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SECTION : B

Assignment No : 02

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Question no : 1

(1) $x^3 y''' + 2x^2 y' + 2y = 10x + 10/x$

Solution:

$$x^3 \frac{d^3y}{dx^3} + 2x^2 \frac{dy}{dx} + 2y = 10x - 10x^{-1}$$

$$x^3 D^3 y + 2x^2 D' + 2y = 10x + 10x^{-1}$$

$$(x^3 D^3 + 2x^2 D + 8) y = 10x + 10x^{-1} \quad \text{—} (1)$$

let $x = e^t \Rightarrow t = \ln x$

$$xD = D$$

$$x^2 D^2 = D(\Delta - 1) = \Delta^2 - \Delta$$

$$x^3 D^2 = \Delta(\Delta - 1) = (\Delta - 2)$$

subtituting into eq (1)

$$(\Delta - 3\Delta^2 + 2\Delta + 2(\Delta^2 - \Delta + 2) y = 10^x + 10x^{-1}$$

$$(\Delta^3 - \Delta^2 + 2) y = 10x + 10x^{-1}$$

$$(m^3 - m^2 + 2) y = 10e^t + 10/e^t$$

using synthetic division.

-1	1	-1	0	2
		-1	2	-2
	1	-2	2	0

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$D^2 - 2D + 2 = 0$

Now using quadratic formula.

$a = 1$, $b = -2$, $c = 2$

$$D = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$D = \frac{-(-2) \pm \sqrt{-2^2 - 4(1)(2)}}{2a}$$

$$D = \frac{2 \pm \sqrt{-4}}{2}$$

$$D = \frac{2 \pm \sqrt{-4}}{2}$$

$$D = \frac{2 \pm \sqrt{-1} \times \sqrt{4}}{2}$$

$$D = \frac{2 + 2i}{2}$$

$$D = \frac{2(1 \pm i)}{2}$$

$D = 1 \pm i$

Since roots are complex.

$y = e^{x}(c_1 \cos t + c_2 \sin t)$

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Now particular integration.

$$yp = \frac{1}{D^3 - D^2 + 2} \cdot 10e^t + \frac{1}{D^3 + D^2 + 2} \cdot 10/e^t$$

$$= \frac{10e^t}{(1)^3 - (1)^2 + 2} + \frac{10e^{-t}}{(1)^3 - (1)^2 + 2}$$

$$= \frac{\overset{5}{\cancel{10}}e^t}{2} + \frac{\overset{5}{\cancel{10}}e^t}{2}$$

$$= 5e^t + 5e^t$$

$$yp = 5e^t + 5e^t$$

General solution.

$$y = y_c + y_p$$

$$y = e^{-x}(c_1 \cos t + c_2 \sin t) + 5e^t + 5e^t$$

put $e^t = x$ and $t = \ln x$

$$y = e^{-x}(c_1 \ln x + c_2 \sin \ln x) + 5e^x + 5e^{-x}$$

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Question no 2

(2) $\frac{x^3 d^3y}{dx} + \frac{4x^2 d^2y}{dx} - 5x\frac{dy}{dx} - 15y = x^4$

Solution:

let $d/dx = D$

$x^3D^3y + 4x^2D^2y - 5xDy - 15y = x^4$

$(x^3D^3 + 4x^2D^2 - 5xD - 15)y = x^4$

let $x = e^t \Rightarrow t = \ln x$

$xD = D$

$x^2D^2 = D(D-1) = D^2 - D$

$x^3D^3 = D(D-1)(D-2) = D^3 - 3D^2 + 2D$

Now subtituting:

$(x^3D^3 + 4x^2D^2 - 5xD - 15)y = x^4$

$(D^3 - 3D^2 + 2D + 4(D^2 - D) - 5(D) - 15)y = e^{4t}$

$(D^3 + D^2 + 7D - 15)y = e^{4t}$

synthetic division.

	1	+1	-7	15
5		3	12	15
	1	4	5	0

$D^2 + 4D + 5 = 0$

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Quadratic Formula:

$$D = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-4 \pm \sqrt{4^2 - 4(1)(5)}}{2}$$

$$= \frac{-4 \pm \sqrt{16 - 20}}{2}$$

$$= \frac{-4 \pm 2i}{2}$$

$$D = \frac{2(-2 \pm i)}{2}$$

$$y_1 = e^4 (c_1 \cos t + c_2 \sin t)$$

for $y_p = ?$

$$y_p = \frac{1}{D^3 + D^2 - 7D - 15} e^{4t}$$

$$= \frac{1}{64 + 16 - 28 - 15} e^{4t}$$

$$= \frac{1}{80 - 43} e^{4t}$$

Hence $y = y_1 + y_p$

$$y = (c_1 \cos t + c_2 \sin t) + \frac{1}{37} e^{4t}$$

again put $t = \ln x$ and $x = \ln x$

$y = e^{3x}(c_1 \cos \ln x + c_2 \sin \log x) + \frac{1}{37}e^{4x}$ Ans

Question no 9

$x^2 y''' + 2xy' - 6y = 10x^2$

Solution:

