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Compute adjoint of;

(i) $A = \begin{bmatrix} 1 & 2 & \text{2nd. ID} \\ 2 & 3 & 1 \\ 3 & 1 & 2 \end{bmatrix}$

$$A = \begin{bmatrix} 1 & 2 & 6 \\ 2 & 3 & 1 \\ 3 & 1 & 2 \end{bmatrix}$$

First of all we find cofactors of "A":

$$A_{ij} = (-1)^{i+j} \cdot m_{ij}$$

$$A_{11} = (-1)^{1+1} \begin{vmatrix} 3 & 1 \\ 1 & 2 \end{vmatrix} = +1(6-1) = \boxed{5}$$

$$A_{12} = (-1)^{1+2} \begin{vmatrix} 2 & 1 \\ 3 & 2 \end{vmatrix} = -1(4-3) = \boxed{-1}$$

$$A_{13} = (-1)^{1+3} \begin{vmatrix} 2 & 3 \\ 3 & 1 \end{vmatrix} = +1(2-9) = \boxed{-7}$$

$$A_{21} = (-1)^{2+1} \begin{vmatrix} 2 & 6 \\ 1 & 2 \end{vmatrix} = -1(4-6) = \boxed{2}$$

$$A_{22} = (-1)^{2+2} \begin{vmatrix} 1 & 6 \\ 3 & 2 \end{vmatrix} = 1(2-18) = \boxed{-16}$$

$$A_{23} = (-1)^{2+3} \begin{vmatrix} 1 & 2 \\ 3 & 1 \end{vmatrix} = -1(1-6) = \boxed{5}$$

$A_{31} = (-1)^{3+1} \begin{vmatrix} 2 & 6 \\ 3 & 1 \end{vmatrix} = 1(2-18) = \boxed{-16}$

$A_{32} = (-1)^{3+2} \begin{vmatrix} 1 & 6 \\ 2 & 1 \end{vmatrix} = -1(1-12) = \boxed{11}$

$A_{33} = (-1)^{3+3} \begin{vmatrix} 1 & 2 \\ 2 & 3 \end{vmatrix} = 1(3-4) = \boxed{-1}$

Now find adjoint.

$$A = \begin{bmatrix} 5 & -1 & -7 \\ 2 & -16 & 5 \\ -16 & 11 & -1 \end{bmatrix}$$

Tating transpose.

$$A = \begin{bmatrix} 5 & -1 & -7 \\ 2 & -16 & 5 \\ -16 & 11 & -1 \end{bmatrix}$$

$$A^t = \begin{bmatrix} 5 & 2 & -16 \\ -1 & -16 & 11 \\ -7 & 5 & -1 \end{bmatrix}$$

This is adjoint of "A".

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$$B = \begin{vmatrix} 3 & 4 & 5 \\ 2 & -1 & 8 \\ 5 & -2 & 8 \end{vmatrix} \rightarrow \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix} \rightarrow \begin{bmatrix} + & - & + \\ - & + & - \\ + & - & + \end{bmatrix}$$

$A_{11} = \begin{vmatrix} -1 & 8 \\ -2 & 8 \end{vmatrix} = -8 + 16 = 8$

$A_{12} = \begin{vmatrix} 2 & 8 \\ 5 & 8 \end{vmatrix} = 16 - 40 = -24$

$A_{13} = \begin{vmatrix} 2 & -1 \\ 5 & -2 \end{vmatrix} = -4 + 5 = 1$

$A_{21} = \begin{vmatrix} 4 & 5 \\ -2 & 8 \end{vmatrix} = 32 + 10 = 42$

$A_{22} = \begin{vmatrix} 3 & 5 \\ 5 & 8 \end{vmatrix} = 24 - 25 = 1$

$A_{23} = \begin{vmatrix} 2 & -1 \\ 5 & -2 \end{vmatrix} = -4 + 5 = 1$

$A_{31} = \begin{vmatrix} 4 & 5 \\ -1 & 8 \end{vmatrix} = 32 + 5 = 37$

$A_{32} = \begin{vmatrix} 3 & 5 \\ 2 & 8 \end{vmatrix} = 24 - 10 = 14$

$A_{33} = \begin{vmatrix} 3 & 4 \\ 2 & -1 \end{vmatrix} = -3 - 8 = -11$

cofactors

$$\begin{bmatrix} 8 & 24 & 1 \\ -42 & 1 & -1 \\ 37 & -14 & -11 \end{bmatrix}$$

$$B^t \begin{bmatrix} 8 & -42 & 37 \\ 24 & 1 & -14 \\ 1 & -1 & -11 \end{bmatrix}$$

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Q2) Find co factors of A_{21}, A_{31}, A_{33}.

