Department of Electrical Engineering
Assignment
Date:13/04/2020

Course Details

Course Title:	Digital Signal Processing	**Module:**	6th
Instructor:	**Pir Meher Ali Sha**	**Total Marks:**	30

Student Details

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<table>
<tr><td rowspan="2">Q1.</td><td>(a)</td><td>Consider the following analog signal

$$x_a(t) = 3\cos 100\pi t + 4\sin 200\pi t$$
i. Determine the minimum sampling rate required to avoid aliasing.
ii. Suppose that the signal is sampled at the rate $F_s = 100\text{Hz}$.What is the discrete-time signal obtained after sampling? Also explain the effect of this sampling rate on the newly generated discrete time signal.
iii. What is the analog signal $y_a(t)$ we can reconstruct from the samples if we use ideal interpolation?</td><td>Marks 5
CLO 1</td></tr>
<tr><td>(b)</td><td>Consider a discrete time signal which is given by

$$x(n) = \begin{cases} 0.5n \ , & n \geq 0 \\ 0, & n < 0 \end{cases}$$
This is signal is sampled at the rate $F_s = 200\text{Hz}$.

i. Draw the sampled signal.
ii. The samples of the signals are intended to carry 3 bits per sample. Determine the quantization level and quantization resolution to quantized the sampled signal achieved in part i .
iii. Perform the process of truncation and rounding off on all the values of the sampled signal and find the quantization error for each of the sampled data. Express your answer in tabular form.</td><td>Marks 5
CLO 1</td></tr>
<tr><td>Q2.</td><td>(a)</td><td>Determine the response of the system to the following input signal with given impulse response

$$x[n] = \{\, 2, \underset{\uparrow}{1}, -2, 3, -4\} \qquad , h[n] = \{\underset{\uparrow}{3}, 1, 2, 1, 4\}$$</td><td>Marks 5
CLO 2</td></tr>
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	(b)	Compute the convolution y(n) of the following signal $\boldsymbol{x}(n)=\begin{cases} \alpha^{n+1}, & -3 \le n \le 5 \\ 0, & elsewhere \end{cases}$ $\boldsymbol{h}(n)=\begin{cases} 2^{n}, & 0 \le n \le 4 \\ 0, & elsewhere \end{cases}$	Marks 5 CLO 2
Q3.		Determine the z- transform of the following signals and also sketch its Region of Convergence (ROC). i) $X(n)\begin{cases} (\frac{1^{n}}{4}) & , n \ge 0 \\ (\frac{1}{3})^{-n} & , n < 0 \end{cases}$ ii) $X(n)=\begin{cases} (\frac{1}{2})^{n} - 3^{n} & , \quad n \ge 0 \\ 0 & , \quad elsewhere \end{cases}$	Marks 10 CLO 2

Q.No.1

Consider the following Analog signal

$$x_a(t) = 3\cos 100\pi t + 4\sin 200\pi t$$

(i) **Describe the minimum sampling rate required to avoid aliasing.**

To avoid aliasing, we must use antialiasing filter or our frequency must be greater than nyquist frequency

we use highest Frequency of the given signal.

$$\omega = 2\pi f$$

$$200\pi = 2\pi f$$

$$f = 100$$

Nyquist theorem

$$f_s \geq 2f \Rightarrow f_s \geq 200$$

This is the minimum frequency required to avoid aliasing.

(ii) Suppose that signal is sampled at rate $f_s = 100\,Hz$. What is the signal and Also explain the effect.

As

$$x_a(t) = 3\cos 100\pi t + 4\sin 200\pi t$$

$$\omega_1 = 2\pi f$$
$$100\pi = 2\pi f$$
$$f_1 = 50\ Hz$$

$$f_1 = F_s f_1'$$
$$f_1' = \frac{50}{100}$$
$$f_1' = 0.5\ Hz$$

Now,

$$\omega_1 = 2\pi f_1'$$
$$= 2\pi \times 0.5$$
$$\omega_1 = \pi\ Hz$$

$$\omega_2 = 200\pi$$
$$2\pi f_2 = 200\pi$$
$$f_2 = 100\ Hz$$

$$f_2 = F_s f_2'$$
$$f_2' = \frac{100}{100}$$
$$f_2' = 1\ Hz$$

$$\omega_2 = 2\pi f_2'$$
$$\omega_2 = 2 \times \pi \times 1$$
$$\omega_2 = 2\pi\ Hz$$

The sampled signal is

$$x_a(t) = 3\cos\pi t + 4\sin 2\pi t$$

Effect:-

Briefly said, there is no overlapping of signal. If we sampled the signal over this frequency, the reconstruction of the signal is possible. We require the same signal at receiving side as we sent at sending side.

(iii) What is the Analog signal $y_a(t)$, we can reconstruct.

