ID# 16950 Muhammad Asif

Q1

(a) $$\frac{3x^4-2x^3+5}{x^3+1}$$

By using deivison rule.

$$\frac{d}{dx}\left(\frac{3x^4-2x^3+5}{x^3+1}\right)=\frac{(x^3+1)\frac{d}{dx}(3x^4-2x^3+5)-3x^4-2x^3+5\,\frac{d}{dx}(x^3+1)}{(x^3+1)^2}$$

$$=\frac{(x^3+1)(12x^3-6x^2)-(3x^4-2x^3+5)(3x^2)}{(x^3+1)^2}$$

$$=\frac{12x^6-\cancel{6x^5}+12x^3-6x^2-9x^6+\cancel{6x^5}-15x^2}{(x^3+1)^2}$$

$$=\frac{3x^6+12x^3-21x^3}{(x^3+1)^2}$$

$$=\frac{3x^2(x^4+4x^3-7)}{(x^3+1)^2}\quad \text{Ans.}$$

(b) $$\frac{(x^3+1)^2}{x^3-1}$$

$$=\frac{(x^3-1)\frac{d}{dx}(x^3+1)^2-(x^3+1)^2\frac{d}{dx}(x^3-1)}{(x^3-1)^2}$$

$$=\frac{(x^3-1)2(x^3+1)\frac{d}{dx}(x^3+1)-(x^3+1)^2\frac{d}{dx}(x^3-1)}{(x^3-1)^2}$$

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$$= \frac{2(x^3-1)(x^3+1)3x^2-(x^3+1)^2 3x^2}{(x^3-1)^2}$$

$$= \frac{2(x^6-1)3x^2-(x^6+1)2x^3)3x^2}{(x^3-1)^2}$$

$$= \frac{3x^2[2x^6-2-x^6-2x^3-1]}{(x^3-1)^2}$$

$$= \frac{3x^2(x^6-2x^3-3)}{(x^3-1)^2}.$$

Q2 find the Integration

(a) $\int \frac{1}{(8x+7)^8} dx$.

$= \int \frac{1}{(8x+7)^8} dx$ — i

let $u = 8x+7$ — (ii)

Apply derivative wrt x

$$\frac{du}{dx} = \frac{d}{dx}(8x+7)$$

$$= 8\frac{dx}{dx} + \frac{d}{dx}7$$

$$= 8 + 0$$

$$du/dx = 8 \text{ — (iii)}$$

ID# 16550 Muhammad Asif (2).

$$dx = \frac{1}{8} du \quad \text{---(4)}$$

put (2) & (4) in equation (i)

$$= \frac{1}{8}\int \frac{1}{u}\, du$$

$$= \frac{1}{8} \ln|u| + c$$

$$= \frac{\ln|u|}{8} + c$$

= putting of u

$$= \frac{\ln(8x+7)}{8} + c.$$

(b) $\int \frac{1}{\sqrt{x^5}}\, dx$

$$\int \frac{1}{\sqrt{x^5}}\, dx$$

$$\int \frac{1}{(x^5)^{1/2}}\, dx$$

$$\int \frac{1}{x^{5/2}}\, dx$$

$$\int x^{-5/2} \qquad a^{\frac{1}{n}} = a^{-m}$$

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Apply integration

$$= \frac{x^{-5/2+1}}{-\frac{5}{2}+1} + c \qquad \int x^m dx = \frac{x^{m+1}}{m+1}$$

$$= \frac{x^{\frac{-3}{2}}}{-3/2} + c$$

$$= \frac{-3}{2} \cdot \frac{1}{x^{3/2}} + c$$

$$= \frac{-3}{2} \cdot \frac{1}{\sqrt{x^3}} + c$$

$$= \frac{-3/2}{\sqrt{x^3}} + c$$

$$= \frac{-3/2}{\sqrt{x^3}} + c$$

$$= \frac{-3}{2\sqrt{x^3}} + c$$

ID# 16950 Muhammad Asif (5)

Q3 find the integration by partial fraction:

