-4(x+2t) + (\pm 2)^2 \sin(x+2t) = 0$$

$$-4 \sin(x+2t) + 4 \sin(x+2t) = 0$$

$$0 = 0$$

then it will be verified for the orbitrary constant $c = 2$

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