If We use ideal interpolation then the same signal $x_a(t)$ is required. Because in ideal case there is no destruction of signal. The required signal is

$$y_a(t) = 3\cos 100\pi t + 4\sin 200\pi t$$

14)

(b)

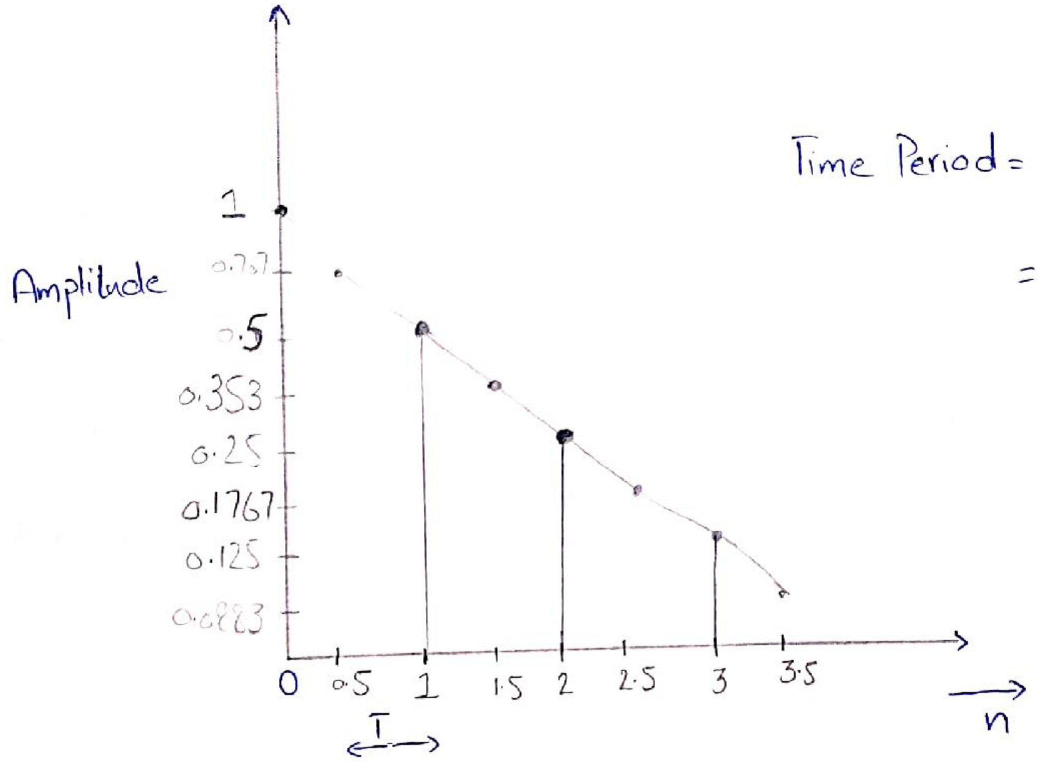
Consider a discrete signal

$$x(n) = \begin{cases} 0.5^n & , n \geq 0 \\ 0 & , n < 0 \end{cases}$$

Sampled rate $f_s = 2Hz$

(i) Draw the sampled Signal.

0.5^n $n =$	$n \geq 0$
0	1
1	0.5
1.5	0.353
0.5	0.707
2	0.25
2.5	0.17677
3	0.125
3.5	0.0883



Time Period $= \frac{1}{2}$

$= 0.5$ sec

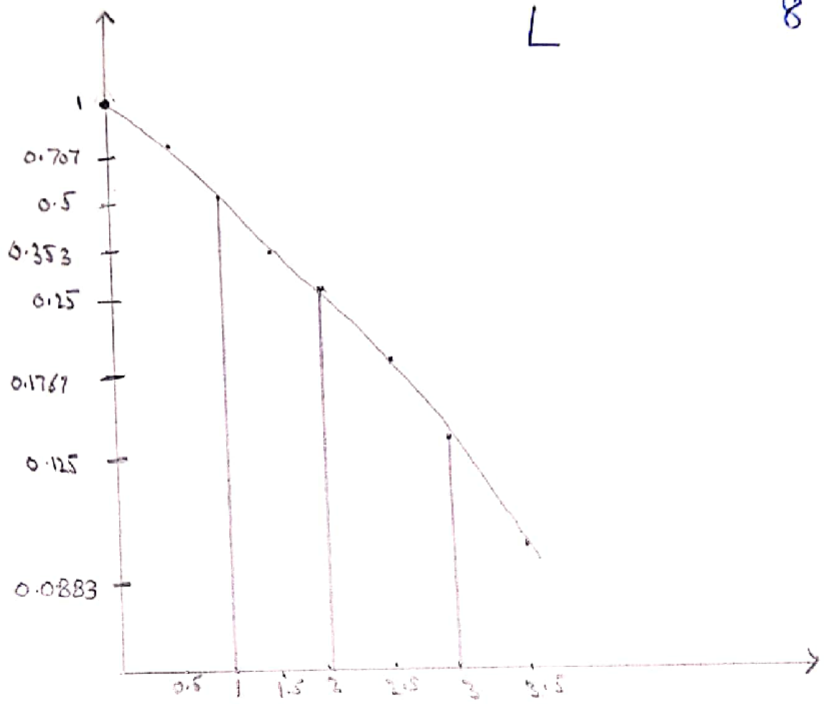
b.)

$L = 2^n$

$n = 3$

$L = 2^3 = 8$

Resolution $= \Delta = \dfrac{x_{max} - x_{min}}{L} = \dfrac{1 - 0}{8} = 0.125$



(iii) Perform the process of truncation and rounding off on all values of the sampled signal and find the quantization Error for each of the sampled data.