$$A = \begin{bmatrix} 1 & -2 & 3 \\ -2 & 3 & 1 \\ 4 & -3 & 2 \end{bmatrix} \longrightarrow \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix}$$

$$A_{21} = \begin{vmatrix} -2 & 3 \\ -3 & 2 \end{vmatrix} = -4 + 9 = 5$$

$$A_{31} = \begin{vmatrix} -2 & 3 \\ 3 & 1 \end{vmatrix} = -2 - 9 = -11$$

$$A_{33} = \begin{vmatrix} 1 & -2 \\ -2 & 3 \end{vmatrix} = 3 + 4 = 7$$

[redacted] Eigen values and Eigen vector

$$A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 3 & 2 \\ -1 & 1 & 2 \end{bmatrix} \text{ and } I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \text{ //}$$

Eigen values:

Step 1: $A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 3 & 2 \\ -1 & 1 & 2 \end{bmatrix}$

Formula: $(A - \lambda I)x = 0$

A = given matrix
λ = constant
I = identity matrix
x = unknown matrix
0 = null matrix

$$\left(\begin{bmatrix} 2 & 1 & 1 \\ 1 & 3 & 2 \\ -1 & 1 & 2 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \right.$$

Step 2: take determinate

$$|A - \lambda I| = 0$$

$$\begin{vmatrix} 2-\lambda & 1 & 1 \\ 1 & 3-\lambda & 2 \\ -1 & 1 & 2-\lambda \end{vmatrix} = 0$$

Step 3: $\lambda^3 - [\text{sum of diagonal elements}]\lambda^2 + [\text{sum of diagonal minors}]\lambda - |A| = 0$

$$\therefore \lambda^3 - 7\lambda^2 + 14\lambda - 8 = 0$$

now take root and you'll get eigen values:

$$\boxed{\lambda = 1, 2, 4}$$

$$\begin{vmatrix} 3 & 2 \\ 1 & 2 \end{vmatrix} = 6 - 2 = 4$$

$$\begin{vmatrix} 2 & 1 \\ -1 & 2 \end{vmatrix} = 4 + 1 = 5$$

$$\begin{vmatrix} 2 & 1 \\ 1 & 3 \end{vmatrix} = 6 - 1 = 5$$

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Part 2 = Eigen vectors.

put value in λ 1

$$\begin{bmatrix} 2-1 & 1 & 1 \\ 1 & 3-1 & 2 \\ -1 & 1 & 2-1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 2 \\ -1 & 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Step 1: ~~using cramer rule~~ using row echelon

$$\sim \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ -1 & 1 & 1 \end{bmatrix} \quad R_2 - (1) R_1$$

$$\sim \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 2 & 2 \end{bmatrix} \quad R_3 + (1) \cdot R_1$$

$R_2 \Longleftrightarrow R_3$

$$\sim \begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 2 \\ 0 & 1 & 1 \end{bmatrix}$$

$R_3 - \frac{1}{2} R_2$

$$\sim \begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 2 \\ 0 & 0 & 0 \end{bmatrix}$$

now Reduce echelon form

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 2 \\ 0 & 0 & 0 \end{bmatrix}$$

$R_2 \leftarrow \frac{1}{2} R_2$

$$\sim \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

concel lead co-ef

$R_1 \leftarrow R_1 - 1 R_2$

$$\sim \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

system with eigenvalue

$$(A - \lambda I)\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} x & 0 & 0 \\ 0 & y & z \\ 0 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} x \\ y+z \\ 0 \end{bmatrix} \qquad \begin{bmatrix} x = 0 \\ y = -z \end{bmatrix}$$

Values in $\eta = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$

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$\therefore \eta = \begin{bmatrix} 0 \\ -z \\ z \end{bmatrix}$; $z \neq 0$
let $z=1$

$\eta = \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix}$ eigen vector of $\lambda = 1$

$\underline{\lambda = 2}$

put 2 in λ in matrix

$= \begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 2 \\ -1 & 1 & 0 \end{bmatrix}$

to solve $(A - 2I)\eta = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 2 \\ -1 & 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{matrix} 0 \\ 0 \\ 0 \end{matrix}$

reduce by row echelon

$\begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 2 \\ -1 & 1 & 0 \end{bmatrix}$

$R_1 \Leftrightarrow R_2$

$\sim \begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & 1 \\ -1 & 1 & 0 \end{bmatrix}$

cancel co-efficient in R_3 $R_3 \leftarrow r_3 + 1R_1$

$\sim \begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & 1 \\ 0 & 2 & 2 \end{bmatrix}$

