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Program B (c)

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Subject Differintail Equation

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Q1

Ans

1) Solution

Order of $A = m \times p$
Order of $B = p \times N$

So order of $A \times B = m \times n$

ii)

The number of non-zero rows in echelon form is called rank of the matrix

for eg

$$A = \begin{bmatrix} 1 & 0 & -2 & 5 & 3 \\ 0 & 0 & 1 & -4 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

since A is in row-reduced form
Since it contain three non zero rows, its row rank is three

iii) If $B = \begin{bmatrix} 1 & 4 \\ 2 & a \end{bmatrix}$ is a singulr Matrix the $a = ?$

Solution

we know that for a singulr Matrix

$|B| = 0$

so

$$|B| = \begin{vmatrix} 1 & 4 \\ 2 & a \end{vmatrix} = 0$$

$$\Rightarrow \overset{a}{\cancel{|B|}} = 1 \times a - 2 \times 4 = 0$$

$$\Rightarrow \quad a - 8 = 0$$

$$\Rightarrow \quad a = 8$$

IV) If $A = \begin{bmatrix} 2i & i \\ i & -i \end{bmatrix}$ then $|A| = ?$

Sol

take Moduls of Matrix A.

$$|A| = \begin{vmatrix} 2i & i \\ i & -i \end{vmatrix}$$

$$|A| = 2i \times (-i) - i \times i \qquad \therefore (i^2 = -1)$$

$$|A| = -2i^2 - i^2$$

$$|A| = -2(-1) - (-1)$$

$$|A| = 2 + 1$$

$$|A| = \boxed{3} \text{ Ans}$$

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V) The Matrix $A = \begin{bmatrix} 9 & 0 \\ 0 & 9 \end{bmatrix}$ is ?

Sol :-

$A = \begin{bmatrix} 9 & 0 \\ 0 & 9 \end{bmatrix}$

Here in matrix diagnol element is same i-e 9. so it is scaler matrix

VI) $dy/dx + 2xy = y$

Sol

$dy/dx + 2xy - y = 0$

$\Rightarrow dy/dx + y(2x-1) = 0$

$\Rightarrow dy/dx = -y(2x-1)$

$\Rightarrow dy/y = -\cancel{2}(2x-1)\,dx$ integrate

$\Rightarrow \ln y = \frac{\cancel{2}x^2}{\cancel{2}} - x + c$

$\Rightarrow \ln y + x^2 + x + C = 0$

$\Rightarrow y + e^{x(x+1)} + c = 0$

VII) Sol

first order third Degree

VIII) Sol

2nd order first Degree

IX) The differential equation $2\,dy/dx + x^2y = 2x+3$

$y = (0) = 5$ is

Sol $2y' + x^2y = x^2 + 3,\ y(0) = 5$

$y' + \left(\frac{x^2}{2}\right)y = \frac{x^2+3}{2}$

$y' + \left(\frac{x^2}{2}\right)y = \frac{1}{2}(x^2+3)$

$\mu = \frac{x^2}{2}$

$e^{\int x^2/2\,dx} = e^{x^3/6}$

$e^{x^3/6}\ y' + e^{x^3/6}\ (x^2/2)y = \frac{1}{2}e^{x^3/6}\ (x^3+3)$

$y(x) = \frac{e^{x^3/6}\ x^2 + 3e^{x^3/6}}{2e^{x^3/6}} + C$

$y(0) = \frac{0+3}{2} = 3/2$

$y(x) = \frac{e^{x^3/6}\ x^2 + 3e^{x^3/6}}{2e^{x^3/6}} + \boxed{3/2}$ Ans

X $\begin{bmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{bmatrix}$ Expand by c1

$\Rightarrow 1\begin{bmatrix} b & b^2 \\ c & c^2 \end{bmatrix} - 1\begin{bmatrix} a & a^2 \\ c & c^2 \end{bmatrix} + 1\begin{bmatrix} a & a^2 \\ b & b^2 \end{bmatrix}$

$\Rightarrow 1(bc^2 - cb^2] - 1(ac^2 + a^2c) + 1(ab^2 - a^2b)$

$= bc^2 - cb^2 - ac^2 - a^2c + ab^2 - a^2b$

$= ab^2 - cb^2 + a^2c - a^2b - ac^2 + bc^2$

$= a^2c - a^2b + ab^2 - cb^2 + bc^2 - ac^2$

$= a^2(c-b) + b^2(a-c) + c^2(b-a)$ Ans

Q2 Part A

Express The Determine

$$\begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \end{vmatrix}$$

Ans

as the Product of factor's Which are linear in a, b, c

Sol

Expand by R_1

$$a = \begin{vmatrix} b^2 & c^2 \\ b^3 & c^3 \end{vmatrix} - b \begin{vmatrix} a^2 & c^2 \\ a^3 & c^3 \end{vmatrix} + c \begin{vmatrix} a^2 \\ a^3 \end{vmatrix}$$