(iii)

Sr. No	Discrete Time Signal	Truncation	Rounding	Error
0.	1	1.0	1.0	0.0
1.	0.8535	0.8	0.9	-0.1
2.	0.707	0.7	0.7	0.0
3.	0.6635	0.6	0.6	0.0
4.	0.5	0.5	0.5	0.0
5.	0.426	0.4	0.4	0.0
6.	0.353	0.3	0.4	-0.1
7.	0.1765	0.1	0.2	-0.1

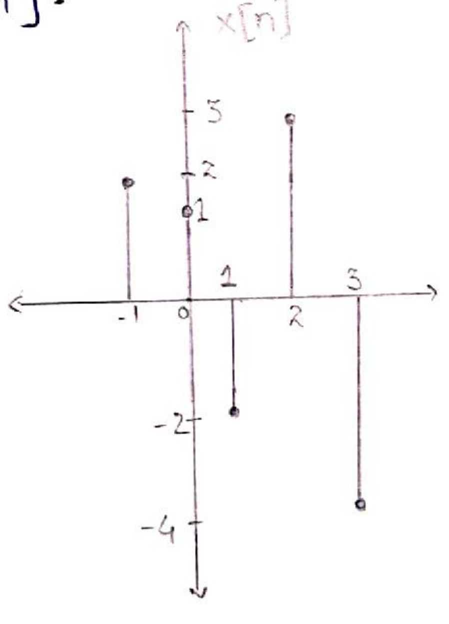
Q No. 2 :-

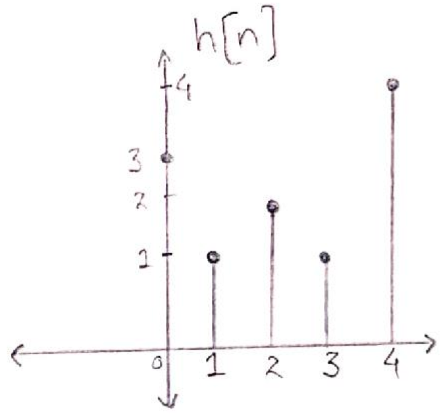
Determine the response of the system of the following input signal with given impulse response :-

$$x[n] = \{2, \underset{\uparrow}{1}, -2, 3, -4\}$$

$$h[n] = \{\underset{\uparrow}{3}, 1, 2, 1, 4\}$$

As we know that, if there is multiplication in one domain then in the other domain there is convolution.
To find $y[n]$, we convolve $x[n]$ and $h[n]$.





$$y[n] = x[n] * h[n]$$

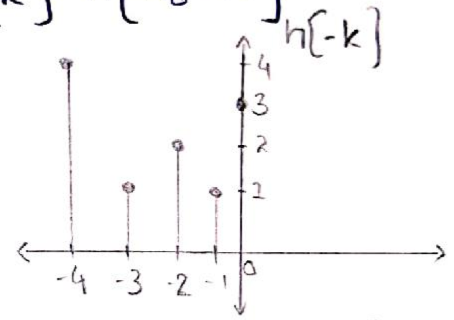
$$y[n] = \sum_{n=-\alpha}^{\alpha} x[n]\, h[n]$$

Replace n with k

$$= \sum_{k=-\alpha}^{\alpha} x[k]\, h[-k]$$

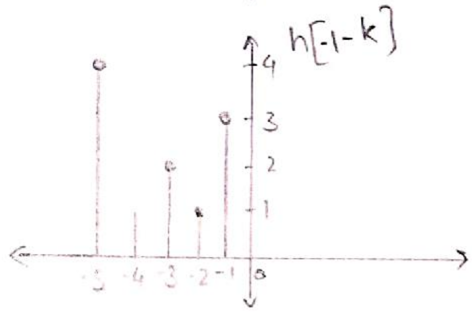
Now, we introduce shifting of n_o in $h[k]$

$$y[n_o] = \sum_{k=-\alpha}^{\alpha} x[k]\, h[n_o - k]$$



For $n_o = 0$

$y[0] = 2 + 3 = 5$



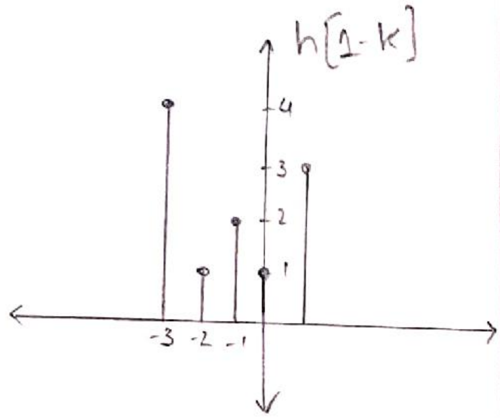
For $n_o = -1$

$y[-1] = 6$

For $n_o \leq -2$

$y[n] = 0$

For $n_o = 1$



$y[1] = -6 + 1 + 4 = -1$

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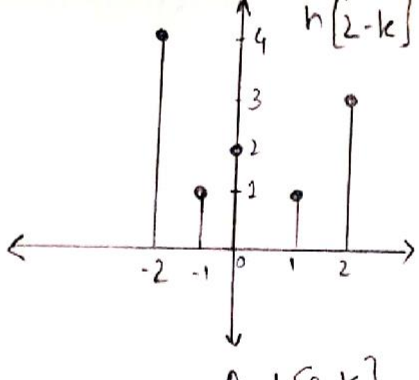
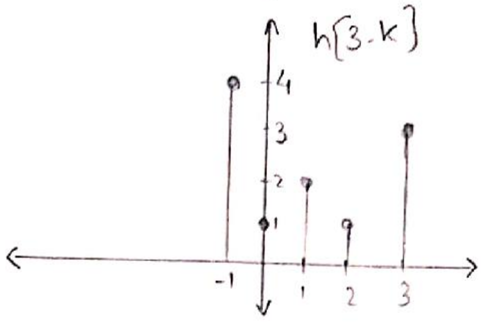
(9)