(a) $\int \frac{-x+9}{2x^2-8x+6}\,dx$

Sol

$$\int \frac{-x+9}{2x^2-8x+6}\,dx$$

$$\frac{1}{2}\int \frac{-x+9}{x^2-4x+3}\,dx$$

$$\frac{1}{2}\int \frac{-x+9}{(x-3)(x-1)}\,dx$$

using partial fraction

$$\frac{1}{2}\left[\frac{x+9}{(x-3)(x-1)} = \frac{A}{x-3} + \frac{B}{x-1}\right]$$

$$-x+9 = A(x-1)+B(x-3)$$

put $x=+3$

$$-(3)+9 = A(3-1)+B(0)$$

$$6 = 2A$$

$$\boxed{3=A}$$

put $x=1$

$$(-1)+9 = A(1-1)+B(1-3)$$

$$8 = B(-2)$$

$$\boxed{-4=B}$$

ID# 16950 Muhammad Asif (6)

$$\frac{1}{2}\int\left(\frac{3}{x-3}+\frac{-4}{(x-1)}\right)dx$$

$$\frac{1}{2}\int\frac{3}{x-3}\,dx-\frac{1}{2}\int\frac{4}{x-1}\,dx$$

$$\frac{3}{2}\int\frac{1}{x-3}\,dx-2\int\frac{1}{x-1}\,dx$$

$$\frac{3}{2}\ln|x-3|-2\ln|x-1|+c \Rightarrow \text{Ans.}$$

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b) $$\int\frac{4x^2+3x}{(x^3)(x^2+2x+3)}\,dx$$

Sol

$$\int\frac{x\quad(4x+8)}{x\cdot x^2(x^2+2x+3)}\,dx$$

$$\int\frac{4x+8}{x^2(x^2+2x+3)}\,dx$$

$$\frac{4x+8}{x^2(x^2+2x+3)}=\frac{A}{x}+\frac{B}{x^2}+\frac{Cx+D}{x^2+2x+3}$$

$$4x+8=A(x)(x^2+2x+3)+B(x^2+2x+3)+(Cx+D)(x^2) \quad -(i)$$

$$4x+8=A(x^3+2x^2+3x)+B(x^2+2x+3)+(Cx^3+Dx^2) \quad -(ii)$$

put $x=0$

$$4(0)+8)=A(0)+B(3)+C(0)+D(0)$$

$$\boxed{B=8/3}$$

compare coefficient of x

$$4 = 3A + 2B$$
$$4 = 3A + 2(8/3)$$
$$A = -4/9$$

compareof coefficient of x^2

$$0 = 2A + B + D$$
$$D = -2A - B$$
$$D = \frac{-16}{9}$$

compare coefficient of x^3

$$0 = A + C$$
$$C = -A$$
$$C = 4/9$$

putting value we get

$$\frac{-4/9}{x} + \frac{8/3}{x^2} + \frac{\frac{4x}{9} - \frac{16}{9}}{x^2+2x+3}$$

$$-4/9\int \frac{1}{x}dx + \frac{8}{3}\int \frac{1}{x^2}dx + \frac{4}{9}\int \frac{x-4}{x^2+2x+3}dx$$

$$\frac{-4}{9}\ln|x| + \frac{8}{3}\int x^{-2} + \frac{2}{9}\int \frac{2x-8}{x^2+2x+3}dx$$

$$+\frac{8}{3}\frac{x^{-2+1}}{-2+1} + \frac{2}{9}\int \frac{2x+2-10}{x^2+2x+3}dx$$

$$-\frac{8}{3}\frac{1}{x} + \frac{2}{9}\int \frac{2x+2dx}{x^2+2x+3} - \frac{20}{9}\int \frac{1}{x^2+2x+3}dx$$

$$+\frac{2}{9}\ln|x^2+2x+3| - \frac{20}{9}\int \frac{1}{x^2+2x+3}dx - \frac{20}{9}\int \frac{1}{(x+1)^2+2}dx$$

$$u = \frac{x+1}{\sqrt{2}} \qquad \frac{du}{dx} = \frac{1}{\sqrt{2}}dx\ \sqrt{2}\,dx$$