$\Rightarrow a(b^2c^3 - b^3c^2) - b(a^2c^3 - a^3c^2)$

$+ \quad c(a^2b^3 - a^3b^3)$

$\Rightarrow ab^2c^3 - ab^3c^2 - a^2bc^3 - a^3bc^3 + a^2cb^3 - a^3b^2c$

taking ABC as Common

$\Rightarrow Abc\,(bc^2 - b^2c - ac^2 - a^2c + ab^2 - a^2b)$

$\Rightarrow Abc\,|bc(c-b) - ac(c-a) + ab(b-a)|$ Ans

Q2 Part B

Ans
$$\begin{bmatrix} 2 & -1 & -1 & 0 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ 0 & -1 & -1 & 2 \end{bmatrix}$$

Sol

characteristic eq $|A - \lambda I| = 0 \longrightarrow ①$

$$\begin{bmatrix} 2 & -1 & -1 & 0 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ 0 & -1 & -1 & 2 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Now we will take determent

$|A - \lambda I| = 0$

$$\begin{vmatrix} 2-\lambda & -1 & -1 & 0 \\ -1 & 3-\lambda & -1 & -1 \\ -1 & -1 & 3-\lambda & -1 \\ 0 & -1 & -1 & 2\lambda \end{vmatrix} = 0$$

Expand by R_1

$$2-\lambda \begin{vmatrix} 3-\lambda & -1 & -1 \\ -1 & 3-\lambda & -1 \\ -1 & -1 & 2-\lambda \end{vmatrix} - (-1) \begin{vmatrix} -1 & -1 & -1 \\ -1 & 3-\lambda & -1 \\ 0 & -1 & 3-\lambda \end{vmatrix}$$

$$-1 \begin{vmatrix} -1 & 3-\lambda & -1 \\ -1 & -1 & -1 \\ 0 & -1 & 2-\lambda \end{vmatrix} = 0 \longrightarrow ②$$

again

$$\begin{vmatrix} 3-\lambda & -1 & -1 \\ -1 & 3-\lambda & -1 \\ -1 & -1 & 2-\lambda \end{vmatrix}$$

Expand by R1

$$\Rightarrow 3-\lambda \begin{vmatrix} 3-\lambda & -1 \\ -1 & 2-\lambda \end{vmatrix} -(-1) \begin{vmatrix} -1 & -1 \\ -1 & 2-\lambda \end{vmatrix} -1 \begin{vmatrix} -1 & 3-\lambda \\ -1 & -1 \end{vmatrix}$$

$$= (3-\lambda) \Big| \big((3-\lambda)(2-\lambda) - (-1)(-1)\big) + 1\big((-1)(2-\lambda) - (-1)(-1)\big) - 1\big((-1)(-1) - (-1)(3-\lambda)\big) \Big|$$

$$\Rightarrow (3-\lambda)(6 - 3\lambda - 2\lambda + \lambda^2 - 1) + (2 + \lambda - 1) - (+1 + 3 - \lambda)$$

$$\Rightarrow (3-\lambda)(\lambda^2 - 5\lambda + 5) + (-3\lambda) - (4-\lambda)$$

$$\Rightarrow 3\lambda^2 - 15\lambda + 15 - \lambda^2 + 5\lambda^2 - 5\lambda - 3 + \lambda - 4 + \lambda$$

$$= \boxed{-\lambda^3 + 8\lambda^2 - 18\lambda + 8} \longrightarrow (1)$$

$$\Rightarrow +1\begin{vmatrix} -1 & -1 & -1 \\ -1 & 3-\lambda & -1 \\ 0 & -1 & 2-\lambda \end{vmatrix}$$

Expand by C_1

$$\Rightarrow -1\begin{vmatrix} 3-\lambda & -1 \\ -1 & 2-\lambda \end{vmatrix} - (-1)\begin{vmatrix} -1 & -1 \\ -1 & 2-\lambda \end{vmatrix} + 0$$

$$\Rightarrow -1(6-3\lambda-2\lambda+\lambda^2-1)+1(-2+\lambda-1)$$

$$\Rightarrow \boxed{-\lambda^2+6\lambda-8} \text{ ——— } (2)$$

$$= -1\begin{vmatrix} -1 & 3-\lambda & -1 \\ -1 & -1 & -1 \\ 0 & -1 & 2-\lambda \end{vmatrix}$$