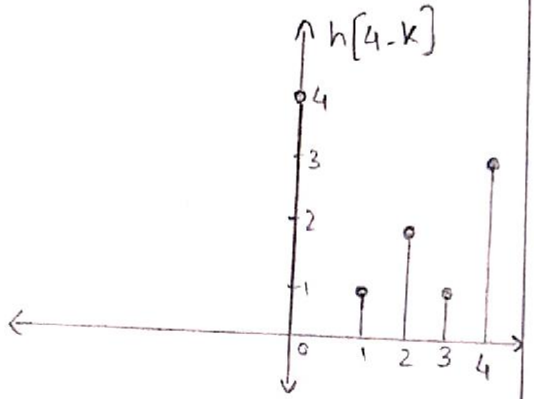
$y[2] = 2+2-2+9 = 11$



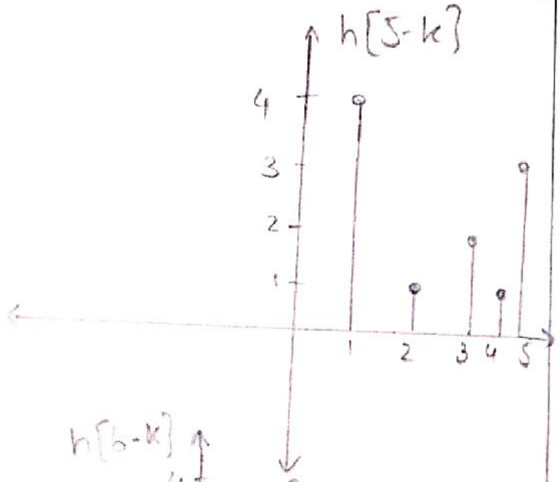
$y[3] = 8+1-4+3-12 = -4$



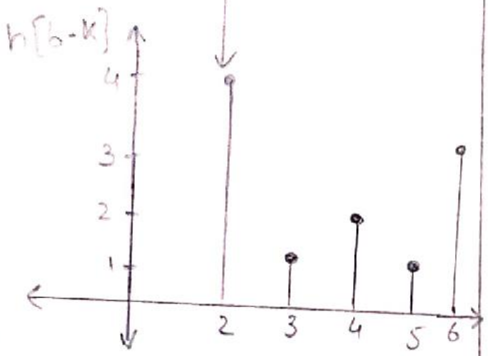
$y[4] = 4 + 6 - 4 - 2 = +4$



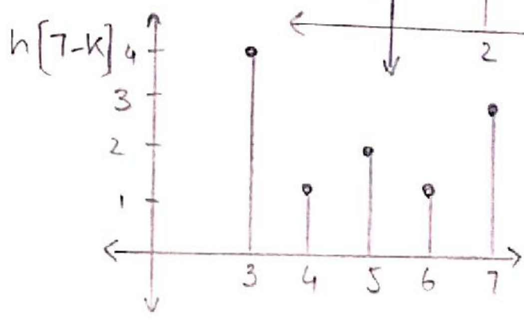
$y[5] = -8+3-8 = -13$



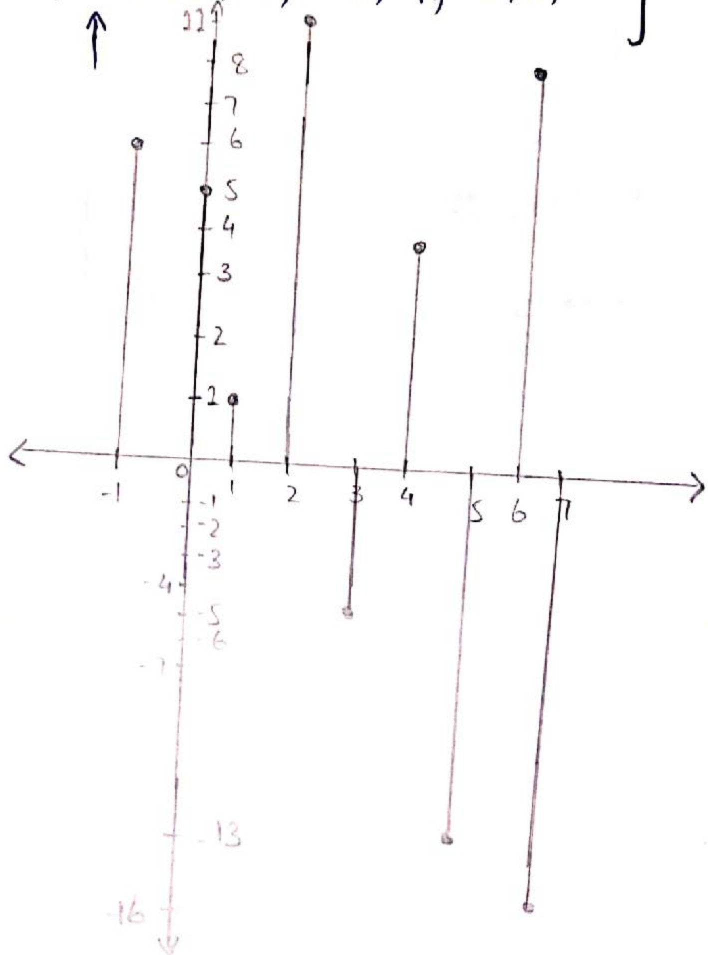
$y[6] = 12-4 = 8$



$y[7] = -16$



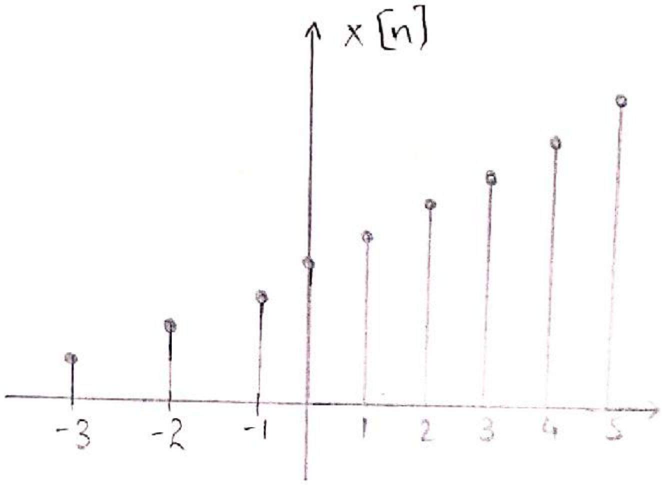
$$y[n] = \{6, \underset{\uparrow}{5}, 1, 11, -4, 4, -13, 8, -16\}$$

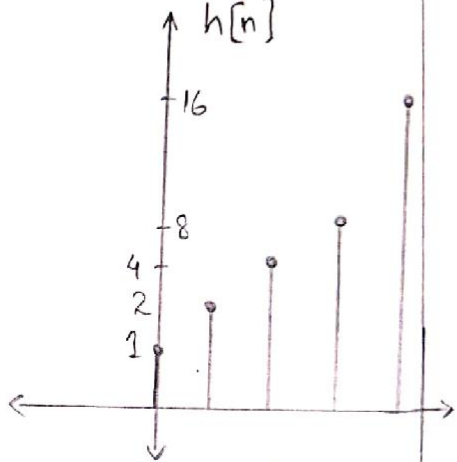


