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Q No 2 Let $E = 10 [\sin(\pi/6) a_x + 5 \sin(\pi/6) a_y + 10 \cos(\pi/6) a_z]$

Solution

(a) $E_p = -10 [\sin(\pi/6) a_x + 5 \sin(\pi/6) a_y + 10 \cos(\pi/6) a_z]$
 $= -[5 a_x + 25 a_y + 50\sqrt{3} a_z]$

(b) $dW_x = -q E \cdot dL a_x$
 $= -2 \times 10^{-9} (-5)(10^{-3}) = 10^{-11} \text{ J}$
 $= \boxed{10 \text{ pJ}}$

(c) of a_y ?
 $dW_y = -q E \cdot dL a_y$
 $= -2 \times 10^{-9} (-25)(10^{-3}) = 50^{-11} \text{ J}$
 $= \boxed{50 \text{ pJ}}$

(d) of a_z
 $dW_z = -q E \cdot dL a_z$
 $= -2 \times 10^{-9} (-50\sqrt{3})(10^{-3})$
 $= \boxed{100\sqrt{3} \text{ pJ}}$

(e) of $(a_x + a_y + a_z)$?

$$dW_{xyz} = -q E \cdot dL \frac{(a_x + a_y + a_z)}{\sqrt{3}}$$
$$= \frac{10 + 50 + 100\sqrt{3}}{\sqrt{3}}$$
$$= \boxed{135 \text{ pJ}}$$

(3)

Thus

$$dW = -(20 \times 10^6) [100a_x - 200a_y + 300a_z] \cdot [0.267a_x - 535a_y + 0.802a_z] \times 6 \times 10^4$$

$$= \boxed{-44.9 \text{ nJ}}$$

(c) In the direction of $G = 2a_x - 3a_y + 4a_z$.

$$a_G = \frac{2a_x - 3a_y + 4a_z}{[2^2 + 3^2 + 4^2]^{1/2}}$$

$$= 0.371a_x - 0.557a_y + 0.743a_z.$$

So now

$$dW = -(20 \times 10^6) [100a_x - 200a_y + 300a_z] \cdot [0.371a_x - 0.557a_y + 0.743a_z] \times 6 \times 10^4$$

$$= -(20 \times 10^6) [37.1(a_x \cdot a_x) - 55.7(a_x \cdot a_y) - 74.3(a_y \cdot a_x) + 111.4(a_y \cdot a_y) + 222.9] (6 \times 10^4).$$

where at $\cos(\theta) = (a_x \cdot a_x) = (a_y \cdot a_y) = \cos(40^\circ) = 0.766$

$$(a_x \cdot a_y) = \sin(40^\circ) = 0.643$$

$$(a_y \cdot a_x) = -\sin(40^\circ) = -0.643$$

Now substituting these results in.

$$dW = -(20 \times 10^6) [28.4 - 35.8 + 47.7 + 85.3 + 222.9] (6 \times 10^4)$$

$$= \boxed{-41.8 \text{ nJ}}$$

QNO1: The value of E at -----

$$\textcircled{c} \phi = 2a_x - 3a_y + 4a_z.$$

Solution:-

① in the direction of $a_p =$ the incremental work is given by $dW = -q_e E \cdot dL$, where in this case $dL = dp a_p = 6 \times 10^6 a_p$. Thus:

$$\begin{aligned} dW &= -(20 \times 10^{-6} \text{C})(100 \text{V/m})(6 \times 10^{-6} \text{m}) \\ &= -12 \times 10^{-9} \text{J} \end{aligned}$$

$$= \boxed{-12 \text{ nJ}}$$

② in the direction of $a_\phi =$ In this case $dL = 2d\phi a_\phi = 6 \times 10^6$ and so,

$$\begin{aligned} dW &= -(20 \times 10^{-6})(-200)(6 \times 10^6) \\ &= 2.4 \times 10^8 \text{J} \end{aligned}$$

$$= \boxed{24 \text{ mJ}}$$

③ in the direction of $a_z =$ Here, $dL = dz a_z = 6 \times 10^6 a_z$.

$$\begin{aligned} dW &= -(20 \times 10^{-6})(300)(6 \times 10^6) \\ &= -3.6 \times 10^8 \text{J} \end{aligned}$$

$$= \boxed{-36 \text{ mJ}}$$

④ In the direction of E .

$$\begin{aligned} a_E &= \frac{100a_p - 200a_\phi + 300a_z}{[100^2 + 200^2 + 300^2]^{1/2}} \end{aligned}$$

$$= 0.267a_p - 0.535a_\phi + 0.802a_z.$$

①

QND 5 Let $G = 3xy^3 a_x + 2zay$. Find.

Solution:-

Let $G = 3xy^3 a_x + 2zay$.

(a) straight line $y = x - 1, z = 1$
$$= \int G \cdot dl = \int_2^4 3xy^2 + \int_1^3 2z dy$$
$$= \int_2^4 3x(x-1)^2 dx + \int_1^3 2(1) dy$$
$$= \underline{\underline{90}}$$

(b) Parabola $6y = x^2 + 2, z = 1$
$$= \int G \cdot dl = \int_2^4 3xy^2 + \int_1^3 2z dy$$
$$= \int_2^4 \frac{1}{12} x(x^2 + 2)^2 + \int_1^3 2(1) dy$$
$$= \underline{\underline{82}}$$

(c)

QNO4 Compute the value $\int_A^P G \cdot dL$.

Solution :-

(a) Straight line segment's $A(1, -1, 2)$ to $B(1, 1, 4)$
to $P(2, 1, 2)$: In general, we would have.

$$\int_A^P G \cdot dL = \int_A^P 2y dx.$$

The change in x occurs when moving b/w
 B & P , during which $y = 1$.

$$\int_A^P G \cdot dL = \int_B^P 2y dx$$

$$= \int_1^2 2(1) dx.$$

(b) Straight line segment $A(1, -1, 2)$ to $C(2, 1, 2)$

to $P(2, 1, 2)$: In this case change in x occurs
when moving from A to C during which $y = -1$

$$= \int_A^P G \cdot dL = \int_C^P 2y dx$$

$$= \int_1^2 2(-1) dx$$

$$= \underline{\underline{-2}}$$

(5)

QNO3

Solution:-

① P(1,2,3) toward Q(2,1,4)

The vector along this direction will be $Q-P = (1, -1, 1)$
from which $a_{PQ} = [a_x - a_y + a_z] / \sqrt{3}$

$$\begin{aligned} dW &= -qE \cdot dL \\ &= -(50 \times 10^6) \left[120 a_p \cdot \frac{(a_x - a_y + a_z)}{\sqrt{3}} \right] (2 \times 10^{-3}) \\ &= -(50 \times 10^6) (120) [a_x \cdot a_p - (a_p \cdot a_y)] \frac{1}{\sqrt{3}} (2 \times 10^{-3}) \end{aligned}$$

At P, $\phi = \tan^{-1}(2/1) = 63.4^\circ$, thus $(a_p \cdot a_x) = \cos(63.4^\circ) = 0.447$

$(a_p \cdot a_y) = \sin(63.4^\circ) = 0.894$

Substituting these, we obtain

$\therefore dW = 3.1 \mu J$

② Q(2,1,4) toward P(1,2,3) A little thought is in order here: Note that the field has only a radial component and does not depend on ϕ or z . And P and Q are at the same radius $(\sqrt{5})$ from z axis. Thus the answer is $dW = 3.1 \mu J$ as in part a. This is also found by going through the procedure as in part a, but with the direction (roles of P & Q) reversed.