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SECTION :- A

Subject :- Differential Equation

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Q No 1

Solve the following objective type Question:

1): The order of Matrix A is m x p and the order of B is p x n. Then the order of Matrix AB is?

$\boxed{m \times n}$

2) The number of non-zero rows in an Echelon form?

$\boxed{\text{One}}$

3) If $B = \begin{bmatrix} 1 & 4 \\ 2 & a \end{bmatrix}$ is a singular matrix then a=?

$$\begin{vmatrix} 1 & 4 \\ 2 & a \end{vmatrix} = a \times 1 - 2 \times 4$$
$$= a - 8 = 0$$
$$\Rightarrow a = 8$$

iv) If $A = \begin{bmatrix} 2i & i \\ i & -i \end{bmatrix}$ then $|A| = ?$

$$|A| = 2i(-i) - i(i)$$
$$= 2i^2 - i^2$$
$$= -2(-1) - (-1)$$
$$= 2 + 1$$
$$= 3$$

v) The matrix $A = \begin{bmatrix} 9 & 0 \\ 0 & 9 \end{bmatrix}$ is?

Scalar matrix.

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vi) Solution of $\frac{dy}{dx} + 2xy = y$?

$\frac{dy}{dx} + 2xy = y$

separating variables

$\Rightarrow \frac{dy}{dx} = y - 2xy$

$= \frac{dy}{dx} = y(1-2x)$

$\frac{dy}{dx} = (1-2x)dx$

$\Rightarrow \int \frac{1}{y} dy = \int 1 dx - \int 2x dx$

$\Rightarrow \ln y = x - \frac{2x^2}{2} + c$

$\Rightarrow \ln y = x - x^2 + c$ Ans

vii) The order and degree of differential equation

~~$\frac{d^2y}{dx^2} - 4xy = \sin\left(\frac{d^2y}{dx^2}\right)$ is~~

$\left(\frac{dy}{dx}\right)^3 = \sqrt{1+\left(\frac{dy}{dx}\right)^2}$ is ?

Order: 1

Degree: 3

viii) The order and degree of differential equation

$\frac{d^2y}{dx^2} - 4xy = \sin\left(\frac{d^2y}{dx^2}\right)$ is ?

order = 2

Degree: 1

Q1)

ix) The differential equation $2\frac{dy}{dx} + x^2 y$ $2x+3$, $y(0) = 5$ is

Solution:

$$2y' + x^2 y = x^2 + 3 \quad , y(0) = 5$$

$$y' + \left(\frac{x^2}{2}\right) y = \frac{x^2+3}{2}$$

$$y' + \left(\frac{x^2}{2}\right) y = \frac{1}{2}(x^2+3)$$

$$a = \frac{x^2}{2}$$

$$e^{\int \frac{x^2}{2} dx} = e^{x^3/6}$$

$$e^{x^3/6} y' + e^{x^3/6}\left(\frac{x^2}{2}\right) y = \frac{1}{2} e^{x^3/6} (x^2+3)$$

$$y(x) = \frac{e^{x^3/6} x^2 + 3e^{x^3/6}}{2e^{x^3/6}} + C$$

$$y(0) = \frac{0+3}{2} = 3/2$$

$$y(x) = \frac{e^{x^3/6} x^2 + 3e^{x^3/6}}{2e^{x^3/6}} + \frac{3}{2}$$ Ans

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Q1
(X)

$$\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$$

Solution

Expand by C_1

$$= 1\begin{vmatrix} b & b^2 \\ c & c^2 \end{vmatrix} - 1\begin{vmatrix} a & a^2 \\ c & c^2 \end{vmatrix} + 1\begin{vmatrix} 1 & b \\ 1 & c \end{vmatrix}$$

$$= 1(bc^2 - cb^2) - 1(ac^2 - a^2c) + (ab^2 - a^2b)$$

$$= bc^2 - cb^2 - ac^2 + a^2c + ab^2 - a^2b$$

$$= ab^2 - cb^2 + a^2c - a^2b - ac^2 + bc^2$$

$$= a^2c - a^2b + ab^2 - cb^2 + bc^2 + bc^2 - ac^2$$

$$= a^2(c-b) + b^2(a-c) + c^2(b-a)$$

Ans.

Q No. 2

i) Express the determinant

$$\begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \end{vmatrix}$$

as the product of factors which are linear in a, b, c.