(b) Compute the convolution of signal:-

$$x[n] = \begin{cases} a^{n+1} & , -3 \leq n \leq 5 \\ 0 & , \text{elsewhere} \end{cases}$$

$$h[n] = \begin{cases} 2^{n} & , 0 \leq n \leq 4 \\ 0 & , \text{elsewhere} \end{cases}$$

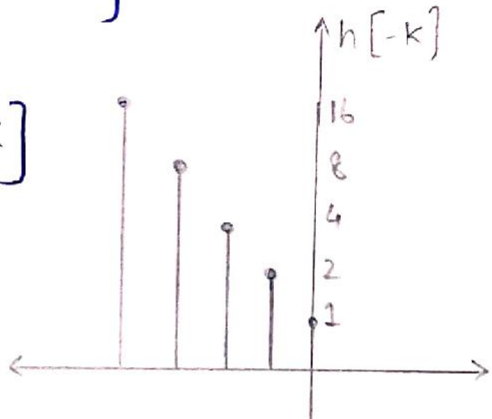


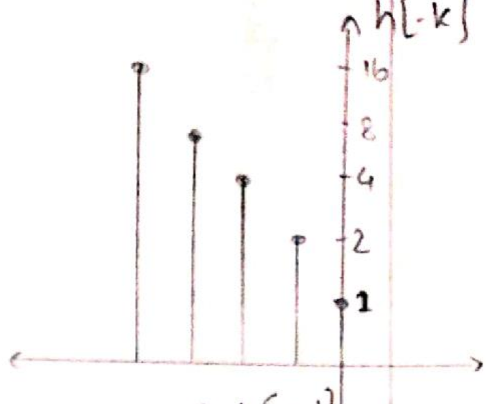


$$x[n] = \left\{ a^{-2}, a^{-1}, 1, \underset{\uparrow}{a}, a^{2}, a^{3}, a^{4}, a^{5}, a^{6} \right\}$$

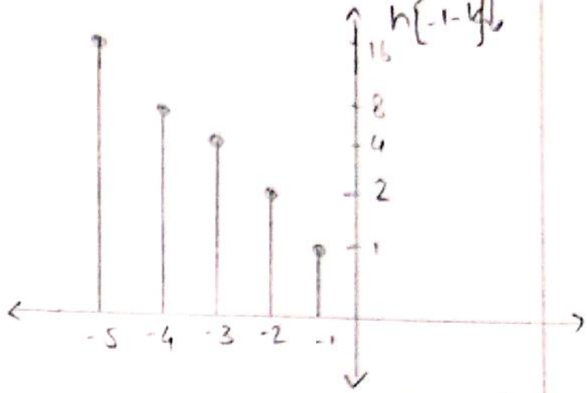
$$h[n] = \left\{ \underset{\uparrow}{1}, 2, 4, 8, 16 \right\}$$