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$$-\frac{20}{9}\int \frac{\sqrt{2}}{2u^2+2}\,du$$

$$-\frac{20}{9\sqrt{2}}\int \frac{1}{u^2+1}\,du$$

$$\frac{-20}{9\sqrt{2}}\tan^{-1}u$$

$$\frac{-20}{9\sqrt{2}}\tan^{-1}\frac{x+1}{\sqrt{2}}$$

final Ans

$$-\frac{4}{9}\ln|x| - \frac{8}{3x} + \frac{2}{9}\ln|x^2+2x+3| - \frac{20}{9\sqrt{2}}\tan^{-1}\frac{x+1}{\sqrt{2}} + C$$

Q4 Solve each of the following matrix equation

(a) $X + \begin{bmatrix} 3 & -1 \\ 2 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 1 \\ -3 & 1 \end{bmatrix}$.

$$x = \begin{bmatrix} 5 & 1 \\ -3 & 1 \end{bmatrix} - \begin{bmatrix} 3 & -1 \\ 2 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 5-3 & 1+1 \\ -3-2 & 1-2 \end{bmatrix}$$

$$x = \begin{bmatrix} +2 & 2 \\ -5 & -1 \end{bmatrix}$$

(b) $X + \begin{bmatrix} -1 & 0 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 2 & 6 \\ 1 & 5 \end{bmatrix} + \begin{bmatrix} -4 & -8 \\ -2 & 0 \end{bmatrix}$

$$X + \begin{bmatrix} -1 & 0 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 2-4 & 6-8 \\ 1-2 & 5+0 \end{bmatrix}$$

$$X + \begin{bmatrix} -1 & 0 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} -2 & -2 \\ -1 & 5 \end{bmatrix}$$

$$X = \begin{bmatrix} -2 & -2 \\ -1 & 5 \end{bmatrix} - \begin{bmatrix} -1 & 0 \\ 0 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} -2+1 & -2-0 \\ -1-0 & 5-2 \end{bmatrix}$$

$$X = \begin{bmatrix} -1 & -2 \\ -1 & 3 \end{bmatrix}$$

(c) $X + 2I = \begin{bmatrix} 3 & -1 \\ 1 & 2 \end{bmatrix}$

$$X = \begin{bmatrix} 3 & -1 \\ 1 & 2 \end{bmatrix} - 2I$$

$$= \begin{bmatrix} 3 & -1 \\ 1 & 2 \end{bmatrix} - 2\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & -1 \\ 1 & 2 \end{bmatrix} - \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 3-2 & -1-0 \\ 1-0 & 2-2 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 1 & 0 \end{bmatrix}$$

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Q5 if $A = \begin{bmatrix} 1 & 4 \\ 2 & 1 \end{bmatrix}$, $B = \begin{bmatrix} -3 & 2 \\ 4 & 0 \end{bmatrix}$, $C = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$

find $A^2 + BC$

Solution

$$A^2 = \begin{bmatrix} 1 & 4 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 4 \\ 2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1\times1+4\times2 & 1\times4+4\times1 \\ 2\times1+1\times2 & 2\times4+1\times1 \end{bmatrix}$$

$$= \begin{bmatrix} 1+8 & 4+4 \\ 2+2 & 8+1 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 9 & 8 \\ 4 & 9 \end{bmatrix}$$

Now $BC = \begin{bmatrix} -3 & 2 \\ 4 & 0 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$

$$= \begin{bmatrix} -3\times1+2\times0 & -3\times0+2\times2 \\ 4\times1+0\times0 & 4\times0+0\times2 \end{bmatrix}$$

$$= \begin{bmatrix} -3+0 & -0+4 \\ 4+0 & 0+0 \end{bmatrix}$$

$$BC = \begin{bmatrix} -3 & 4 \\ 4 & 0 \end{bmatrix}$$

Now $A^2 + BC$

$$A^2+BC = \begin{bmatrix} 9 & 8 \\ 4 & 9 \end{bmatrix} + \begin{bmatrix} -3 & 4 \\ 4 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 9-3 & 8+4 \\ 4+4 & 9+0 \end{bmatrix}$$

$$A^2+BC = \begin{bmatrix} 6 & 12 \\ 8 & 9 \end{bmatrix}$$