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SEC - B.

Sub - Calculus.

To,

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Q-No-01

Given:

$$g(t) = 0 \; ; \; t < 0$$
$$= t^2 \; ; \; 0 \leq t \leq 3$$
$$= 2t + 3 \; ; \; 3 < t \leq 4$$
$$= 4 \; ; \; t > 0$$

Since the definition of the funtion changes at the point $t = 3$.
therefore we check, the discontinuity at that point, at $t = 3$

$$g(t) = t^2 \quad ; \text{ by definition of funtion}$$

$$\Rightarrow g(3) = 3^2$$

$$g(3) = 9 \quad \text{— } 1$$

Now

$$\text{Limit}_{t \to 3} g(t) = Lt_{t \to 3} (t^2)$$
$$= 3^2$$

$$\underset{t \to 0}{Lt}\ g(t) = 9 \quad - (2)$$

From eq (1) & (2) we have

$$g(3) = \underset{t \to 3}{Lt}\ g(t)$$

$\Rightarrow$ $g(t)$ is continous at each point

Again we check at $t = 0$

$g(t) = t^2$; by definition of funtion

$$g(3) = 3^2$$

$$g(3) = 9 \quad - (1)$$

Now

$$\underset{t \to 3}{Lt}\ g(t) = \underset{t \to 3}{Lt}\ 0$$

$$g(3) = 0 - (2)$$

From (1) & (2) we get

$$g(t) \neq \underset{t \to 3}{Lt}\ g(t)$$

So the funtion is discontinous at $t = 0$.

$$\lim_{t\to 3} g(t) = \lim_{t\to 3} (t)^2$$

$$= 3^2$$

$$\lim_{t\to 3} g(t) = 9.$$

Q-No-02

$$y(x) = x^2 + \sin x$$

Solution:

By Maclaurin's series expantion, we have,

$$f(x) = f(0) + x f'(0) + \frac{x^2 f''(0)}{2!} + \frac{x^3 f'''(0)}{3!} + \cdots\cdots$$

Now,

$$f(x) = y(x) = x^2 + \sin x$$

$$f'(x) = 2x + \cos x$$

$$f''(x) = x - \sin x$$

$$f'''(x) = -\cos x$$

Thus,

$$f(0) = 0^2 + \sin(0) = 0$$

$$f'(0) = 2(0) + \cos(0) = 1$$

$$f''(0) = 2 - \sin(0) = 2$$

$$f'''(0) = -\cos(0) = -1$$

Hence by Maclaurin's Expansion,

$$f(x) = y(x) = 0 + x(1) + \frac{x^2(2)}{2!} + \frac{x^3(-1)}{3!} + \dots$$

$$= 0 + x + 0 - \frac{x^3}{3!} + \dots\dots$$

$$= x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \frac{x^9}{9!} \dots\dots$$

is the required Maclaurins Expansion.

Q. No. 03

(i) Find y'' given

$$1+xy=x^2+y^2 \quad \text{--- (1)}$$

Solution:

diff (1) w.r.t x.

$$0+x\frac{dy}{dx}+y(1)=2x+2y\frac{dy}{dx}$$

$$x\frac{dy}{dx}-2y\frac{dy}{dx}=2x-y$$

$$(x-2y)\frac{dy}{dx}=2x-y$$

$$\Rightarrow \frac{dy}{dx}=\frac{2x-y}{x-2y} \quad \text{--- (A)}$$

Again diff w.r.t x

$$\frac{d^2y}{dx^2}=\frac{(x-2y)\frac{d}{dx}(2x-y)-(2x-y)\frac{d}{dx}(x-2y)}{(x-2y)^2}$$

$$= \frac{(x-2y)\left(2(1) - \frac{dy}{dx}\right) - (2x-y)\left(1 - 2\frac{dy}{dx}\right)}{(x-2y)^2}$$

$$= \frac{(x-2y)\left(2 - \frac{2x-y}{x-2y}\right) - (2x-y)\left(1 - 2\frac{(2x-y)}{x-2y}\right)}{(x-2y)^2}$$

$$= \frac{(x-2y)\left(\frac{2(x-2y) - (2x-y)}{x-2y}\right) - \left(\frac{(2x-y)(x-2y-2(2x-y))}{x-2y}\right)}{(x-2y)^2}$$

$$= \frac{(x-2y)(\cancel{2x} - 4y - \cancel{2x} + y) - (2x-y)(x - \cancel{2y} - 4x + \cancel{2y})}{(x-2y)(x-2y)^2}$$

(ii) Find y' by using logarithmic differentiation

$$Y = x^3(1+x)^9 e^{6x}$$

Solution:

$$Y = x^3(1+x)^9 e^{6x}$$

take ln both sides

$$\ln y = \ln(x^3(1+x)^9 e^{6x})$$

$$\ln y = \ln x^3 + \ln(1+x)^9 + \ln e^{6x}$$

$$\ln y = 3\ln x + 9\ln(1+x) + \ln e^{6x}$$

Now diff w.r.t 'x'

$$\frac{1}{y}\frac{dy}{dx} = 3\left(\frac{1}{x}\right) + 9\left(\frac{1}{1+x}\right)dx(1+x) + \frac{1}{e^{6x}}\frac{d}{dx}e^{6x}$$

$$\frac{dy}{dx} = y\left(\frac{3}{x} + \frac{9}{1+x}(0+1) + \frac{1}{\cancel{e^{6x}}}\cancel{e^{6x}}\frac{d}{dx}6x\right.$$

$$\frac{dy}{dx} = y\left(\frac{3}{x} + \frac{9}{1+x}(0+1) + 6(1)\right)$$

$$\frac{dy}{dx} = y\left(\frac{3}{x} + \frac{9}{1+x} + 6\right)$$

$$\frac{dy}{dx} = x^3 (1+x)^9 \cdot e^{6x} \left(\frac{3}{x} + \frac{9}{1+x} + 6\right)$$ Ans.