$$y[n_0] = \sum_{k=-\infty}^{\infty} x[k]\, h[n_0 - k]$$

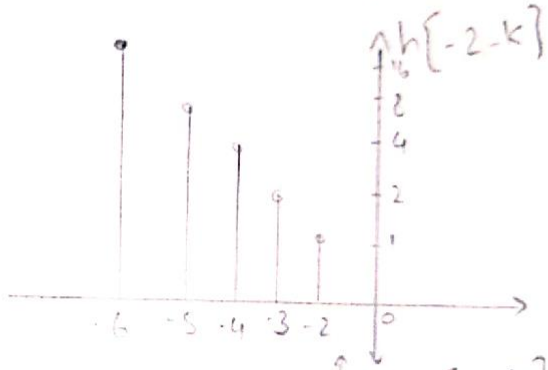




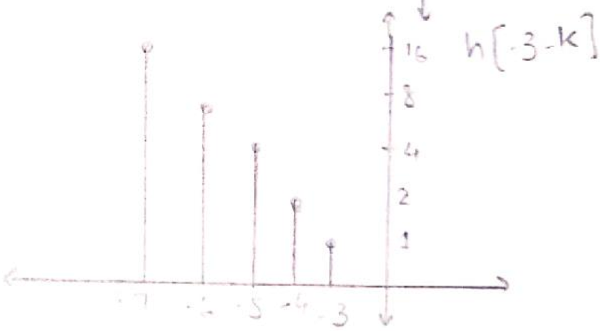
$$y[0] = a - 2 + 4a^{-1} + 8a^{-2}$$



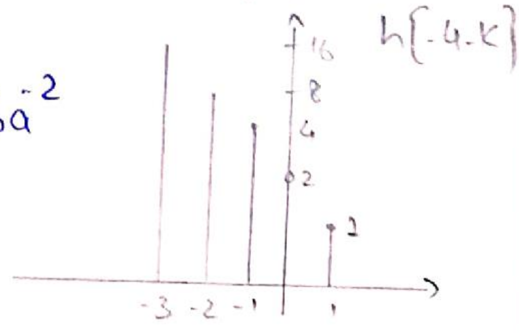
$$y[-1] = 1 + 2a^{-1} + 4a^{-2}$$



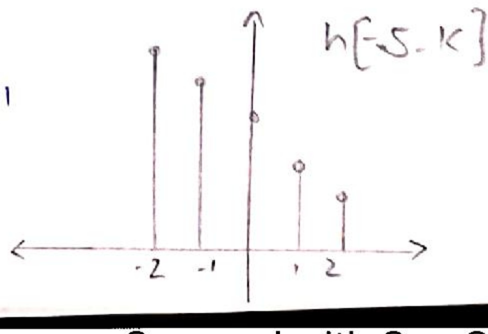
$$y[-2] = 2a^{-2} + a^{-1}$$



$$y[-3] = a^{-2}$$

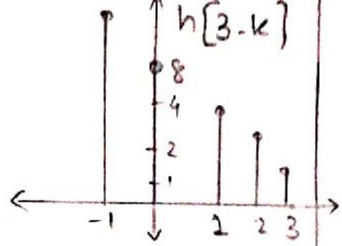


$$y[1] = a^{2} + 2a + 4 + 8a^{-1} + 16a^{-2}$$

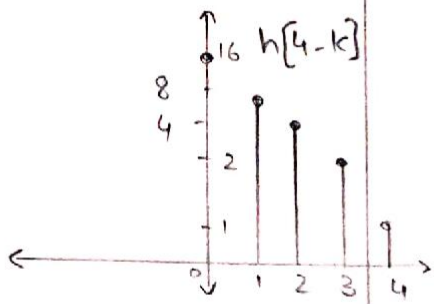


$$y[2] = a^{3} + 2a^{2} + 4a + 8 + 16a^{-1}$$

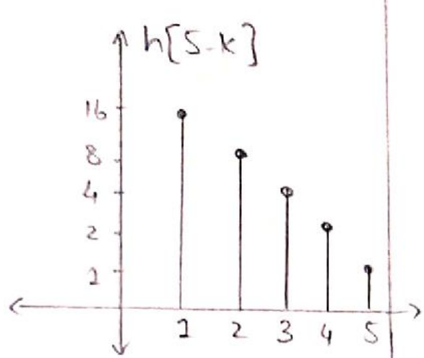
$$y[3] = a^4 + 2a^3 + 4a^2 + 8a + 16$$



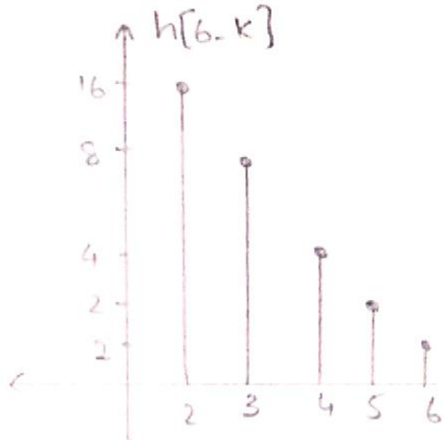
$$y[4] = a^5 + 2a^4 + 4a^3 + 8a^2 + 16a$$