$y(1) = 1$ and $y'(1) = -6$

$\frac{x^2 d^2y}{dx^2} + 2x\, dy/dx - 6y = 10x^2$

$\Rightarrow \left(x^2 \frac{d^2}{dx^2} + 2x \frac{d}{dx} - 6\right) y = 10x^2$

put $xD = D \Rightarrow x^2D^2 = D(D-1) = D^2 - D$

$x = e^t$ and $\log x = t$

$(D^2 - D + 2D - 6)y = 10e^{2t}$

$(D^2 + D - 6)y = 10e^{2t}$

The Characteristic equation.

$D^2 + D - 6 = 0$

$D^2 + 3D - 2D - 6 = 0$

$D(D+3) - 2(D+3) = 0$

$(D+3)(D-2) = 0$

$D + 3 = 0, \quad D - 2 = 0$

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$D = 2$, $D = -3$

Since roots are real and distinct

For $y_c = ?$

$y_c = c_1 e^{-3t} + c_2 e^{3t}$

For $y_p = ?$

$y_p = \frac{1}{D^2 - D - 6} \cdot 10 e^{2t}$

$10 \cdot \frac{1}{0} e^{2t}$

Now

$10 \frac{1}{d/dD (D^2 + D - 6)} e^{2t}$

$\Rightarrow 10 \frac{t}{2D^{t}} e^{2t}$

$= 10 \frac{1 \cdot t e^{2t}}{4+1}$

$y_p = 2 t e^{2t}$.

General Solution

$y = y_c + y_p$

$= c_1 e^{-3t} + c_2 e^{2t} + 2te^{2t}$

$y = c_1 x^{-3} + c_2 x^2 + 2(\log x) x^2$ —(B)

put $y(1) = 1$ i.e $x = 1$ $y = 1$ in (B)

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$\sqrt{b^2-4ac}$

$1 = c_1(1)^{-3} + c_2(1)^2 + 2\log(1).$

$1 = c_1 + c_2$ —— (2)

Now differentiate eq 'B' w.r.t x.

$y' = -3c_1 x^{-4} + 2c_2 x + \frac{2}{x}(x^2) + 4x\log x$

Now put $y'(1) = -6$ i-e $y' = -6$ and $x = 1$

$-6 = -3(1)c_1 + 2c_2 + 2 + 0$

$-6 = -3c_1 + 2c_2 + 2$

$-6 - 2 = -3c_1 + 2c_2 + 2$

$-8 = -3c_1 + 2c_2$ —— (D)

Solving eq (1) with (2) and subtracting from D.

$2c_1 + 2c_2 = 2$

$\mp 3c_1 \pm 2c_2 = \mp 8$

$5c_1 = 10$

$c_1 = 10/5 \quad c_1 = 2$

$-8 = -3(2) + 2c_2$

$-8 = -6 + 2c_2$

$2c_2 = -8+6$

$2c_2 = -2$

$c_2 = -2/2.$

$c_2 = -1$

Now put the value of c_1 and c_2 in eq (B).

$y = 2x^{-3} - x^2 + 2\ln x \; x(x^2)$

$y = 2/x^3 - x^2 + 2x^2 \log x$ Ans

Question 4

$x^2 y'' + 7xy' + 5y = x^5$

$y(1) = 2$ and $y'(1) = 2$

Solution

$$x^2 \frac{dy^2}{dx^2} + 7x\, dy/dx + 5y = x^5$$

$\Rightarrow \left(x^2 \frac{d^2}{dx^2} + 7x \frac{d}{dx} + 5\right) y = x^5$ — (A)

Put $xD = D \Rightarrow x^2 D^2 = D(D-1) = D^2 - D$

$x = e^t \Rightarrow \log x = t$ in eq(A)

$(D^2 - D + 7D + 5) y = e^{5t}$

$(D^2 + 6D + 5) y = e^{5t}$

By quadratic formula.

$$D = \frac{b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$D = \frac{-6 \pm \sqrt{6^2 - 4(1)(5)}}{2(1)}$$

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Since root are real and distinct.