Expand by C_2

$$-\left[-1\begin{vmatrix} -1 & -1 \\ -1 & 2-\lambda \end{vmatrix} - (-1)\begin{vmatrix} 3-\lambda & -1 \\ -1 & 2-\lambda \end{vmatrix} + 0\right]$$

$$\Rightarrow -\left[-(-2+\lambda-1)+1(6-3\lambda-2\lambda^2-1)\right.$$

$= -(3-\lambda+\lambda^2-5\lambda+5)$

$= \lambda^2+5\lambda-5-3+\lambda$

$= \boxed{-\lambda^2+6\lambda-8}$ — c

Put ⓐ ⓑ and c in Ⓑ

$(2-\lambda)\left[-\lambda^3+3\lambda^2-18\lambda+8\right]-\lambda^2+6\lambda-8-\lambda^2+6\lambda-8$

$= -2\lambda^3+16\lambda^2-36\lambda+\lambda^4-8\lambda^3+16\lambda^2-8\lambda-\lambda^2+6\lambda$
$-8-\lambda^2+16\lambda-8$

$\Rightarrow \lambda^4-2\lambda^3-8\lambda^3+16\lambda^2+16\lambda^2-\lambda^2-\lambda^2-36\lambda$
$-8\lambda+6\lambda+6\lambda+16-16$

$\Rightarrow \lambda^4-10\lambda^3+32\lambda^2-32\lambda=0$

By synthatic division

We know that

$\lambda(\lambda-2)(\lambda^2-8\lambda+16)=0$

$(\lambda=0)$

$\lambda-2=0$

$\Rightarrow \lambda=2$

$\lambda^2-8\lambda+16=0$

By factrazation method

$\lambda^2-4\lambda-4\lambda+16=0$

$\lambda(\lambda-4)-4(\lambda-4)=0$

$(\lambda-4)(\lambda-4)$

$\lambda=4,\ \lambda=4$

$\lambda=0,\ \lambda_2=2,\ \lambda_3=3,\ \lambda_4=4$ Ans

Q3 $(x^2+3y^2)\,dx - 2xy\,dy = 0$

$x = 2,\ y = 6$

sol

$(x^2+3y^2)\,dx - 2xy\,dy = 0$

$\Rightarrow (x^2+3y)\,dx = 2xy\,dy$

Dividing both side by $2xy\,dx$ we will get

$$dy/dx = \frac{x^2+3y^2}{2xy}$$

$\Rightarrow dy/dx = \cancel{x^2}/\cancel{2}xy + 3y\cancel{^2}/2x\cancel{y}$

$\Rightarrow dy/dx = 1/2\left[x/y + 3y/x\right] \longrightarrow 1$

$\Rightarrow$ let $y = vx$

Diff $dy = v\,dx + x\,dv$

dividing by dx

$dy/dx = v + dv/dx \longrightarrow (a)$

Put (a) in (1)

$v + x\,dv/dx - 1/2$

$$\left[\frac{\cancel{x}}{\cancel{x}v} + 3 \quad \frac{v\cancel{x}}{\cancel{x}} \right]$$

$$v + \frac{x\,dv}{dx} = \frac{1}{2}\left[\frac{1}{v} + 3v\right]$$

Multiflying both side by '2'

we know that

$$2\cancel{v} + 2x\,dv/dx = 1/v + 3v$$

$$2x\,dv/dx = 1/v + 3v - 2v$$

$$2x\,dv/dx = 1/v + v$$

$$2x\,dv/dx = \frac{1+v^2}{v}$$

Multiflying both side by dx/dv

we know that

$$2x\,dv = \frac{1+v^2}{v}\,dx$$

Multify both side $\frac{v}{x(1+v^2)}$

we know that

$$\frac{v}{1+v^2}\,dv = 1/\cancel{x}\,dx$$

take "$\int$" on both side

$$\int \frac{2v}{1+v^2} \, dv = \int \frac{1}{x} \, dx + c$$

$$\ln |1+v^2| = \ln x + \ln c$$

Take "e" on both sides

$$e^{\ln |1+v^2|} = e^{\ln |xc|}$$

$$1+v^2 = xc$$

$$1 + v^2 = xc$$

Put $v = y/x$

$$1 + (y/x)^2 = xc$$

$$\frac{x^2+y^2}{x^2} = xc$$

$$x^2+y^2 = x^3 c \longrightarrow (2)$$

Put $x=2$, $y=6$ in eq (2)

$$(4)+(36) = 8c$$

$$c = 40/8$$

$c = 5$ Put in eq(2)

$$x^2+y^2 = 5x^3$$

$$y^2 = 5x^3 - x^2$$

$$y^2 = x^2(5x-1)$$

taking "$\sqrt{}$" on both sides

$$y = +x\sqrt{5x-1}, \quad y = -x\sqrt{5x-1}$$

$$y = \pm x\sqrt{5x-1}$$