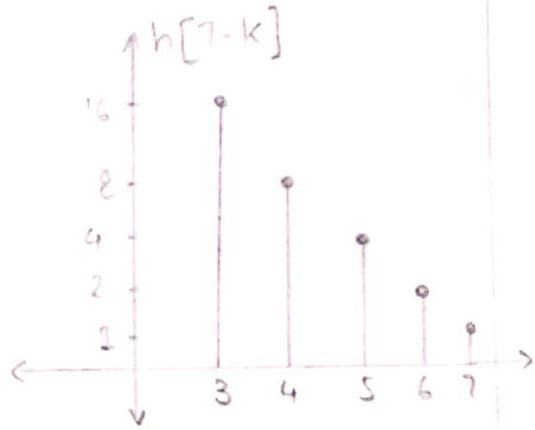
$$y[5] = 16a^2 + 8a^3 + 4a^4 + 2a^5 + a^6$$



$$y[6] = 16a^3 + 8a^4 + 4a^5 + 2a^6$$



$$y[7] = 16a^4 + 8a^5 + 4a^6$$



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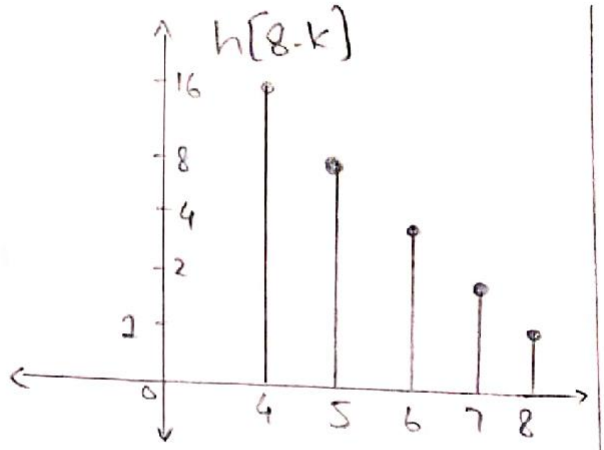
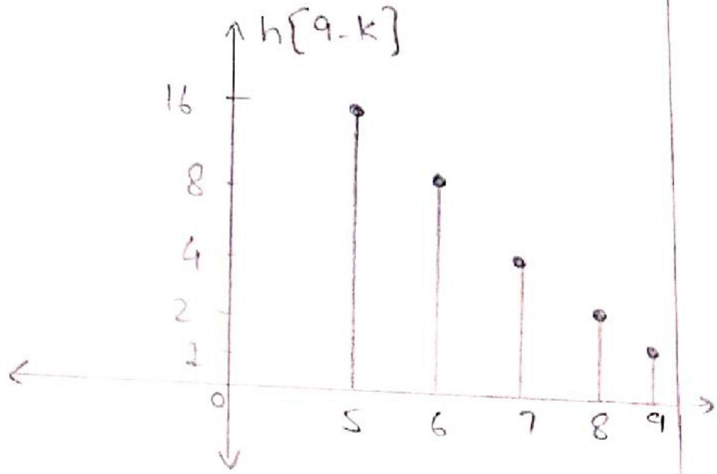
$$y[8] = 16a^5 + 8a^6$$



$$y[9] = 16a^6$$



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Qno 3:-

Determine the region of convergence and z-transform of the following signal:-

(i)
$$x(n) = \begin{cases} \left(\frac{1}{4}\right)^n , & n \geq 0 \\ \left(\frac{1}{3}\right)^{-n} , & n < 0 \end{cases}$$

$$x_1(n) = \left(\frac{1}{4}\right)^n$$

$$X_1(z) = \sum_{n=-\infty}^{\infty} x[n] \, z^{-n}$$

$$x_1[z] = \sum_{n=-\infty}^{\infty} \left[\frac{1}{4}\right]^n z^{-n}$$

$$x_1[z] = \frac{1}{1-\frac{1}{4}z^{-1}} \qquad |ROC| = |z| > \frac{1}{4}$$

$$x_2(n) = \left(\frac{1}{3}\right)^{-n}$$

$$x_2(z) = \sum_{n=-\infty}^{\infty} \left(\frac{1}{3}\right)^{-n} z^{-n} \quad , \; n<0$$

$$x_2[z] = \frac{1}{1-\frac{1}{3}z} \qquad ROC = |z| > 3$$

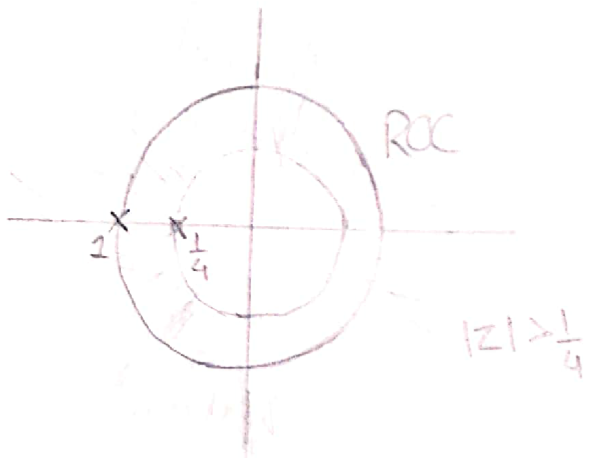
$$x[z] = x_1(z) + x_2(z)$$

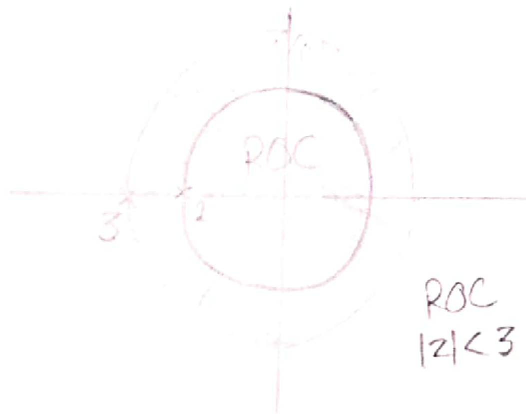
$$x[z] = \frac{z}{\left(z - \frac{1}{4}\right)} + \frac{1}{\left(1 - \frac{1}{3}z\right)} - 1$$

