Date: 13-April-2020

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SUBJECT : DISCRETE STRUCTURES
CLASS TIMING : TUESDAY & 8:00 — 11:00
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DEPARTMENT : BS (CS-2)

Q1: (A) Let p be the statement "DATAENDFLAG is off", q the statement "ERROR equals 0", and r the statement "SUM is less than 1,000". Express the following sentences in symbolic notation.

a) DATAENDFLAG is off, ERROR equals 0, and SUM is less than 1,000.

Solution:-

(a) p = "DATAENDFLAG is off".
q = "ERROR equals 0".
r = "SUM is less than 1,000"
We can then rewrite the given sentence as:
"DATAENDFLAG is off, and ERROR equals 0 and SUM is less than 1,000".
Replacing the statements by p, q and r and replacing "and" by $\wedge$, we then obtain:

$$p \wedge (q \wedge r)$$

b.

b. DATAENDFLAG is off but ERROR is not equal to 0.

(b)

p = "DATAENDFLAG is off"
q = "ERROR equals 0"
r = "SUM is less than 1,000"

Note that the word "but" in the given sentence actually implies "and", thus we can then rewrite the given sentence as:

"DATAENDFLAG is off and ERROR does not equal 0".

Replacing the statements by p, q and r, replacing "and" by $\wedge$ and replacing "not" by $\sim$, we then obtain:

$p \wedge \sim q$.

. DATAENDFLAG is off; however ERROR is not 0 or SUM is greater than or equal to 1,000.

p = "DATAENDFLAG is off"
q = "ERROR equals 0"
r = "SUM is less than 1,000".

We can then rewrite the given sentence as:

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"DATAENDFLAG is off and, ERROR does not 0 or SUM is not less than 1,000"

Replacing the statements by p, q and r, replacing "and" by ∧, replacing "or" by ∨ and replacing "not" by ~, we then obtain:

$p \wedge (\sim q \vee \sim r)$

d. DATAENDFLAG is on and ERROR equals 0 but SUM is greater than or equal to 1,000.

(d)

p = "DATAENDFLAG is off"

q = "ERROR equals 0"

r = "SUM is less than 1,000"

Note that the word "but" in the given sentence actually implies "and", thus we can then rewrite the given sentence as:

"DATAENDFLAG is not off and ERROR equals 0, and SUM is not less than 1,000"

Replacing the statements by p, q and r, replacing "and" by ∧ and replacing "not" by ~, we then obtain:

$(\sim p \wedge q) \wedge \sim r$.

e. Either DATAENDFLAG is on or it is the case that both ERROR equals 0 and SUM is less than 1,000.

(e)

p = "DATAENDFLAG is off"
q = "ERROR equals 0"
r = "SUM is less than 1,000"

We can then rewrite the given sentence as:

"DATAENDFLAG is not off or, ERROR equals 0 and SUM is less than 1,000".

Replacing the statements by p, q and r, replacing "and" by $\wedge$, replacing "or" by $\vee$ and replacing "not" by $\sim$, we then obtain:

$$\sim p \vee (q \wedge r).$$

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Q1(b) Show that $p \vee q \to r \equiv (p \to r) \wedge (q \to r)$.

Ans.

So, from column (5) and (8), all values are same. So hence, proved, that $p \vee q \to r \equiv (p \to r) \wedge (q \to r)$.

L.H.S = R.H.S.

p	q	r	$p \vee q$	$p \vee q \to r$	$p \to r$	$q \to r$	$(p \to r) \wedge (q \to r)$
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	F
T	F	T	T	T	T	T	T
T	F	F	T	F	F	T	F
F	T	T	T	T	T	T	T
F	T	F	T	F	T	F	F
F	F	T	F	T	T	T	T
F	F	F	F	T	T	T	T

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Q2 (A)

Write the Converse, Inverse, and Contrapositive of the following:-

(a)

If Howard can swim across the lake, the Howard can swim to the island.

Ans. **CONVERSE:** If Howard can swim to the island, then Howard can swim across the lake.

INVERSE: If Howard cannot swim across the lake, then Howard cannot swim to the island.

CONTRAPOSITIVE: If Howard cannot swim to the the island, then Howard cannot swim across the lake.

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(b) If today is Easter, then tomorrow is Monday.

Ans

CONVERSE:

If tomorrow is Monday, then today is Easter.

INVERSE:

If today is not Easter, then tomorrow is not Monday.

CONTRAPOSITIVE:-

If tomorrow is not Monday, then today is not Easter.

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Q2

B) Use truth tables to determine whether the argument forms are valid. Indicate which columns represent the premises and which represent the conclusion.

(a) p

$p \rightarrow q$

$\sim q \vee r$

$\therefore r$

p	q	r	$\sim q$	$p \rightarrow q$	$\sim q \vee r$	
T	T	T	F	T	T	Premise
T	T	F	F	T	F	Premise
T	F	T	T	F	T	Premise
T	F	F	T	F	F	
F	T	T	F	T	T	
F	T	F	F	T	F	
F	F	T	T	T	T	
F	F	F	T	T	T	Conclusion

Ans The first row is the only row in which all Premises are 'True' and the conclusion is also 'true' there. There is no other raw where premises are all 'true'. So this argument is 'Valid'.

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b) $p \wedge q \rightarrow \neg r$
$p \vee \neg q$
$\neg q \rightarrow p$
$\therefore \neg r$

In the third row, all Premises are 'True' but conclusion is 'False'. So this argument is 'invalid'.

p	q	r	$\neg r$	$p \wedge q$	$p \wedge q \rightarrow \neg r$	$p \vee \neg q$	$\neg q \rightarrow p$
T	T	T	F	T	F	T	T
T	T	F	T	T	T	T	T
T	F	T	F	F	T	T	T
T	F	F	T	F	T	T	T
F	T	T	F	F	T	F	T
F	T	F	T	F	T	F	T
F	F	T	F	F	T	T	F
F	F	F	T	F	T	T	F

Premise - 1
Premise - 2
Premise - 3
Conclusion

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Q3: In the back of an old cupboard you discover a note signed by a pirate famous for his bizzarre sense of humor and love of logical puzzles. In the note he wrote that he had hidden treasure somewhere on the property. He listed five true statements (a-e below) and challenged the reader to use them to figure out the location of the treasure.

(a) If this house is next to lake, then the treasure is not in the kitchen.

(b) If the tree in the front yard is an elm, then the treasure is in the kitchen.

(c) This house is next to lake.

(d) The tree in the front yard is an elm or the treasure is buried under the flagpole.

(e) If the tree in the back yard is an oak, then the treasure is in the garage.

Where is the treasure hidden?

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Ans.

SOLUTION:-

Assuming,

P = This house is next to a lake.
q = The treasure is in the kitchen.
r = The tree in the front yard is an elm.
s = The treasure is buried under the flagpole.
t = The tree in the back yard is an oak.
u = The treasure is in the garage.

So,

1. $P \rightarrow \neg q$ = Premise
2. $r \rightarrow q$ = Premise.
3. P = Premise.
4. $r \vee s$ = Premise.
5. $t \rightarrow u$ = Premise
6. $\neg q$ = Modus Ponens from (1) and (3).
7. $\neg r$ = Modus tollens from (2) and (6).
8. s = Elimination of (1) and (7).

So step '8' meaning 's' is true.
So the treasure is buried under the flagpole.