Solution:

$$\begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \end{vmatrix}$$

Expand by R_1

$$a\begin{vmatrix} b^2 & c^2 \\ b^3 & c^3 \end{vmatrix} - b\begin{vmatrix} a^2 & c^2 \\ a^3 & c^3 \end{vmatrix} + c\begin{vmatrix} a^2 & b^2 \\ a^3 & b^3 \end{vmatrix}$$

$$= a\,(b^2c^3 - b^3c^2) - b\,(a^2c^3 - a^3c^2)$$
$$+ c\,(a^2b^3 - a^3b^2)$$

$$= ab^2c^3 - ab^3c^2 - a^2bc^3 - a^3bc^2$$
$$+ a^2cb^3 - a^3b^2c$$

common abc

$\Rightarrow abc\,(bc^2 - b^2c - ac^2 + a^2c + ab^2 - a^2b)$

$= abc\,[bc(c-b) - ac(c-a) + ab(b-a)]$

Ans

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Question No 2
Part #2

Find the Eigen value

$$\begin{bmatrix} 2 & -1 & -1 & 0 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ 0 & -1 & -1 & 2 \end{bmatrix}$$

Solution:

$$\begin{bmatrix} 2 & -1 & -1 & 0 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ 0 & -1 & -1 & 2 \end{bmatrix}$$

characteristic eq $|A - \lambda I| = 0 \rightarrow$ A

$$\begin{bmatrix} 2 & -1 & -1 & 0 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ 0 & -1 & -1 & 2 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Now take determinant

$$|A - \lambda I| = 0$$

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$$\begin{vmatrix} 2-\lambda & -1 & -1 & 0 \\ -1 & 3-\lambda & -1 & -1 \\ -1 & -1 & 3-\lambda & -1 \\ 0 & -1 & -1 & 2-\lambda \end{vmatrix} = 0$$

Expand by R_1

$$2-\lambda \begin{vmatrix} 3-\lambda & -1 & -1 \\ -1 & 3-\lambda & -1 \\ -1 & -1 & 2-\lambda \end{vmatrix} - (-1) \begin{vmatrix} -1 & -1 & -1 \\ -1 & 3-\lambda & -1 \\ 0 & -1 & 2-\lambda \end{vmatrix}$$

$$-1 \begin{vmatrix} -1 & 3-\lambda & -1 \\ -1 & -1 & -1 \\ 0 & -1 & 2-\lambda \end{vmatrix} = 0 \rightarrow \text{(B)}$$

Again

$$\begin{vmatrix} 3-\lambda & -1 & -1 \\ -1 & 3-\lambda & -1 \\ -1 & -1 & 2-\lambda \end{vmatrix} \quad \text{Expand by } \underline{R_1}$$

$$= 3-\lambda \begin{vmatrix} 3-\lambda & -1 \\ -1 & 2-\lambda \end{vmatrix} - (-1) \begin{vmatrix} -1 & -1 \\ -1 & 2-\lambda \end{vmatrix}$$

$$-1 \begin{vmatrix} -1 & 3-\lambda \\ -1 & -1 \end{vmatrix}$$

$$= (3-\lambda)\Big[(3-\lambda)(2-\lambda) - (-1)(-1) + 1\big((-1)(2-\lambda) - (-1)(-1)\big)$$

$$-1\big((-1)(-1) - (-1)(3-\lambda)\big)\Big]$$

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Multiplying both sides by $\frac{dx}{dv}$
we get

$$2x\,dv = \frac{1+v^2}{v}\,dx$$

Multiplying both sides by $\frac{v}{x(1+v^2)}$
we get

$$\frac{v}{1+v^2}\,dv = \frac{1}{x}\,dx$$

Take "$\int$" on both sides

$$\int \frac{2v}{1+v^2}\,dv = \int \frac{1}{2x}\,dx + C$$

$$\ln|1+v^2| = \ln x + \ln c$$

Take "e" on both sides

$$e^{\ln}(1+v^2) = e^{\ln|xc|}$$

$$1 + v^2 = Xc.$$

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$$-\lambda \begin{vmatrix} -1 & 3-\lambda & -1 \\ -1 & -1 & -1 \\ 0 & -1 & 2-\lambda \end{vmatrix}$$