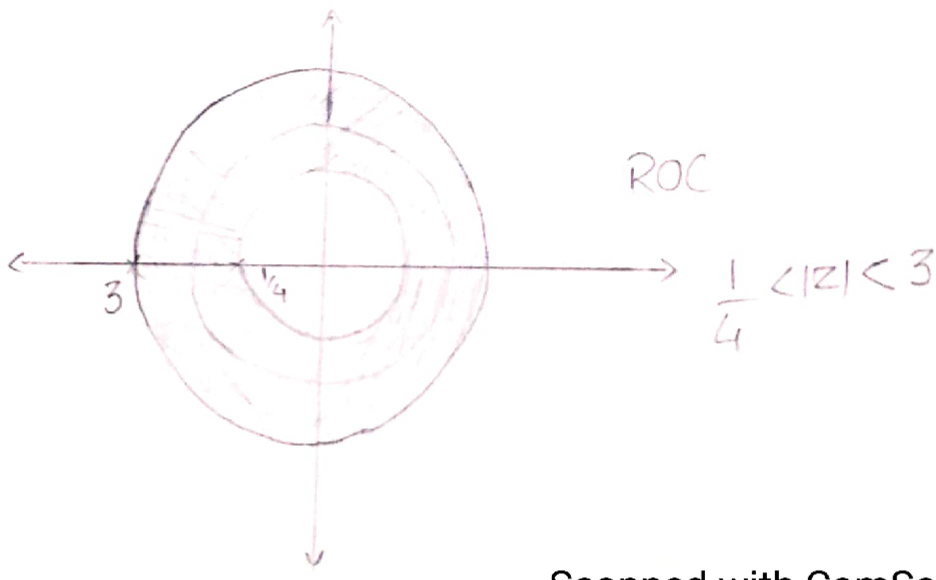
$$= \frac{z - \frac{1}{3}z^2 + z - \frac{1}{4} - z + \frac{1}{3}z^2 + \frac{1}{4} - \frac{1}{12}z}{\left(z - \frac{1}{4}\right)\left(1 - \frac{1}{3}z\right)}$$

$$x[z] = \frac{z - \frac{1}{12}z}{\left(z - \frac{1}{4}\right)\left(1 - \frac{1}{3}z\right)}$$

$$\text{ROC} = \frac{1}{4} < |z| < 3$$







(ii)

$$x(n) = \begin{cases} \left(\frac{1}{2}\right)^n - 3^n & , n \geq 0 \\ 0 & , \text{elsewhere} \end{cases}$$

$$X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n}$$

$$= \sum_{n=-\infty}^{\infty} \left[\left(\frac{1}{2}\right)^n - 3^n\right] z^{-n}$$

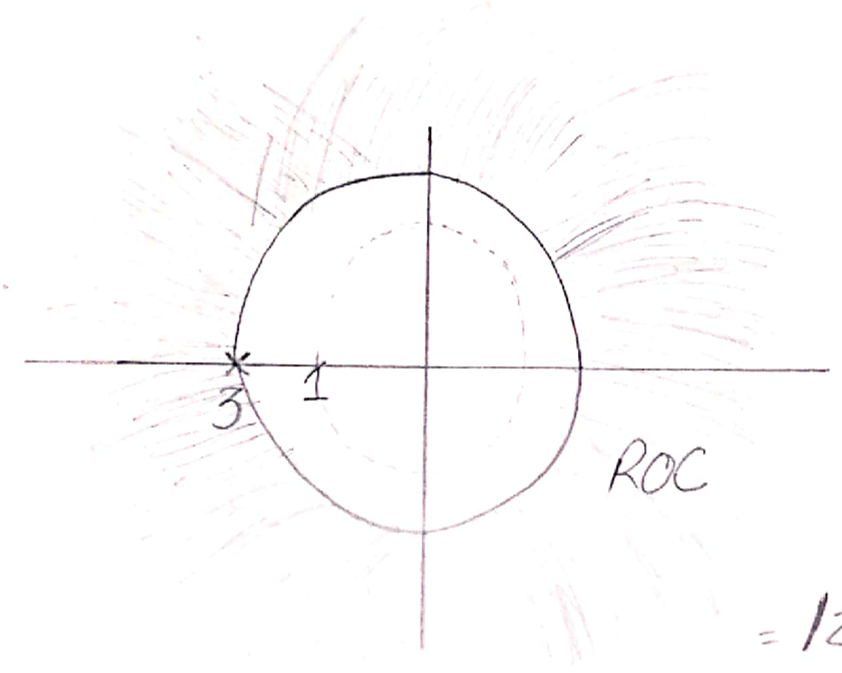
$$= \sum_{n=-\infty}^{\infty} \left(\frac{1}{2}\right)^n z^{-n} - \sum_{n=-\infty}^{\infty} 3^n z^{-n}$$

$$= \frac{z}{z-\frac{1}{2}} - \frac{z}{z-3}$$

$$= \frac{z^2 - 3z - z^2 + \frac{1}{2}z}{\left(z-\frac{1}{2}\right)(z-3)}$$

$$= \frac{-5/2\, z}{\left(z-\frac{1}{2}\right)(z-3)}$$

ROC $= |z| > 3$



$= |z| > 3$