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$R_2 \longleftrightarrow R_3$

$$\sim \begin{bmatrix} 1 & 1 & 2 \\ 0 & 2 & 2 \\ 0 & 1 & 1 \end{bmatrix}$$

cancel leading co-feccient R_3

$R_3 \leftarrow R_3 - \frac{1}{2} R_2$

$$\sim \begin{bmatrix} 1 & 1 & 2 \\ 0 & 2 & 2 \\ 0 & 0 & 0 \end{bmatrix}$$

Reduce Row echelon

$$\begin{bmatrix} 1 & 1 & 2 \\ 0 & 2 & 2 \\ 0 & 0 & 0 \end{bmatrix}$$

$R_2 \leftarrow 1/2 \, R_2$

$$\sim \begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

cancel leading co-efificient by

$R_1 \leftarrow R_1 - 1R_2$

$$\sim \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

system

$$(A-2I)\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$(A-2I)\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

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$$=(A-2I)\begin{bmatrix} x \\ y \\ z \end{bmatrix} (\eta) = \begin{bmatrix} x & 0 & z \\ 0 & y & z \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} x+z \\ y+z \\ 0 \end{bmatrix}$$

$$\begin{cases} x+z=0 \\ y+z=0 \end{cases} = \begin{matrix} x=-z \\ y=-z \end{matrix}$$

$$\eta = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$= \eta = \begin{bmatrix} -z \\ -z \\ z \end{bmatrix} \quad z \neq 0$$

$z=1$

$\eta = \begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix}$. vector of $\lambda=2$

$\lambda=4$ put 4 in λ in matrix

$$= \begin{bmatrix} -2 & 1 & 1 \\ 1 & -1 & 2 \\ -1 & 1 & -2 \end{bmatrix}$$

reduce matrix

$$A-4I = \begin{bmatrix} -2 & 1 & 1 \\ 1 & -1 & 2 \\ -1 & 1 & -2 \end{bmatrix}$$

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Row echelon form.

$$A-4I=\begin{bmatrix}-2 & 1 & 1\\ 1 & -1 & 2\\ -1 & 1 & -2\end{bmatrix}$$

cancel leading co-efficient in R_2
$R_2 \leftarrow R_2 + \frac{1}{2}R_1$

$$\sim\begin{bmatrix}-2 & 1 & 1\\ 0 & -\frac{1}{2} & \frac{5}{2}\\ -1 & 1 & -2\end{bmatrix}$$

cancel co-eff in
$R_3 \leftarrow R_3 - \frac{1}{2}R_1$

$$\sim\begin{bmatrix}-2 & 1 & 1\\ 0 & -\frac{1}{2} & \frac{5}{2}\\ 0 & \frac{1}{2} & -\frac{5}{2}\end{bmatrix}$$

$R_3 \leftarrow R_3 + 1R_2$

$$\sim\begin{bmatrix}-2 & 1 & 1\\ 0 & -\frac{1}{2} & \frac{5}{2}\\ 0 & 0 & 0\end{bmatrix} \rightarrow \text{Row echelon of } A-4I$$

reduce echelon form

$$\therefore\begin{bmatrix}-2 & 1 & 1\\ 0 & -\frac{1}{2} & \frac{5}{2}\\ 0 & 0 & 0\end{bmatrix}$$

$R_2 \leftarrow -2R_2$

$$\sim\begin{bmatrix}-2 & 1 & 1\\ 0 & 1 & -5\\ 0 & 0 & 0\end{bmatrix}$$

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$R_1 \leftarrow R_1 - 1R_2$

$$\sim \begin{bmatrix} -2 & 0 & 6 \\ 0 & 1 & -5 \\ 0 & 0 & 0 \end{bmatrix}$$

$R_1 \leftarrow -1/2\, R_1$

$$\sim \begin{bmatrix} 1 & 0 & -3 \\ 0 & 1 & -5 \\ 0 & 0 & 0 \end{bmatrix} \rightarrow \text{reduced}$$

system

$$= \begin{bmatrix} 1 & 0 & -3 \\ 0 & 1 & -5 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} x & 0 & 3z \\ 0 & 1y & -5z \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$x - 3z = 0 \;=\; x = 3z$

$y - 5z = 0 \;=\; y = 5z$

plus values into

$\eta = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$

$\eta = \begin{bmatrix} 3z \\ 5z \\ z \end{bmatrix} \quad z \neq 0$

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$Z=1$

$$\eta = \begin{bmatrix} 3 \\ 5 \\ 1 \end{bmatrix}$$

eigen vectors for $A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 3 & 2 \\ -1 & 1 & 2 \end{bmatrix}$

are;

$$\begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ 5 \\ 1 \end{bmatrix}$$

or

$$\begin{bmatrix} 2 & 1 & 1 \\ 1 & 3 & 2 \\ -1 & 1 & 2 \end{bmatrix} = \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ 5 \\ 1 \end{bmatrix}$$