$y_c = c_1 e^{-5t} + c_2 e^{-t}$

$y_p = ?$

$$y_p = \frac{1}{D^2 + 6D + 5}$$

$$= \frac{1}{(5)^2 + 6(5) + 5} e^{5t}$$

$$= \frac{1}{60} e^{5t}$$

Now General solution is

$$y = y_c = y_p$$

$$y = c_1 e^{-5t} + c_2 e^{-t} + \frac{1}{60} e^{5t}$$

$$y = c_1 x^{-5} + c_2 x^{-1} + \frac{1}{60} x^5 \quad \text{—— (B)}$$

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$x = 0$ put in this equation

No in eq B $e^0 = 1$

put $y(0) = 2$ i-e $y = 2$ and $x = 2$

$2 = c_1 (2)^{-5} + c_2 (2)^{-1} + \frac{1}{60}(2)^5$

$2 = -32c_1 - 2c_2 + \frac{1}{60}(32)$

$2 = -32c_1 - 2c_2 + 8/15$

$2(1 - 8/15) = -32c_1 - 2c_2$.

$22/15 = -32c_1 - 2c_2$ —(1)

Now differentiate eq (B) w.r.t x

$y' = -5c_1 x^{-6} - c_2 x^{-2} + \frac{1}{12} x^4$ -

put $y'(1) = 2$ i-e $y' = 2$

and $x = 2$ in above equation

$2 = -5(1) c_2^{-6} - c_2 (2)^{-2} + \frac{1}{12}(2)^4$

$2 = -5c_1 (-64) - c_2 (4) + \frac{1}{12} (16)$

$2 = 320c_1 + 4c_2 + 4/3$

$2 - 4/_{13} = 320c_1 + 4c_2$

$0/_3 = 320c_1 + 4c_2$ — (D)

x'ing eq(1) with 2 and the

- ing eq c from D.

$-44/_{15} = 64c_1 + 4c_2$

$\frac{-44}{15} = 64c_1 + 4c_1$

$\underline{\mp 2/_3 = +320c_1 + 4c_2}$

$34/_{15} = -256c_1$

$c_1 = 580.$

put the value of c_1 in eq (1)

$\frac{22}{15} = -32(580) + 2c_2.$

$\Rightarrow \frac{22}{15} = -18560 - 2c_2.$

$\frac{22}{15} + 18560 = -2c_2$

$-9280 = c_2$

Now put the value of c_1 and c_2 in eq B.

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$y = 580x^{-8} - 9280x^{-1} + \frac{1}{60}x^5$

$y = \frac{580}{x^5} - \frac{9280}{x} + \frac{1}{60}x^8$

Ans.

Question no 8

$(x+1)^2 y'' - 3(x+1)' y' + 4y = x^2$

Solution

$(x+1)^2 \frac{d^2y}{dx^2} - 3(x+1)\, dy/dx + 4y = x^2$

$(x+1)^2 \frac{d^2}{dx^2} - 3(x+1)\, \frac{d}{dx} + 4)\, y = x^2$

$(x+1)^2\, D^2 - 3(x+1)\, D + 4]\, y = x^2$ — (A)

put $(x+1)D = \mathcal{D} \Rightarrow (x+1)^2 D^2 = \mathcal{D}(\mathcal{D}-1) = \mathcal{D}^2 - \mathcal{D}$

$x = e^t$ in eq (A)

$[\mathcal{D}^2 - \mathcal{D} - 3\mathcal{D} + 4]\, y = e^{2t}$

$[\mathcal{D}^2 - 4\mathcal{D} + 4]\, y = e^{2t}$

$(\mathcal{D}^2 - 4\mathcal{D} + 4)\, 2 = e^{2t}$

for y_c we find the roots

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$D^2 - 4D + 4 = 0$

$D^2 - 2D - 2D + 4 = 0$

$(D - 2) - 2(D - 2) = 0$

$D - 2 = 0$, $D = 2$

$D - 2 = 0$, $D = 2$

So the roots are real and ~~di~~ repeat.

the General solution:

$y = (c_1 + c_2 x)^{2x}$

$y = (c_1 + c_3 x)^{23}$

for $y_p = ?$

$$y_p = \frac{1}{D^2 - 4D + 4}$$

$$y_p = \frac{2}{2D - 4} e^{2t}$$

If we put 2^{-3}

$2D - 4 \Rightarrow 2(2) - 4 = 0$

~~co.~~

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(15)

we take again derivative.

$y_P = \frac{?}{2} e^{2t}$

$y = (c_1 + c_2 x) e^{2t} + e^{2t}$ Ans.

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