Expand by C_1

$$\Rightarrow -\left[-1 \begin{vmatrix} -1 & -1 \\ -1 & 2-\lambda \end{vmatrix} - (-1) \begin{vmatrix} 3-\lambda & -1 \\ -1 & 2-\lambda \end{vmatrix} + 0\right]$$

$$\Rightarrow -(3-\lambda+\lambda^2-5\lambda+5)$$

$$-\lambda^2+5\lambda-5-3+\lambda$$

$$\boxed{-\lambda^2+6\lambda-8} \rightarrow c$$

Put a, b and c in B

$$(2-\lambda)\left[-\lambda^3+8\lambda^2-18\lambda+8\right]$$

$$-\lambda^2+6\lambda-8-\lambda^2+6\lambda-8.$$

$$= -2\lambda^3+16\lambda^2-36\lambda+16+\lambda^4-8\lambda^3+18\lambda^2-8\lambda$$

$$-\lambda^2+6\lambda-8-\lambda^2+16\lambda-8$$

$$\Rightarrow \lambda^4-2\lambda^3-8\lambda^3+16\lambda^2+16\lambda^2-\lambda^2-\lambda^2$$

$$-36\lambda-8\lambda+6\lambda+6\lambda+16-16$$

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$$\Rightarrow \lambda^4 - 10\lambda^3 + 32\lambda^2 - 32\lambda = 0$$

By Synthathic division

we get :-

$$\lambda(\lambda-2)(\lambda^2 - 8\lambda + 16) = 0$$

$$(\lambda = 0)$$

$$\lambda - 2 = 0 \Rightarrow \boxed{\lambda = 2}$$

$$\lambda^2 - 8\lambda + 16 = 0$$

By Factorization Method

$$\lambda^2 - 4\lambda - 4\lambda + 16 = 0$$

$$\lambda(\lambda-4) - 4(\lambda-4) = 0$$

$$(\lambda-4)(\lambda-4)$$

$$\lambda = 0, \lambda = 0$$

$$\boxed{\lambda_1 = 0, \lambda_2 = 2, \lambda_3 = 4, \lambda_4 = 4 \quad \text{Ans}}$$

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Question No#3

The rate of change in the form of differential equation is given by

$(x^2+3y^2)dx - 2xy\,dy = 0$. Find the general Solution at $x=2$ and $y=6$

Soln

$$(x^2+3y^2)dx - 2xy\,dy = 0$$

$$(x^2+3y^2)dx = 2xy\,dy$$

Divide both sides by $2xy\,dx$ we get

$$\frac{dy}{dx} = \frac{x^2+3y^2}{2xy}$$

$$\frac{dy}{dx} = \frac{x^2}{2xy} + \frac{3y^2}{2xy}$$

$$\frac{dy}{dx} = \frac{1}{2}\left[\frac{x}{y} + \frac{3y}{x}\right] \rightarrow \circledast$$

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let $y = vx$

Diff $dy = v\,dx + x\,dv$

Dividing by dx

$$\frac{dy}{dx} = v + x\frac{dv}{dx} \rightarrow (a)$$

Put (a) in (A)

$$v + \frac{dv}{dx} = \frac{1}{2}\left[\frac{x}{xv} + 3\frac{vx}{x}\right]$$

$$v + \frac{x\,dv}{dx} = \frac{1}{2}\left[\frac{1}{v} + 3v\right]$$

Multiplying both sides by "2"

$$2v + 2x\frac{dv}{dx} = \frac{1}{v} + 3v$$

$$2x\frac{dv}{dx} = \frac{1}{v} + 3v - 2v$$

$$2x\frac{dv}{dx} = \frac{1}{v} + v$$

$$2x\frac{dv}{dx} = \frac{1+v^2}{v}$$

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Multiplying both sides by $\frac{dx}{dv}$ we get

$$2x\,dv = \frac{1+v^2}{v}\,dx$$

Multiplying both sides by $\frac{v}{x(1+v^2)}$ we get

$$\frac{v}{1+v^2}\,dv = \frac{1}{x}\,dx$$

Take "$\int$" on both sides

$$\int \frac{2v}{1+v^2}\,dv = \int \frac{1}{2x}\,dx + C$$

$$\ln|1+v^2| = \ln x + \ln c$$

Take "e" on both sides

$$e^{\ln(1+v^2)} = e^{\ln |xc|}$$

$$1+v^2 = xe.$$

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$1+v^2 = xc$

Put $v = y/x$

$1+(y/x)^2 = xc$

$\frac{x^2+y^2}{x^2} = xc$

$x^2+y^2 = x^3c \rightarrow$ ★★

Put $x=2, y=6$ in eq ★★

$(4)+(36) = 8c$

$c = \frac{40}{8}$

$\boxed{c=5} \rightarrow$ Put in ★★

So

$x^2+y^2 = 5x^2$

$y^2 = 5x^3 - x^2$

$y^2 = x^2(5x-1)$

Taking "$\sqrt{\ }$" on b·sides

$y = +x\sqrt{5x-1}$, $y = -x\sqrt{5x-1}$

or

$$\boxed{y = \pm x\sqrt{5x-1}}$$

Ans

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